

## ECE 132

Semiconductors are the class of materials that exhibit conductivity between the high values for metals and the very low values for insulators. The properties of crystalline materials are a function of the nature of the bonding between the atoms that constitute the lattice.

### Bonding Forces in Solids

#### 1. IONIC BONDING

Ionic bonding between two atoms occurs when metallic elements (example: *Na*) donate an electron to the more electronegative element (example *Cl*) thus creating an  $Na^+$  and  $Cl^-$  ion pair. The electrostatic attractive force creates the ionic bond between the elements. The electrons are in full orbits and tightly bound to both the *Na* and the *Cl* atoms and hence there are no atoms available for current flow.

*NaCl* is thus a good insulator.

#### 2. METALLIC BONDING

In the case of metals, the outer shell is only partially filled. These electrons are very loosely bound to the atoms and hence are free to move through the metal when subjected to an  $\vec{E}$  field.

Thus metals are excellent conductors

#### 3. COVALENT BONDING

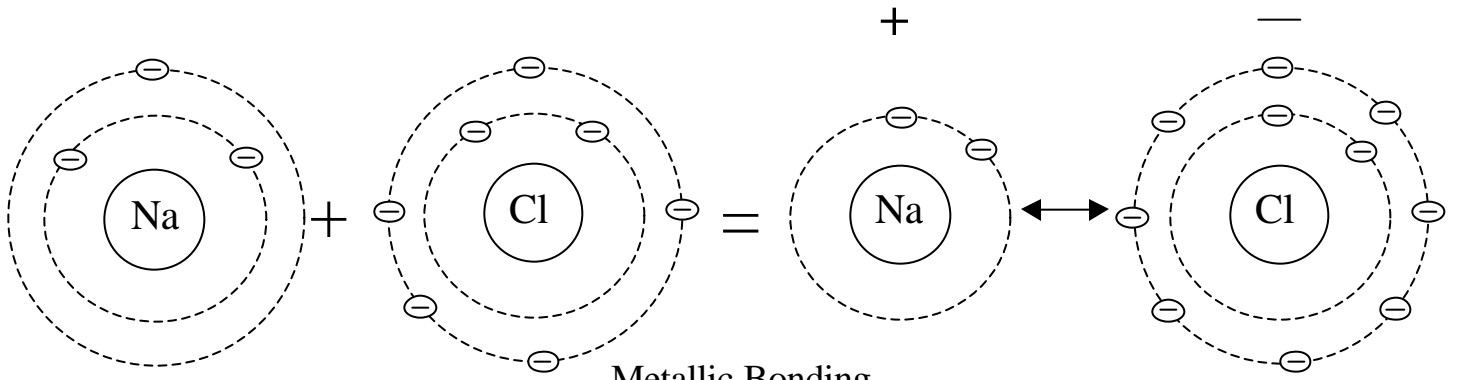
Elemental semiconductors such as *Si* and *Ge* exhibit this type of bonding. Compound semiconductors such as *GaAs* have an ionic component to a dominantly covalent bond because of the differing electronegativity of the two elements.

S/C	% ionic
<i>Si</i>	0
<i>Ge</i>	0
<i>SiC</i>	18
<i>GaAs</i>	32
<i>InP</i>	44

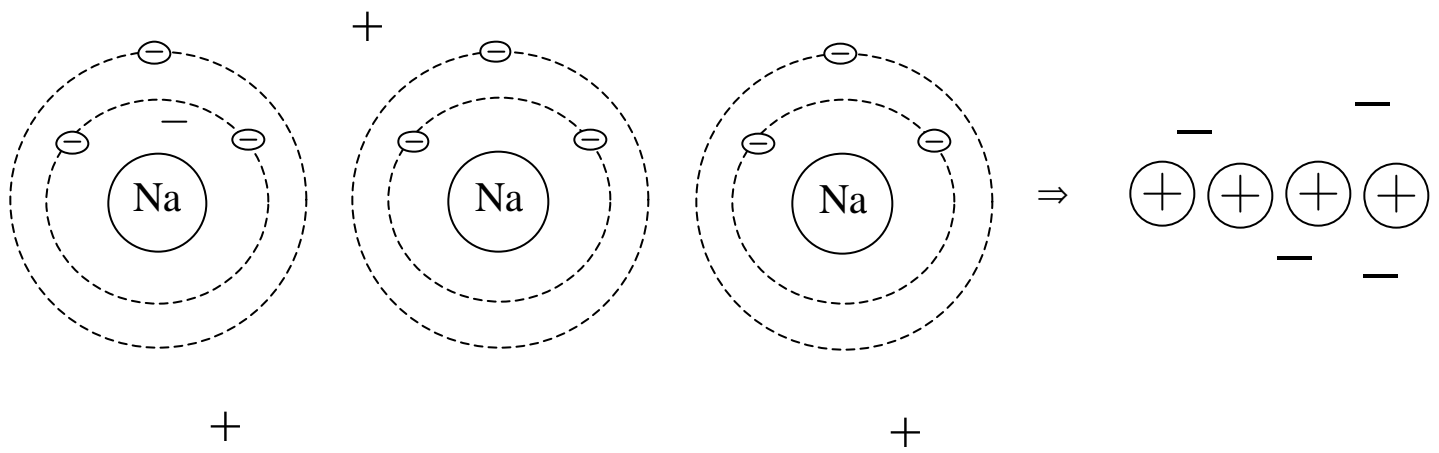
(Physical Properties of Semiconductors)  
Wolfe, Holonyak, & Stillman

The important semiconductors today are bonded into a diamond lattice, which minimizes the energy of the lattice. The bonding is tetrahedral in nature. This can be understood by considering say *Si*. The four outer electrons in *Si* exist in  $sp^3$  hybrid orbitals. These bonds overlap and the electrons are shared between two atoms fulfilling the required condition of two electrons per orbital.

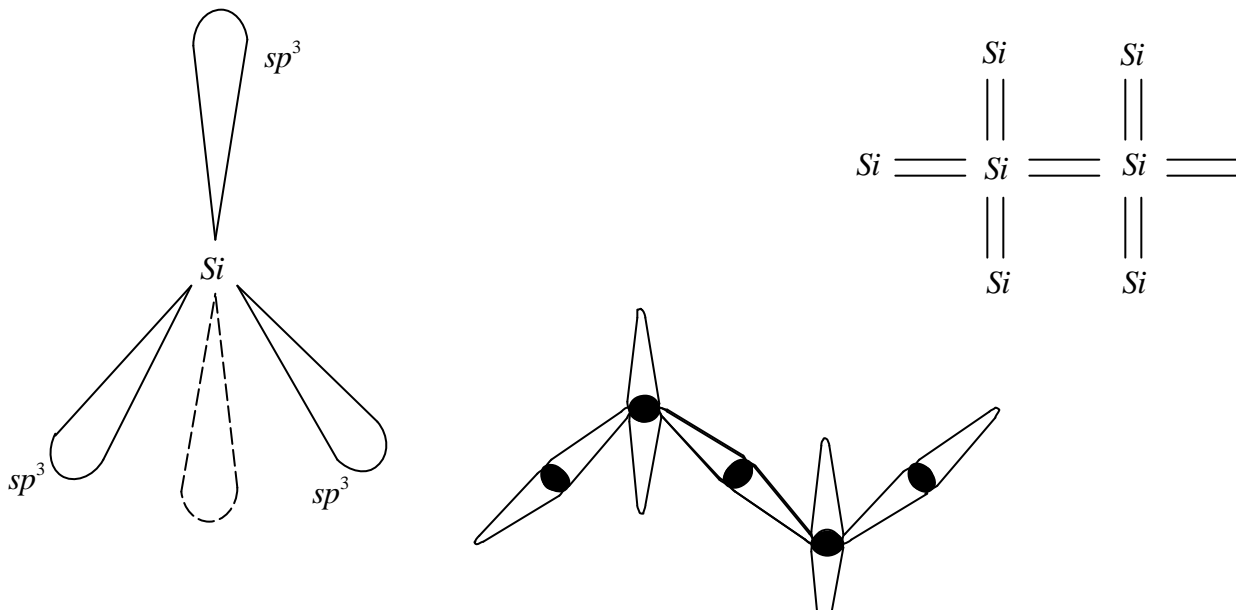
Ionic Bonding



Metallic Bonding

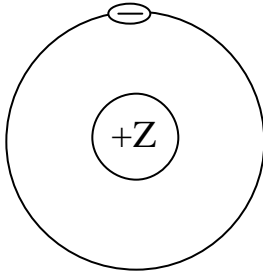


Covalent Bonding



### Another Pictorial Representation

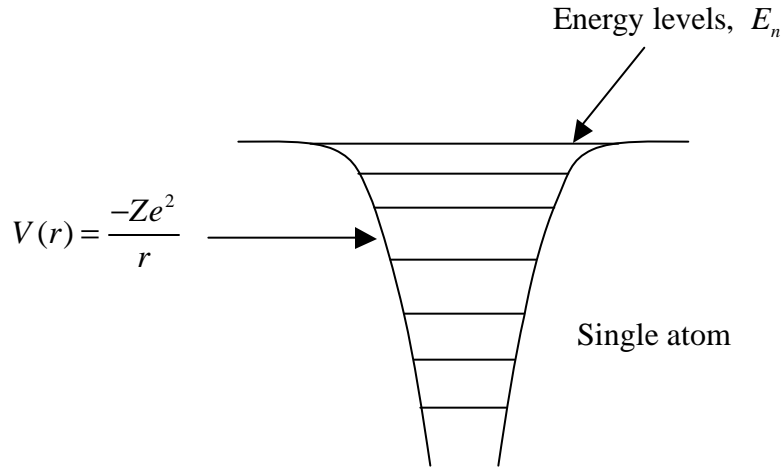
The energy levels of a hydrogen atom is obtained as



$$E_n = -\frac{m_r e^4 z^2}{2\hbar^2 n^2} = -\frac{13.6eV}{n^2}$$

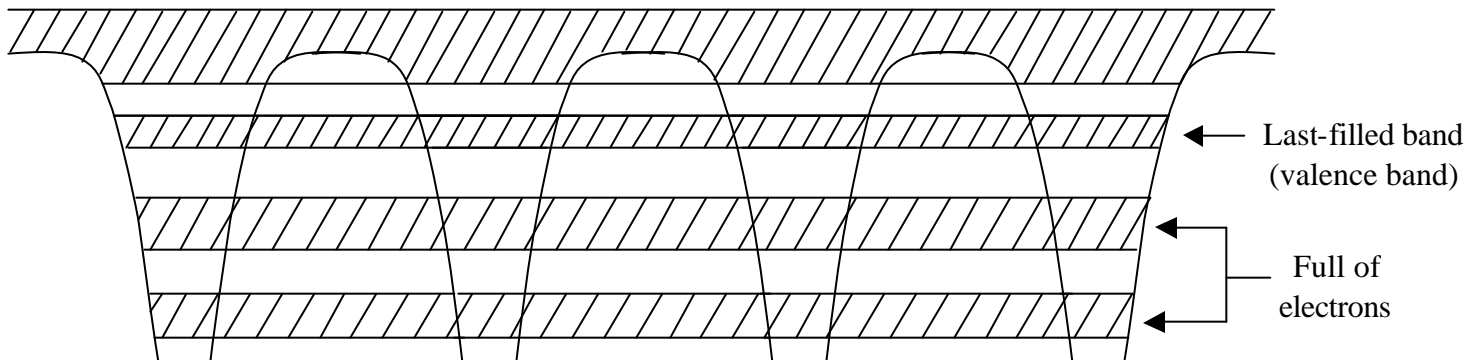
$z$  is the atomic number

This is obtained by solving the Schrödinger equation for a coulombic potential.



### Solid

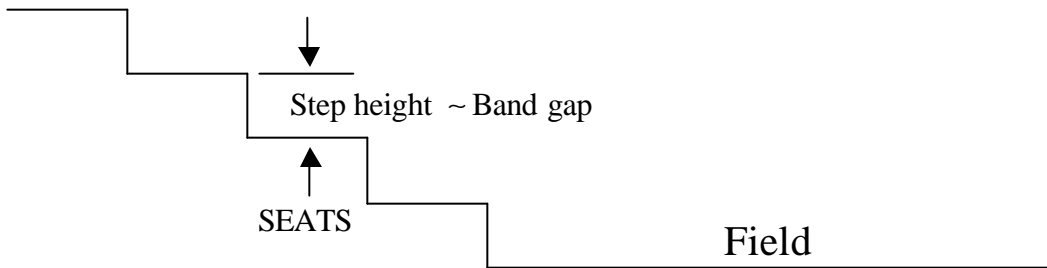
Next band is empty  
(conduction band)



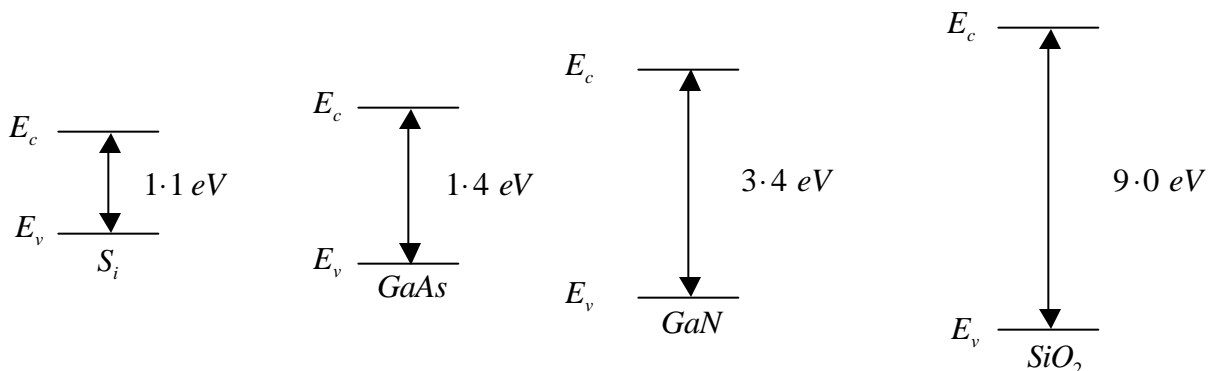
Note that the energy gaps between the quantized energy levels in a single atom are the origin of the band gap for solids. Typical numbers for band gap for various solids:

Semiconductor	$E_g$
<i>Si</i>	1.1 eV
<i>GaAs</i>	1.4 eV
<i>Ge</i>	0.7 eV
<i>SiO<sub>2</sub></i>	9.0 eV
<i>GaN</i>	3.4 eV

This is the MINIMUM energy required to allow an electron to leave the (mostly) full valence band and occupy a state in the conduction band. An analogy is say a football stadium.



The step height is analogous to the band gap in that an energy difference less than the step height will not get you to the next step. The energy required to move an electron from the valence band (VB) to the conduction band (CB) is related to the bond strength and can be provided by one of several means; thermal, light (photons), high energy particles, etc. Thermal energy exists in each solid at temperatures  $T > 0K$ . A measure of thermal energy is  $k_B T$  where  $k_B$  is the Boltzmann's constant and  $k_B T \sim 25.9 meV$  at room temperature (300K by convention).  $1 eV =$  Energy required to move one electron up a potential energy of  $1V = 1.6 \times 10^{-19}$  coulombs  $1V = 1.6 \times 10^{-19}$  Joules.



At room temperature pure *Si* has an intrinsic carrier concentration of  $1.5 \times 10^{10} \text{ cm}^{-3}$  or 1 electron/ $10^{12}$  *Si* atoms.

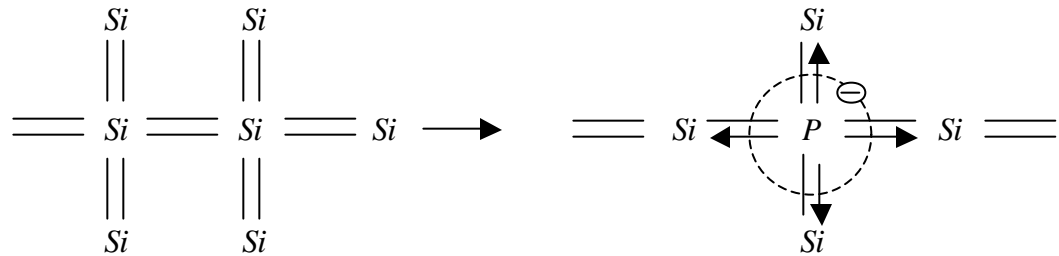
To increase the conductivity of the semiconductors add impurities to make the s/c extrinsic.

### Extrinsic Semiconductors

Creation of free electrons or holes by the addition of impurities is called doping.

#### N-Type Case (Electrons are majority carriers)

If a group V atom is introduced into the crystal so that it replaces a *Si* atom. (i.e. SUBSTITUTIONAL IMPURITY) then the *Si* lattice looks like such



The P atom incorporates into the lattice through tetrahedral covalent bonding but has an excess electron that is unpaired and weakly bonded. P in a *Si* lattice now looks hydrogenic.

The electron at finite temperatures is stripped from the P atom or DONATED to the crystal and is free. Hence the name DONOR atom to group V elements like P in semiconductors like *Si*. When the electron is no longer bound to the P atom the P atom has a net positive charge.

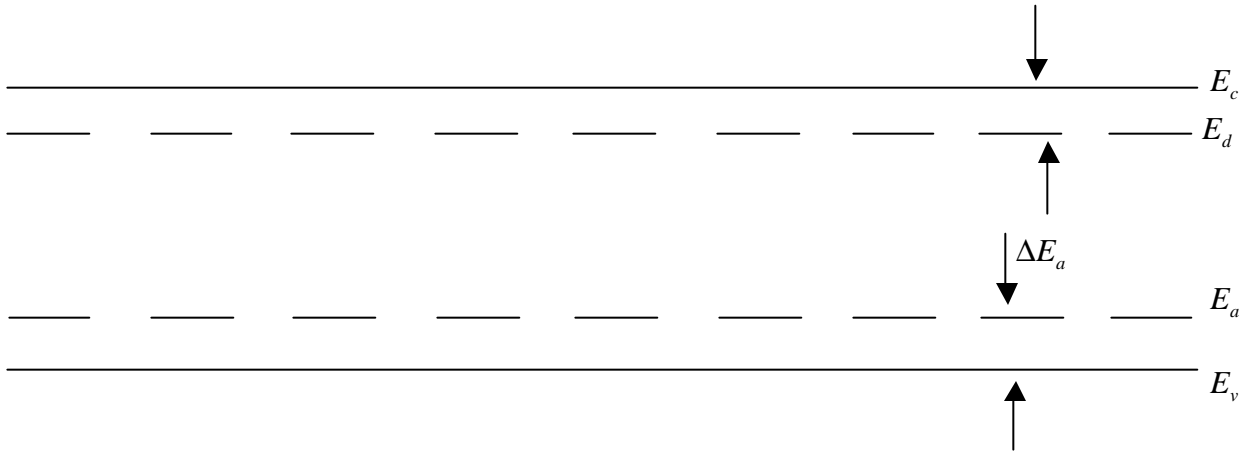
The binding energy of the electron can be calculated as

$$\Delta E_{d_n} = (E_c - E_d) = \frac{q^4 z^2 m^*}{2n^2 (4\pi \epsilon_r \epsilon_0 h)^2}$$

the energy required to make the electron free  
i.e. to place it in the conduction band.

$$\Delta E_d = \frac{13.6}{n^2} \left( \frac{z}{\epsilon_r} \right)^2 \left( \frac{m^*}{m} \right) eV$$

Recall:  $E_n = \frac{13.6}{n^2}$  for a hydrogen atom



### Intrinsic Semiconductor

As stated previously an intrinsic semiconductor has no free carriers at  $0K$  and hence is then an insulator. At any finite temperature, however, the covalent bonds break producing electron-hole pairs.

$\therefore$  At steady state

$$n \equiv p \equiv n_i \leftarrow \text{Intrinsic carrier concentration}$$

Since the electrons can be recaptured by the orbital lacking an electron, this is referred to as to RECOMBINE WITH A HOLE.

At steady state

$$r_i \equiv g_i$$

$$EHP C_m^{-3} s^{-1}$$

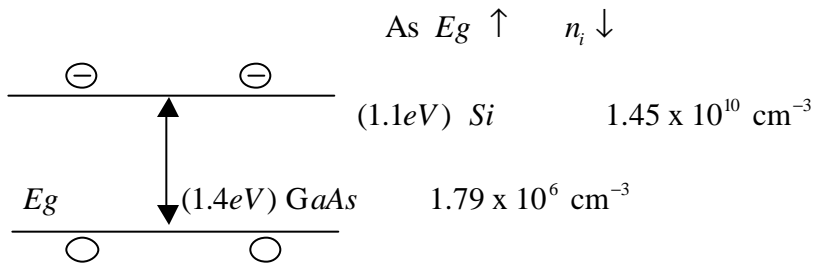
At all temperatures

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Recombination rate

Generation rate

$$\text{As } T \uparrow: g_i \uparrow \Rightarrow r_i \uparrow$$



Study the nature of  $f(E)$

At  $T = 0K$

$$k_B = 8.62 \times 10^{-5} \text{ eV/K}$$

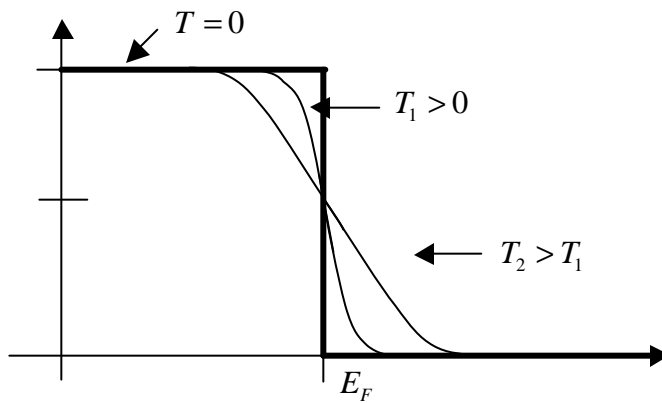
$$= 1.38 \times 10^{-23} \text{ J/K}$$

$$f(E)(T=0) = \begin{cases} 1 & \text{for } E < E_F \\ 0 & \text{for } E > E_F \end{cases}$$

$$k_B T @ T = 300K = 26 \text{ meV}$$

It is impossible for electrons to occupy energy above  $E_F$  at  $T = 0K$

At finite  $T$

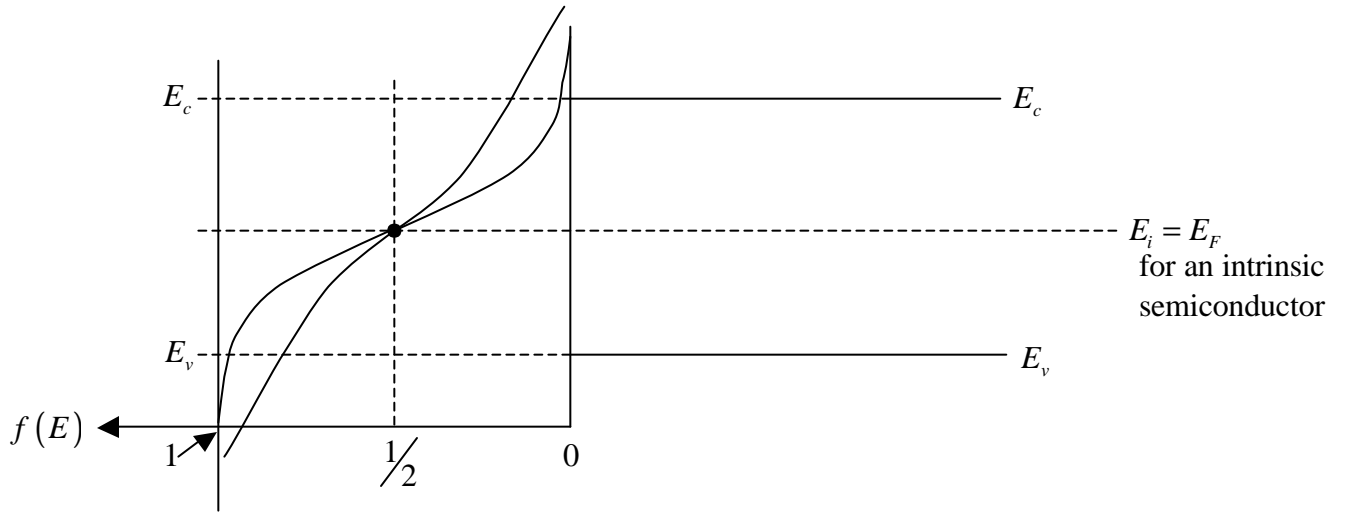


$$N_d^+ = N_d \frac{1}{1 + \frac{1}{2} e^{\left[ \frac{E_d - E_F}{kT} \right]}}$$

$$N_a^- = N_a \frac{1}{1 + 2e^{\frac{E_a - E_F}{kT}}}$$

$f(E)$  is the probability of occupancy of an AVAILABLE STATE.  $\therefore$  if no state exists and even is  $f(E)=1$  there will be no electron at that energy.

In a semiconductor



At finite  $T$  there is a creation of a hole population (or a lack of an electron population) in the valence band.  $\therefore f(E) < 1$

NOTE:

- $f(E)$  has a finite value in the forbidden gap but no electron exists because there are no states.
- $f(E)$  is symmetrical about  $E_F$ .
- $f(E)$  is the probability of occupancy of a state by an electron  
 $\boxed{1 - f(E)}$  is the probability of lack of occupancy of a state or the probability of occupancy by a hole.

### Rewriting

$$\boxed{f_n(E) = f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

$$f_p(E) = 1 - f(E) = 1 - \frac{1}{1 + e^{(E - E_F)/kT}} = \frac{e^{(E - E_F)/kT}}{1 + e^{(E - E_F)/kT}}$$

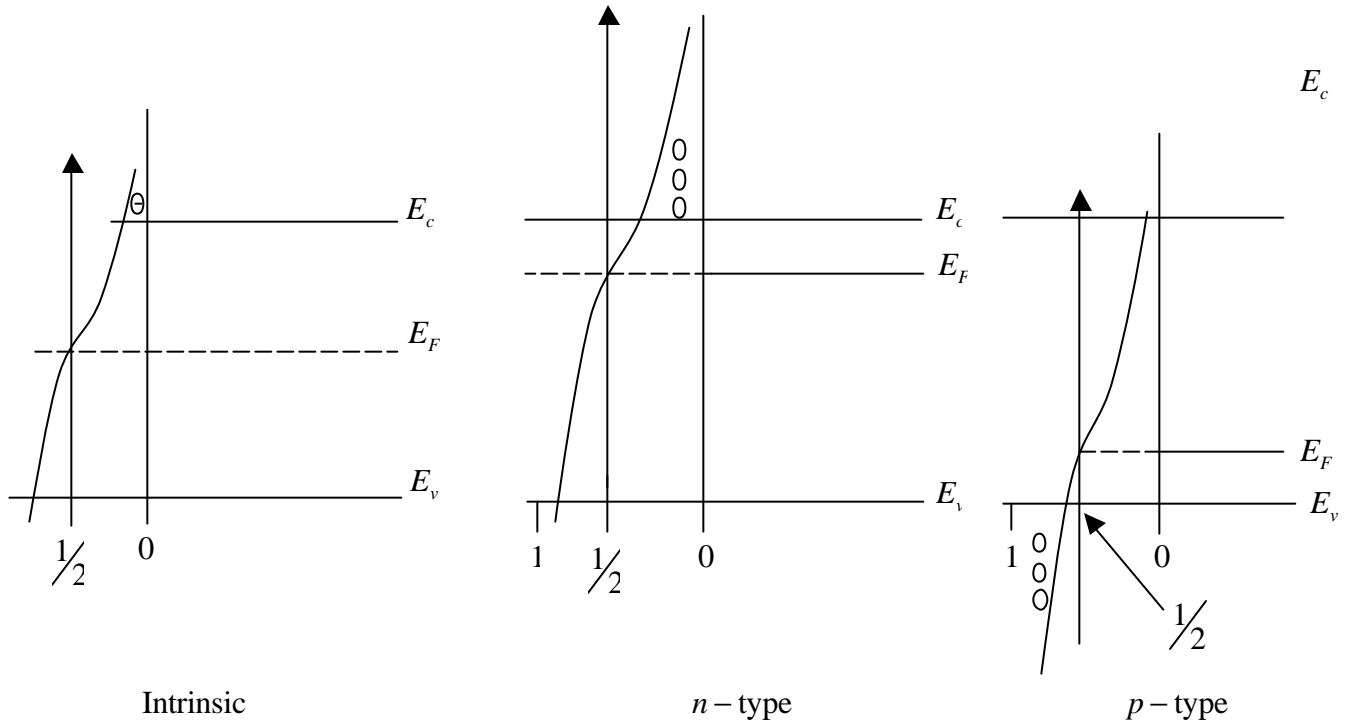
or  $\boxed{f_p(E) = \frac{1}{1 + e^{(E_F - E)/kT}}$

NOTE THE SYMMETRY OF ELECTRON AND HOLE PROBABILITY ABOUT  $E_F$ .



In Intrinsic Si

$n_o = p_o$  Since the concentration is equal, the Fermi level should lie close to the center of the gap



$\therefore E_F$  is closer to  $E_c$  in n-type material and closer to

**Equilibrium Electron and Hole Densities**

(for this class the derivation is not required)

To calculate the number of electrons in the CB we must know

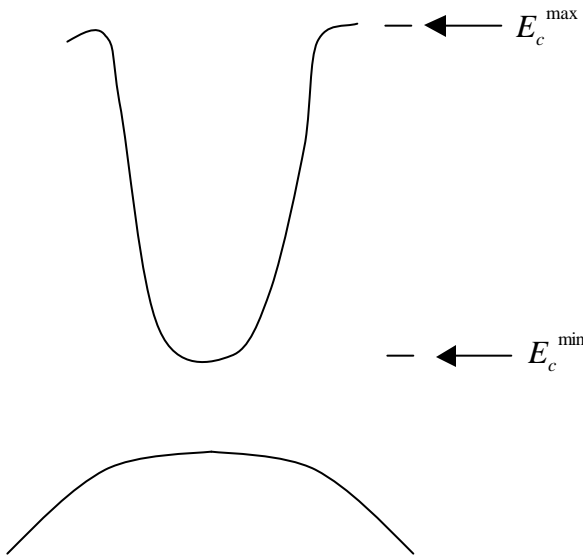
- 1) How many states are available
- 2) What is the probability of occupancy

We thus introduce

$N(E)dE \triangleq$  DENSITY OF STATES ( $cm^{-3}$ ) available for occupancy in the energy range  $dE$

$\therefore$  Total number of electrons in the CB can be written as

$n = \int_{\text{Bottom of CB}}^{\text{Top of CB}} f(E) \cdot N(E) dE \quad (\text{electrons } cm^{-3})$
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We know that

$$f(E) = \frac{1}{1 + e^{(E-E_F)/RT}}$$

(Fermi-Dirac Statistics)

If  $E - E_F > 4kT$

$$\frac{1}{1 + e^{(E-E_F)/kT}} \approx e^{-(E-E_F)/kT}$$

Since

$$e^{\left(\frac{E-E_F}{kT}\right)} = e^4 = 55 \left[ \begin{array}{l} e^5 = 148 \\ e^6 = 406 \\ \text{etc.} \end{array} \right]$$

At room temperature  $kT = 0.026\text{eV}$

We saw that if  $E_C - E_F = 4kT$  then the probability of occupancy of states at  $E_C^{\min}$

$$f(E_C) = e^{-(E_C-E_F)/kT} = e^{-4} = 0.0183$$

For higher energies the  $f(E)$  drops sharply

Example:  $e^{-(E-E_F)/kT} = e^{-10} = 0.000045 = 45 \times 10^{-6}$

$\therefore$  the contribution to the integral at higher  $E$  is reduced to the point that

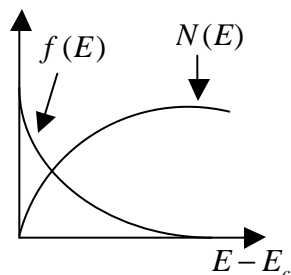
Top of  $CB$

$$n = \int_{B \text{ of } cB} N(E) f(E) d(E) = \int_{B \text{ of } cB}^{\infty} N(E) f(E) d(E)$$

Using quantum mechanics and including

Pauli's Principle

$$N(E) = \frac{1}{2\pi^2} \left( \frac{2m^* e}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2}$$



$$\hbar = \frac{h}{2\pi}$$

$$h = 6.63 \times 10^{-34} \text{ J-s} \\ = 4.14 \times 10^{-15} \text{ eV-s}$$

Here  $m_e^*$  is the density of state effective mass

$$n = \int_{E_c}^{\infty} \frac{1}{2\mathbf{p}^2} \cdot \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} \cdot (E - E_c)^{1/2} \cdot e^{-(E - E_F)/kT} dE$$

Multiplying and dividing by

$$= (kT)^{3/2} \quad \text{we get}$$

$$n = \frac{1}{2\mathbf{p}^2} \cdot \left( \frac{2m_e^*}{\hbar^2} \right)^{3/2} \cdot \int e^{-(E - E_F)/kT} \cdot \frac{(E - E_c)^{1/2}}{\sqrt{kT}} \cdot \frac{dE}{kT}$$

Define  $\frac{E - E_c}{kT} \triangleq u$

$$\therefore n = \frac{1}{2\mathbf{p}^2} \left( \frac{2m_e^* kT}{\hbar^2} \right)^{3/2} \cdot e^{(E_F - E_c)/kT} \int e^{-\left(\frac{E - E_c}{kT}\right)} \cdot \left( \frac{E - E_c}{kT} \right)^{1/2} d \left( \frac{E - E_c}{kT} \right)$$

$$\therefore n = \frac{1}{2\mathbf{p}^2} \left( \frac{2m_e^* kT}{\hbar^2} \right)^{3/2} \cdot e^{(E_F - E_c)/kT} \int_0^{\infty} e^{-u} \cdot u^{1/2} du$$

$$\int_0^{\infty} e^{-u} \cdot u^{1/2} du = \frac{\sqrt{\mathbf{p}}}{2} \quad (\text{From definite integral tables})$$

$$n = 2 \cdot \left( \frac{2\mathbf{p}m_e^* kT}{\hbar^2} \right)^{3/2} \cdot e^{(E_F - E_c)/kT} \quad \text{or} \quad \boxed{n = N_c e^{-(E_c - E_F)/kT}}$$

$$\boxed{N_c \triangleq 2 \left( \frac{2\mathbf{p}m_e^* kT}{\hbar^2} \right)^{3/2} = \text{Effective density of states in the CB}}$$

i.e. Instead of considering the occupancy by electrons of all the states from  $E_c^{\min}$  to  $E_c^{\max}$ ,  $N_c$  gives us the number of electrons in the CB by considering all the states in the band to be effectively at the band edge populated with a probability function  $e^{-(E_c - E_F)/kT}$ .

Likewise, proceeding in a similar manner for the holes in the VB we obtain

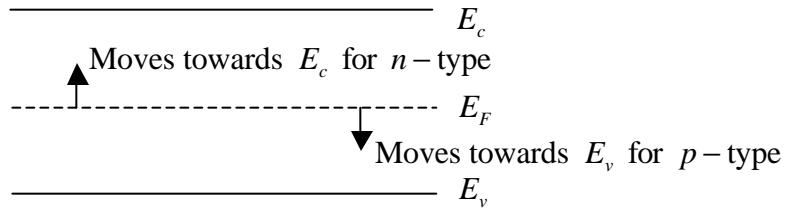
$$p = N_v e^{-(E_F - E_v)/RT}$$

$$\text{where } N_v = 2 \left[ \frac{2p m_p^* kT}{h^2} \right]^{3/2}$$

$N_v = \text{Effective density of states in the VB}$

It is clear now that in  $n$  type material  $E_F$  is close to  $E_c$  and as  $(E_c - E_F) \downarrow$   $n \uparrow$ .

Similarly as  $(E_F - E_v) \downarrow$   $p \uparrow$  OR



As  $E_F \uparrow$   $n \uparrow$  and  $p \downarrow$   
 and  $E_F \downarrow$   $n \downarrow$  and  $p \uparrow$

Examine the product  $n \cdot p$

$$n \cdot p = N_c N_v e^{-(E_c - E_F)/kT} \cdot e^{-(E_F - E_v)/kT}$$

$$= N_c N_v e^{(-E_c + E_F - E_F + E_v)/kT}$$

$$np = N_c N_v e^{(-E_c - E_v)/kT}$$

or  $np = N_c N_v e^{-(E_c - E_v)/kT}$

$$E_c - E_v = E_g$$

$$np = N_c N_v e^{-E_g/kT}$$

The expressions for  $n$  and  $p$  are applicable for all types of semiconductors:  $n$ ,  $p$  or  $i$

In an intrinsic material the Fermi Level is identified as  $E_{Fi}$

$$n(\text{intrinsic}) = n_i = N_c e^{(E_{Fi} - E_c)/kT}$$

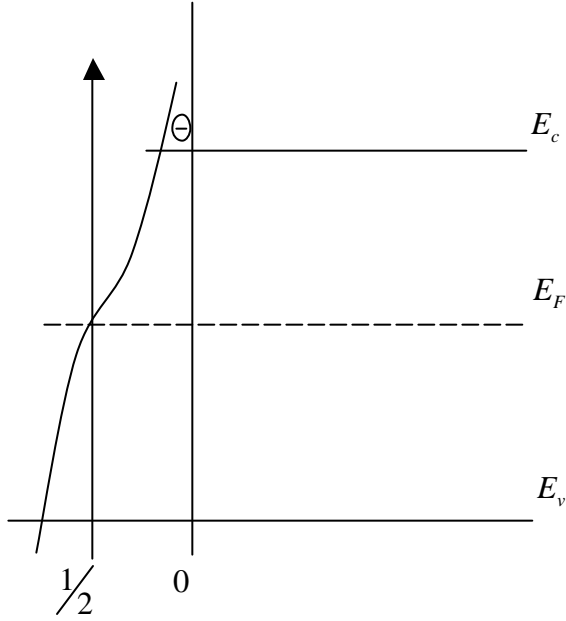
$$\text{and } p(\text{intrinsic}) = p_i = N_v e^{-(E_{Fi} - E_v)/kT}$$

$$\therefore n_i p_i = n_i^2 = N_c N_v e^{-(E_c - E_v)/kT}$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

Note that

$$np = n_i^2 N_c N_v e^{-(E_c - E_v)/kT}$$



Now that we can relate  $E_F$  to the nature of the  $CB$  and  $VB$  through  $m_n^*$  and  $m_p^*$  we can recalculate,  $E_{Fi}$ , more accurately. Any asymmetry in  $n$  &  $p$  distribution has to be introduced through differences in  $N_c$  &  $N_v$ .

$$n_i = N_c e^{-(E_c - E_{Fi})/kT} = \sqrt{N_c N_v} e^{-E_g/2kT}$$

$$\Rightarrow e^{-(E_c - E_{Fi})/kT} = \sqrt{\frac{N_v}{N_c}} e^{-E_g/2kT}$$

$$\therefore \exp\left(\frac{-E_c + E_{Fi}}{kT}\right) \cdot \exp\left(\frac{E_g}{2kT}\right) = \sqrt{\frac{N_v}{N_c}}$$

$$\Rightarrow \exp\left(\frac{-E_c + E_{Fi} + \frac{E_g}{2}}{kT}\right) = \sqrt{\frac{N_v}{N_c}}$$

Taking  $\ln$  of both sides

$$-E_c + E_{Fi} + \frac{E_g}{2} = kT \ln \sqrt{\frac{N_v}{N_c}}$$

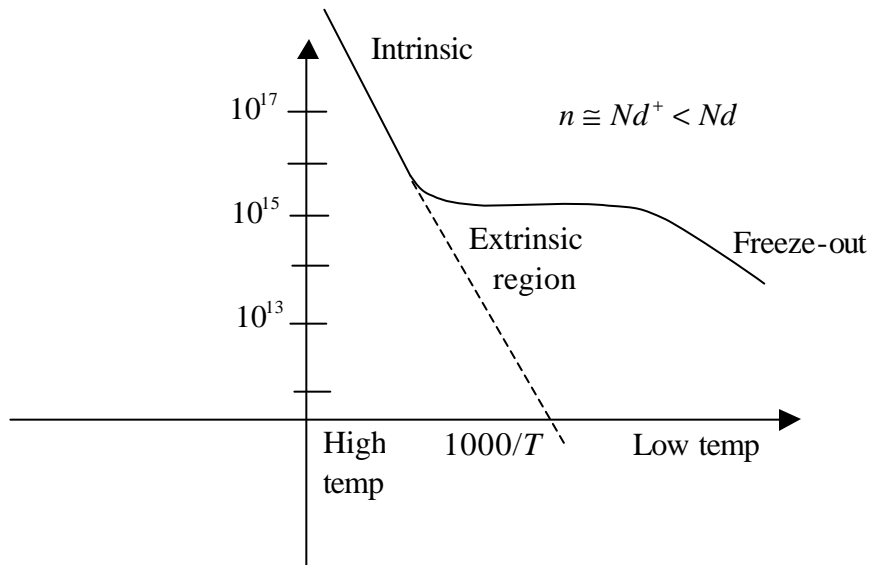
Since  $E_g = E_c - E_v$

$$E_{Fi} = E_c - \frac{E_c}{2} - \frac{E_v}{2} + kT \ln \sqrt{\frac{N_v}{N_c}}$$

$$\therefore E_{Fi} = \frac{E_c + E_v}{2} + \frac{1}{2} kT \ln \frac{N_v}{N_c}$$

The second term is the derivation of  $E_{Fi}$  from the middle of the gap.

So what is the dependence of  $n$  vs  $T$  ?



### Space Charge Neutrality

We can dope a semiconductor with both donors and acceptors

Donate electrons ↙

↘ Accept electrons  
i.e. remove electrons

$$n_{\text{effective}} \cong N_d - N_A \quad \text{for n-type material}$$

$$p_{\text{effective}} \cong N_a - N_d \quad \text{for p-type}$$

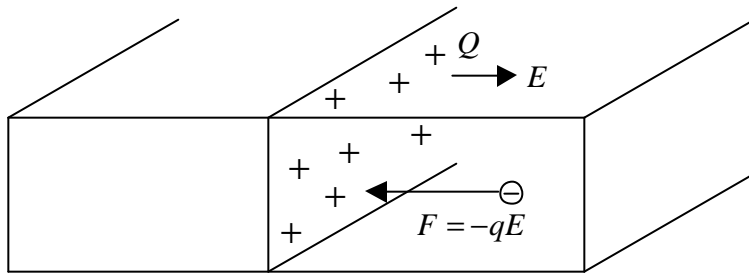
These materials are called compensated semiconductors.

$$n = N_c e^{-(E_c - E_F)/kT}$$

$$\text{and } p = N_v e^{-(E_F - E_v)/kT}$$

are still valid relations.

In a bulk semiconductor there must be charge neutrality everywhere because any excess charge will immediately set up an electric field.



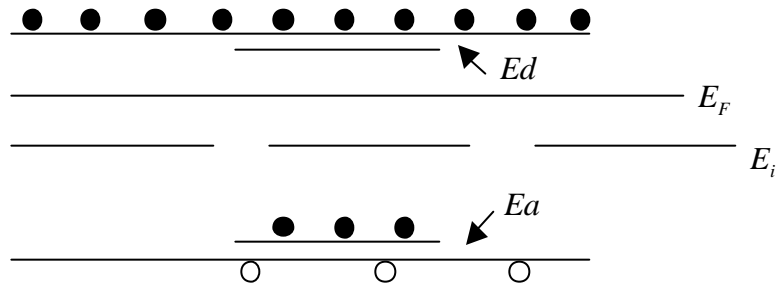
$$E = \frac{Q}{e} \quad F = -q\vec{E}$$

The electrons will flow towards the positive charge until the value  $Q$  is reduced to 0. Then  $E \rightarrow 0$ .  $\Rightarrow$  The attractive force,  $F \rightarrow 0$ .

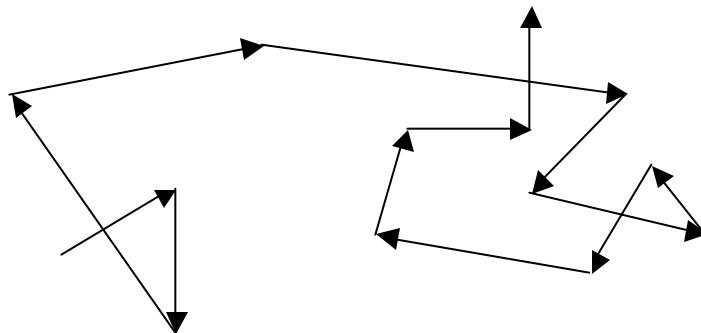
$$p_o + N_D^+ = n_o + N_A^- \quad \text{Bulk semiconductor at equilibrium}$$

$$n_o = \left( N_D^+ - N_A^- \right) + p_o$$

or neglecting  $p_o$   $n_o \approx N_D^+ - N_A^-$



### Conductivity and Mobility



At any temperature the individual electrons show randomized motion. Scattering with the lattice is the dominant randomizing process. If we apply a field,  $E_x$  then

$$F \text{ /per electron} = -qE_x$$

$\therefore$  Net change of momentum of the electron ensemble=

$$\frac{dp}{dt} = -nqE_x$$

where  $n = \#$  of electrons/cm<sup>3</sup>

We want to use this equation to determine the response of an electron to an applied electric field.

i.e. The relationship between its velocity and the applied field.

Assume:

- 1) The probability of collision is constant
- 2)  $N_o = \#$  of electrons in the group at  $t = 0$
- 3)  $N(t) = \#$  of electrons which have not undergone collisions by time  $t$

Let the scattering rate or number of collisions/sec be  $K$ .

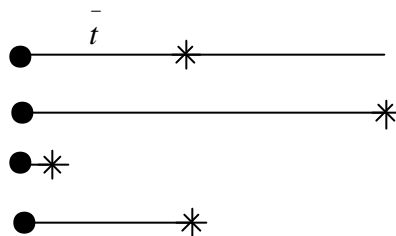
$\therefore$  The rate of decrease of  $N(t)$ , the scattered population is given by

$$\frac{dN(t)}{dt} = -KN(t) \quad (\text{assumption \#1})$$

$$\therefore \frac{dN(t)}{N(t)} = -Kdt \quad \Rightarrow \ln N(t) = -Kt + c$$

Since  $N(t=0) = N_o$ ,  $c = \ln N_o$

$$\therefore \boxed{N(t) = N_o e^{-Kt}}$$





Let  $\bar{t}$  be the mean time between collisions

$$\bar{t} \triangleq \frac{\int_0^{\infty} t N(t) dt}{\int_0^{\infty} N(t) dt} = \frac{N_o \int_0^{\infty} t e^{-Kt} dt}{N_o \int_0^{\infty} e^{-Kt} dt} \quad \text{so} \quad \boxed{\bar{t} = 1/K} \quad \text{and} \quad \boxed{N(t) = N_o e^{-t/\bar{t}}}$$

The probability that an electron will experience a collision in an interval  $dt$  is  $\frac{dt}{\bar{t}}$ . This

is equivalently the fraction of electrons that undergo a collision. Let  $p_x$  be the momentum in the  $x$ -direction.

$$\therefore dp_x = \boxed{-p_x \frac{dt}{\bar{t}}}$$

↑ because momentum decreases

or  $\boxed{\frac{dp_x}{dt} = \frac{-p_x}{\bar{t}}}$  rate of decrease of  $p_x$  due to collisions

$$\boxed{\frac{dp_x}{dt} = -qnE_x}$$
 rate of increase of  $p_x$  due to  $E_x$

At steady state rate of momentum increase due to  $E_x$  = rate of momentum decrease due to collision.

$$\therefore \frac{P_x}{\bar{t}} = -qnE_x \Rightarrow \boxed{P_x = -nq\bar{t}E_x}$$

$$\therefore \langle P_x \rangle \triangleq \frac{P_x}{n} = -q\bar{t}E_x, \text{ the average momentum/electron}$$

$$\therefore \boxed{\langle V_x \rangle = \frac{\langle P_x \rangle}{m_n^*} = \frac{-q\bar{t}E_x}{m_n^*}}$$

Introduce the concept of mobility

$$\langle V_x \rangle_n = -m_n E_x \Rightarrow \mathbf{m} = \frac{q\bar{t}_n}{m_n^*} \quad cm^2V^{-1}s^{-1}$$

$$\langle V_x \rangle_p = m_p E_x \Rightarrow \mathbf{m} = \frac{q\bar{t}_p}{m_p^*}$$

$\langle V_x \rangle_n$  is the net drift of the electron distribution under the influence of  $E_x$ .

The current density resulting from the drift velocity is

$$J_{nx} = -qn \langle V_x \rangle_n$$

$$J_{px} = +qp \langle V_x \rangle_p$$

$$\therefore J_{nx} = -qn \left[ \frac{-q\tau}{m_n^*} \right] E = \frac{+q^2 n \tau}{m_n^*} E_x.$$

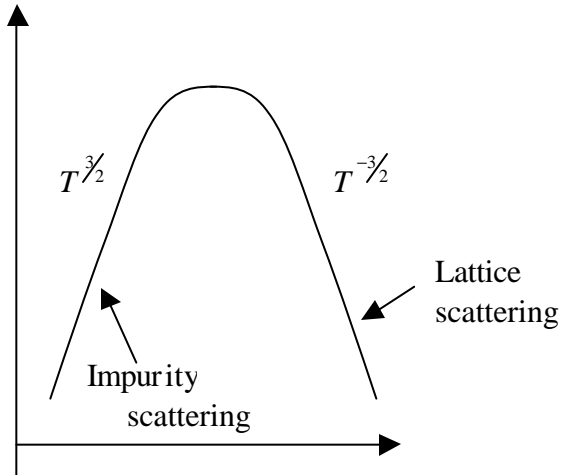
Rewriting as conventional Ohm's Law

$$J_{nx} = s_n E_x \quad s_n \triangleq \frac{nq^2 \tau}{m_n^*}$$

$$\text{Note } s_n = qn m_n : s_p = qp m_p$$

If both electrons and holes are present

$$J_x = J_{px} + J_{nx} = q(n m_n + p m_p) E_x$$



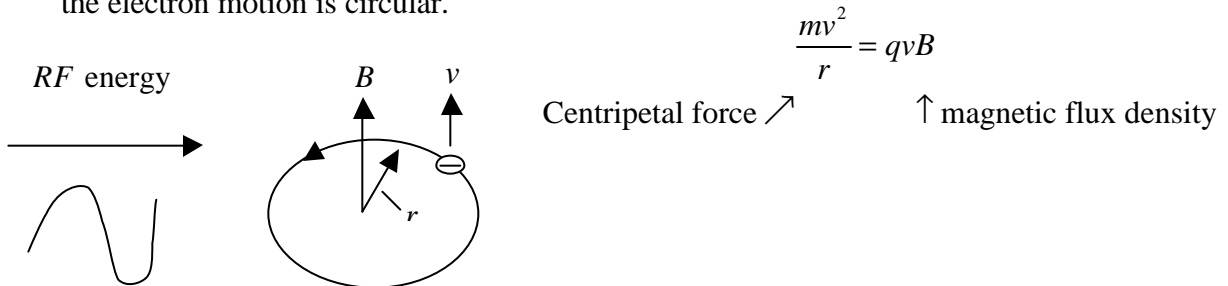
Since the probabilities of scattering add  $K = K_1 + K_2$

$$\text{Since } K_i = \frac{1}{\tau_i} \propto \frac{1}{m_i}$$

$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2} \quad \text{Matthiesen's Rule}$$

## Measurement of Semiconductor Parameters

1) Effective mass  $m^*$  can be determined by electron resonance. Under a magnetic field the electron motion is circular.

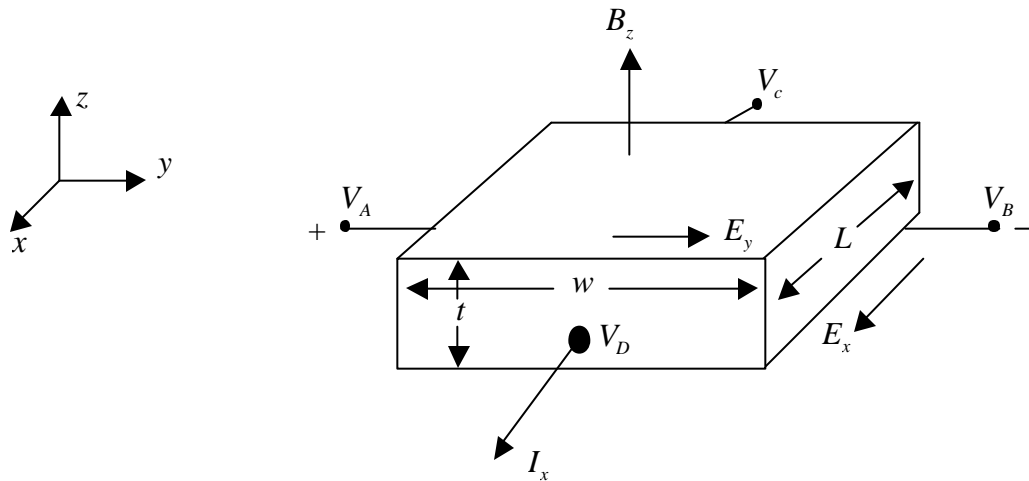


$$\therefore \nu = \frac{qBr}{m^*} \quad \text{or} \quad \omega_c = \frac{qB}{m^*} = \text{Cyclotron resonance frequency}$$

If  $RF$  energy is incident on the sample it will be transmitted for all frequencies except  $\omega_c$  when resonance absorption of energy occurs. (Reduced transmission)

By measuring  $\omega_c$  we can calculate  $m_n^*$  or  $m_p^*$ .

## 2) Mobility (Hall Effect) and Carrier Concentration



Consider a p-type bar, which is subject to an electric field  $E_x$  and a magnetic field  $B_z$ . The force on a single hole

$$F = q \left( \vec{E} + \vec{v} \times \vec{B} \right)$$

$$\therefore F_y = q (E_y - v_x B_z)$$

$E_y$  is the electric field that is set up by the holes that were deflected under the influence of the force  $v_x B_z$ . Since no current can flow in the y-direction.

$$F_y = 0 \Rightarrow \boxed{E_y = v_x B_z}$$

The  $v_x$  can be obtained from  $J_x$  as

$$v_x = \frac{J_x}{qp_o} \quad \text{where } p_o = \text{hole concentration}$$

$$\therefore E_y = \frac{J_x}{qp_o} \cdot B_z = R_H J_x B_z$$

Where  $R_H = \frac{1}{qp_o} =$  Hall Coefficient

$$J_x = \frac{I_x}{wt}$$

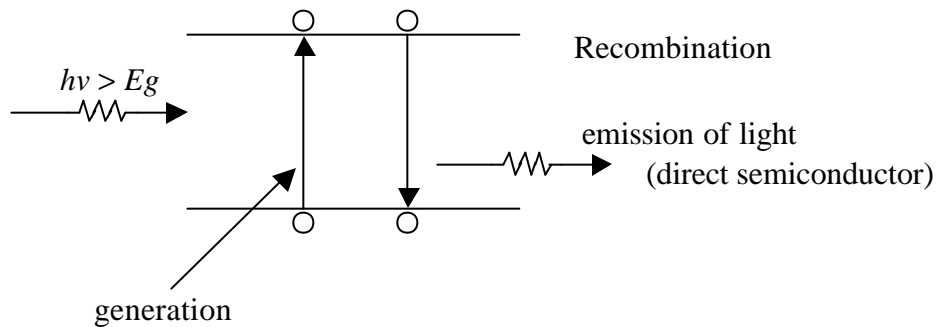
$$\therefore p_o = \frac{1}{qR_H} = \frac{J_x B_z}{qE_y}$$

$$E_y = \frac{V_A - V_B}{w} = \frac{V_{AB}}{w}$$

$$p_o = \frac{I_x B_z}{qtV_{AB}}$$

$$m = \frac{s}{qp_o} = \frac{1}{qp_o} \quad \text{where} \quad = \frac{Rwt}{L} = \frac{V_{CD} / I_x}{L / wt}$$

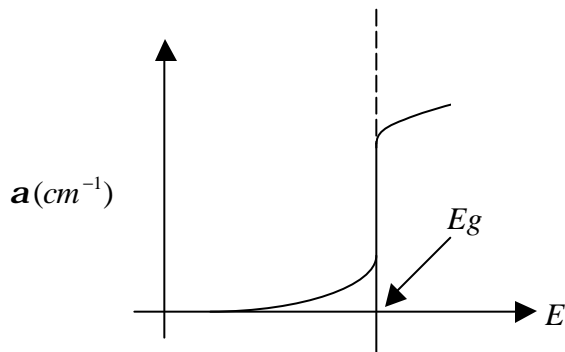
### Excess Carriers in Semiconductors :



The beam is absorbed

$$-\frac{dI(x)}{dx} = \mathbf{a}I(x) \quad \text{absorption coefficient}$$

$$\Rightarrow I(x) = I_o e^{-ax}$$



$$hv = E = \frac{hc}{\lambda} = \frac{1.24(\text{eV})}{\lambda(\text{mm})}$$

is the energy of the photon in eV.

## Carrier Lifetime & Photoconductivity

Let us calculate the rate of change in the electron population when I switch the light off. We first assume that the rate of recombination,  $R$ , is proportional to the product of the electron and hole population. For example:  $R = a_r p(t) n(t)$ . In the absence of light the thermal generation rate should be equal to the thermal recombination rate.

$$\text{or } G_{th} = R_{th} = a_r n_i p_i = a_r n_i^2$$

$$\text{Now } \frac{dn(t)}{dt} = G - R = G_{th} - R \quad (\text{In the absence of light})$$

$$\frac{dn(t)}{dt} = \underset{\substack{\uparrow \\ \text{Thermal generation rate}}}{a_r n_i^2} - a_r n(t) p(t)$$

Recognizing that on shining light equal numbers of  $e$  and hole pairs are created ( $dn(t) = dp(t)$ ), and since  $a_r n_i^2 = a_r n_o p_o$ ,

$$\begin{aligned} \frac{d}{dt} dn(t) &= a n_i^2 - a_r [n_o + dn(t)][p_o + dp(t)] \\ &= -a_r [(n_o + p_o) dn(t) + dn^2(t)] \end{aligned}$$

$\therefore$  Low level injection  
P-type material

$$\Rightarrow n_o \ll p_o \text{ and } dn^2(t) < dn(t) \cdot p_o$$

$$\Rightarrow \frac{d}{dt} dn(t) = -a_r p_o dn(t)$$

$$\Rightarrow \boxed{dn(t) = \Delta n e^{-a_r p_o t} = \Delta n e^{-t/t_n}}$$

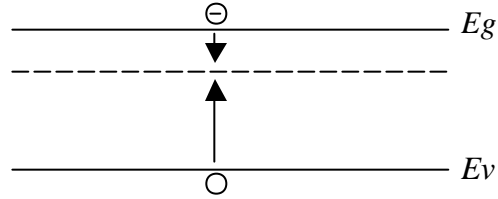
$$\boxed{(a_r p_o)^{-1} = t_n = \text{Recombination time}}$$

$$\boxed{\text{Also the minority carrier lifetime}}$$

THIS ANALYSIS IS VALID ONLY FOR DIRECT RECOMBINATION

Not valid for recombination via traps

In general traps reduce the minority carrier lifetime even further



Quasi-Fermi level [IMREF]

$g(t) = \mathbf{a}_r n_o p_o$ ; in steady state since generation rate = recombination rate

We know with light  $g(T) + g_{op} = \mathbf{a}_r np = \mathbf{a}_r (n_o + \mathbf{d}n)(p_o + \mathbf{d}p)$

In steady state with no trapping  $\mathbf{d}n = \mathbf{d}p$

$$g(T) + g_{op} = \mathbf{a}_r n_o p_o + \mathbf{a}_r [(n_o + p_o)(\mathbf{d}n) + \mathbf{d}n^2]$$

$$g_{op} = \mathbf{a}_r [p_o] \mathbf{d}n \quad \text{[Low level injection in a } p\text{-type material} \\ \Rightarrow \mathbf{d}n < p_o \text{ and } n_o < p_o]$$

$$= \frac{\mathbf{d}n}{\mathbf{t}_n}$$

$$\Rightarrow \boxed{\mathbf{d}n = g_{op} \mathbf{t}_n} \quad \text{In general } \mathbf{d}n = \mathbf{t}_n g_{op}$$

$$\mathbf{d}p = \mathbf{t}_p g_{op}$$

If  $\mathbf{t}_n \neq \mathbf{t}_p$

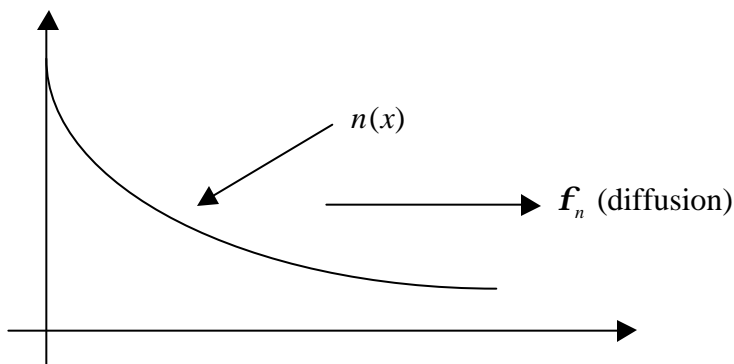
$$n_o + \mathbf{d}n = n = n_i e^{(E_{Fn} - E_i)}$$

$$p_o + \mathbf{d}p = p = n_i e^{(E_i - E_{fp})/kT}$$

$E_{Fn}$  = Electron IMREF

$E_{fp}$  = Hole IMREF

### Diffusion of Carriers



$$f_n(x) = \boxed{-D_n \frac{dn(x)}{dx}}$$

↑

The negative sign represents diffusion from higher to lower concentration

$$f_p(x) = -D_p \frac{dp(x)}{dx}$$

The diffusion current  $J_n = (-q) \cdot \left( -D_n \frac{dn}{dx} \right)$

$$\boxed{J_n = qD_n \frac{dn}{dx}}$$

$$\boxed{J_p = -qD_p \frac{dp}{dx}}$$

In the presence of an electric field

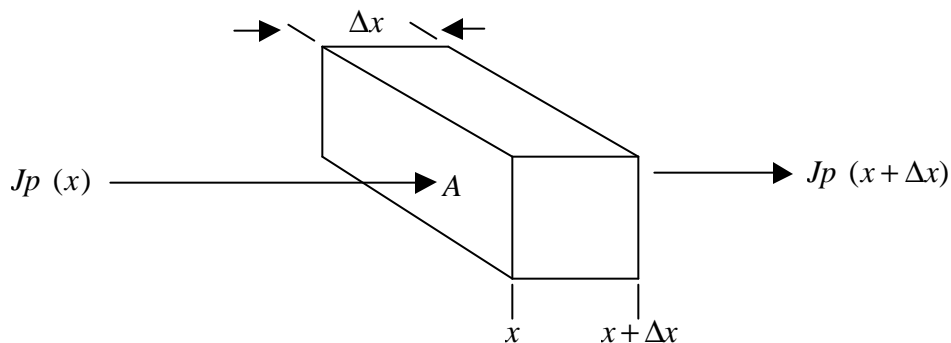
$$J_n(x) = J_{n, \text{drift}} + J_{n, \text{diffusion}}$$

$$J_n(x) = qm_n n(x) E(x) + qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = qm_p p(x) E(x) - qD_p \frac{dp(x)}{dx}$$

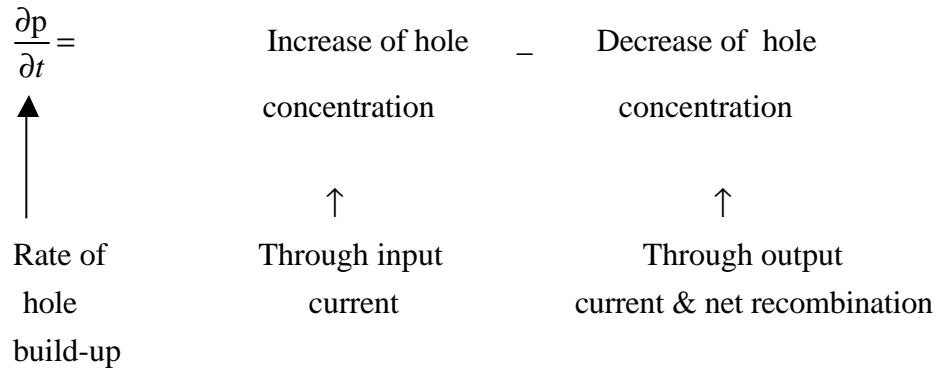
$J(x) = J_n(x) + J_p(x)$  or the total current is the sum of all electron and hole currents

### Continuity Equation



$A =$  Area in  $\text{cm}^2$  and  $\Delta x$  is the length of the box in centimeters.

Conservation of mass



Input current =  $J_p(x) \Rightarrow$  (charge) x (input # of carriers/unit area/second)

$$\begin{aligned} \text{Input hole flux} &= \frac{\text{Input \# carriers/second}}{\text{unit volume}} = \mathbf{f}_p(x) \\ &\Rightarrow \frac{J_p(x)}{q\Delta x} = \mathbf{f}_p(x) \end{aligned}$$

$$\mathbf{f}_p(x + \Delta x) = \frac{J_p}{q\Delta x}(x + \Delta x)$$

$$\frac{\partial p}{\partial t} = \frac{1}{q} \frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} - \frac{\mathbf{d}p}{\mathbf{t}_p}$$

$$\therefore \frac{\partial p(x,t)}{\partial t} = \boxed{\begin{aligned} \frac{\partial \mathbf{d}p}{\partial t} &= -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\mathbf{d}p}{\mathbf{t}_p} \\ \frac{\partial \mathbf{d}n}{\partial t} &= +\frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\mathbf{d}n}{\mathbf{t}_n} \end{aligned}}$$

**Continuity equation for electrons and holes.**

For purely diffusive currents.



$$\frac{\partial \mathbf{d}n}{\partial t} = D_n \frac{d^2 \mathbf{d}n}{dx^2} - \frac{\mathbf{d}n}{\mathbf{t}_n}$$

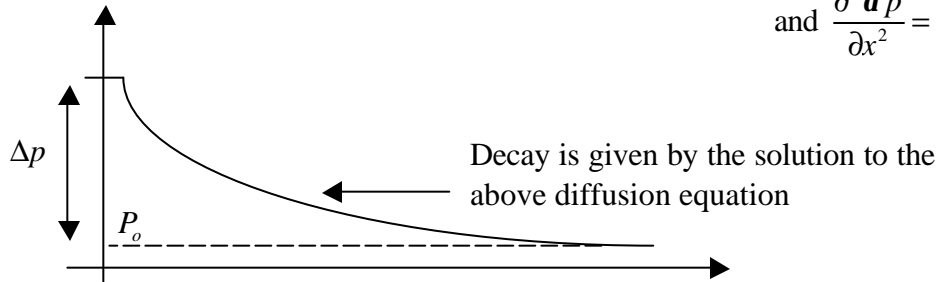
$$\frac{\partial \mathbf{d}p}{\partial t} = D_p \frac{\partial^2 \mathbf{d}p}{\partial x^2} - \frac{\mathbf{d}p}{\mathbf{t}_p}$$

### Diffusion Length

Consider steady state or  $\frac{\partial}{\partial t} = 0$

$$\Rightarrow \frac{\partial^2 \mathbf{d}n}{\partial x^2} = \frac{\mathbf{d}n}{D_n \mathbf{t}_n} \equiv \frac{\mathbf{d}n}{L_n^2}$$

$$\text{and } \frac{\partial^2 \mathbf{d}p}{\partial x^2} = \frac{\mathbf{d}p}{D_p \mathbf{t}_p} \equiv \frac{\mathbf{d}p}{L_p^2}$$



$$\therefore \mathbf{d}p(x) = c_1 e^{x/L_p} + c_2 e^{-x/L_p}$$

As  $x \rightarrow \infty$  we know  $\mathbf{d}p \rightarrow 0$  or  $c_1 = 0$

$$\therefore \mathbf{d}p(x) = c_2 e^{-x/L_p}$$

$L_p$  = the distance where the distribution drops to  $\frac{1}{e}$  x initial value

AND

$L_p$  = average distance a hole travels before recombining.

The hole diffusion current can be calculated at any  $x$  as

$$J_p(x) = -\frac{qD_p \mathbf{d}p(x)}{dx} = -qD_p \frac{d}{dx}(\mathbf{d}p)$$

$$= \frac{qD_p}{L_p} \mathbf{d} p(x)$$

Currents in relation to  $E_{F_n}$  &  $E_{F_p}$

$$J_n(x) = q\mathbf{m}_n n(x) E(x) + qD_n \frac{dn(x)}{dx}$$

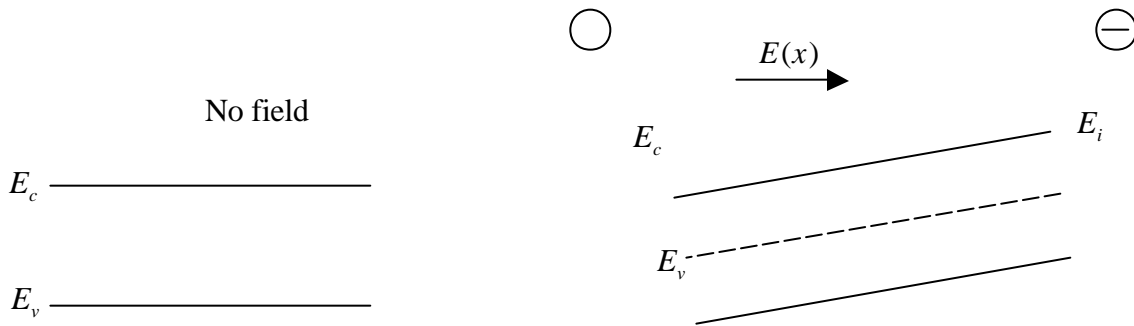
Since  $\frac{dn(x)}{dx} = \frac{d}{dx} \left[ n_i e^{\left( \frac{E_{F_n} - E_{F_i}}{RT} \right)} \right] = \frac{n(x)}{RT} \left[ \frac{dE_{F_n}}{dx} - \frac{dE_{F_i}}{dx} \right]$

AND since  $Dn = \mathbf{m}_n RT$ , we get

$$J_n(x) = q\mathbf{m}_n n(x) E(x) + \mathbf{m}_n n(x) \left[ \frac{dE_{F_n}}{dx} - \frac{dE_{F_i}}{dx} \right]$$

But  $\frac{dE_i}{dE_x} = qE(x)$

$\therefore J_n(x) = \mathbf{m}_n n(x) \frac{dE_{F_n}}{dx}$  total electron current



$$E(x) = -\frac{dV(x)}{dx}$$

But  $V(x) = \left( \frac{E(x)}{-q} \right) \Rightarrow E(x) - \frac{d}{dx} \left[ \frac{E_{F_i}}{-q} \right] = \frac{1}{q} \frac{dE_i}{dx}$

What is the relationship between  $D$  &  $m$ ?

At equilibrium no current flows in the semiconductor. So, assuming a  $p$ -type semiconductor

$$J_p(x) = q m_p p E(x) - q D_p \frac{dp(x)}{dx} = 0$$

$$E(x) = \frac{D_p}{m_p} \frac{1}{p(x)} \frac{dp(x)}{dx}$$

$$p(x) = n_i e^{(E_i - E_F)/RT}$$

$$\Rightarrow E(x) = \frac{D_p}{m_p} \frac{1}{RT} \left( \left( \frac{dE_i}{dx} \right) - \frac{dE_F}{dx} \right)$$

Einstein's relation

$$\Rightarrow \boxed{\frac{D}{m} = \frac{kT}{q}}$$