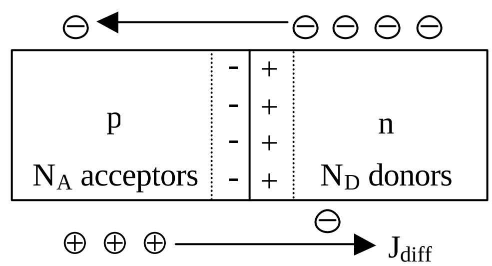
p-n junctions

A p-n junction is a metallurgical and electrical junction between p and n materials. When the materials are the same the result is a HOMOJUNCTION and if they are dissimilar then it is termed a HETEROJUNCTION.

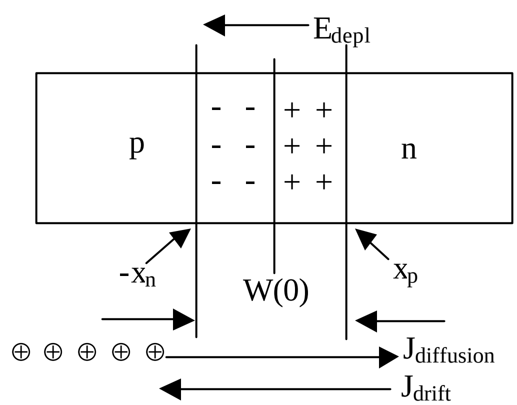
Junction Formation:



1. Majority carriers diffuse [holes from p to n and electrons from n $\rightarrow$ p]

2. Bare ionized dopants are exposed on either side of the junction. Positively charged donors on the n-side and negatively charged acceptors on the p-side.

3. The dopant ions are contained in a region of reduced carrier concentration (as the mobile majority charges have diffused as stated in (1)). This region is therefore called the depletion region.

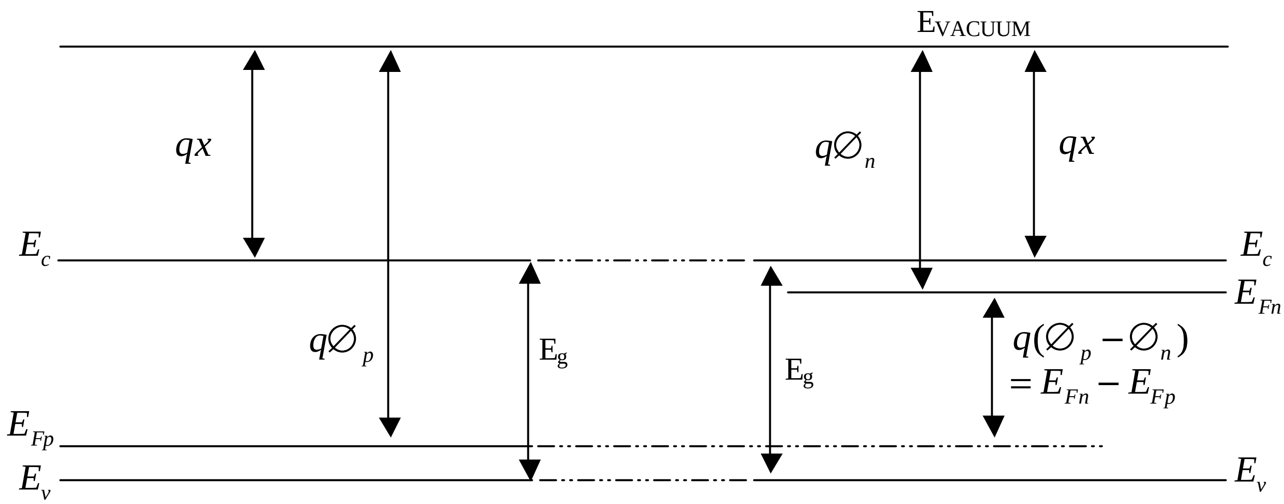


4. The process of diffusion continues until the depletion region expands to a width, W(0), such that the electric field in the depletion region E_{depl} is large enough to repel the diffusing carriers. More precisely,

$J_{diffusion} = J_{drift}$ FOR EACH CARRIER SEPARATELY	(once equilibrium has been established)

5. The driving force for carrier motion is the ELECTRO-CHEMICAL POTENTIAL DIFFERENCE that exists between the two semiconductors in the bulk prior to junction formation

In band diagram terms here are the before and after pictures:



BEFORE: THE TWO MATERIALS ARE SEPARATE

Definitions:

a) qx = Electron affinity in units of energy. (use eV or Joules)

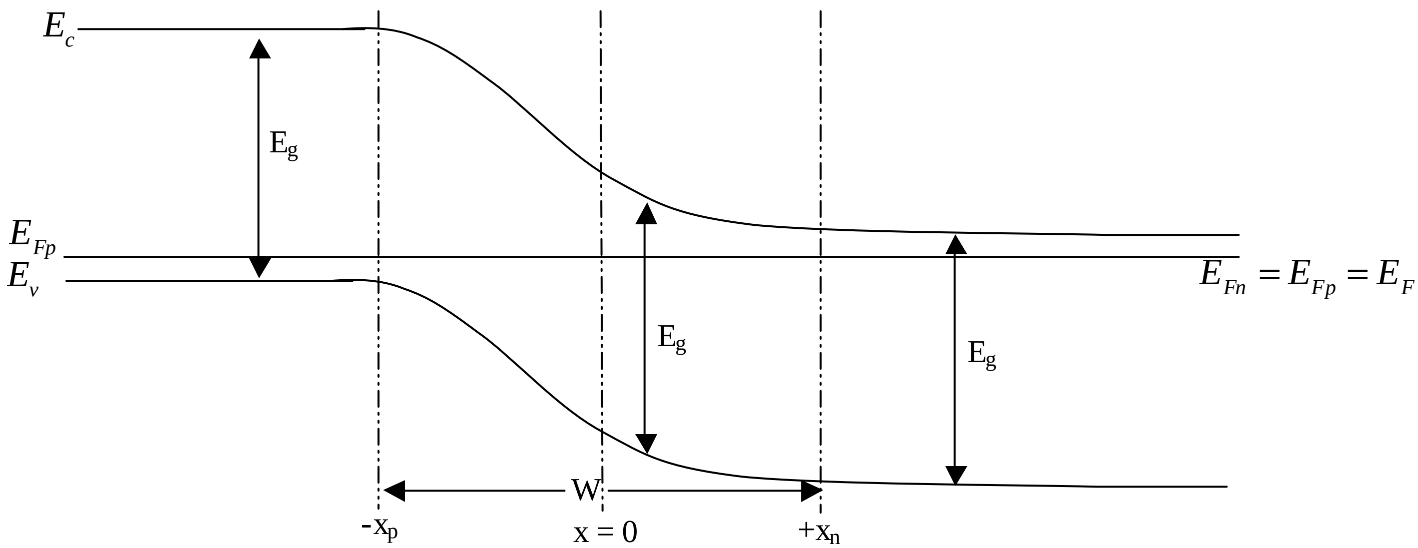
b) E_{Fp} = Fermi Level in the p-type material or electro-chemical potential of the p-type material.

c) E_{Fn} = Fermi Level in the n-type material or electro-chemical potential of the n-type material.

NOTE: THIS IS THE ELECTRO-CHEMICAL POTENTIAL OF ELECTRONS IN BOTH CASES

d) $q\varnothing_p$ and $q\varnothing_n$ are the work functions of the two materials (p and n) respectively.

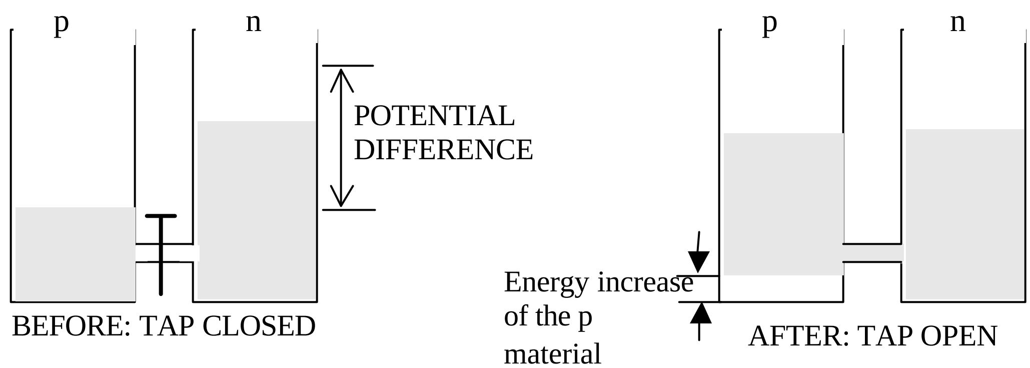
e) Note that the work function difference between the two materials $q\left(\varnothing_p - \varnothing_n\right)$ is the difference between the electro-chemical potentials of the bulk materials E_{Fn} and E_{Fp}.



AFTER: The materials are brought together to form a junction. The fermi levels E_{Fn} and E_{Fp} now equalize or $E_{Fn} = E_{Fp} = E_F$ (IN EQUILIBRIUM)

a) Assume the p-material is kept at a constant potential (say ground).

b) The p-material has to increase its electro-chemical potential of electrons (upward motion of the bands) until the fermi levels line up as shown in the diagram below where the effect is simulated using two beakers of water in equilibrium with different amounts of water in each beaker.



c) The lowering of the electron energy of the n-type semiconductor is accompanied by the creation of the depletion region.

d) The depletion region has net charge and hence the bands have curvature following Gauss' Law:

$$\frac{\partial E}{\partial x} = \frac{r}{\in}$$ $E =$ Electric Field

OR $$\frac{\partial^2 V}{\partial x^2} = -\frac{r}{\in}$$ $$E = -\frac{\partial V}{\partial x}$$

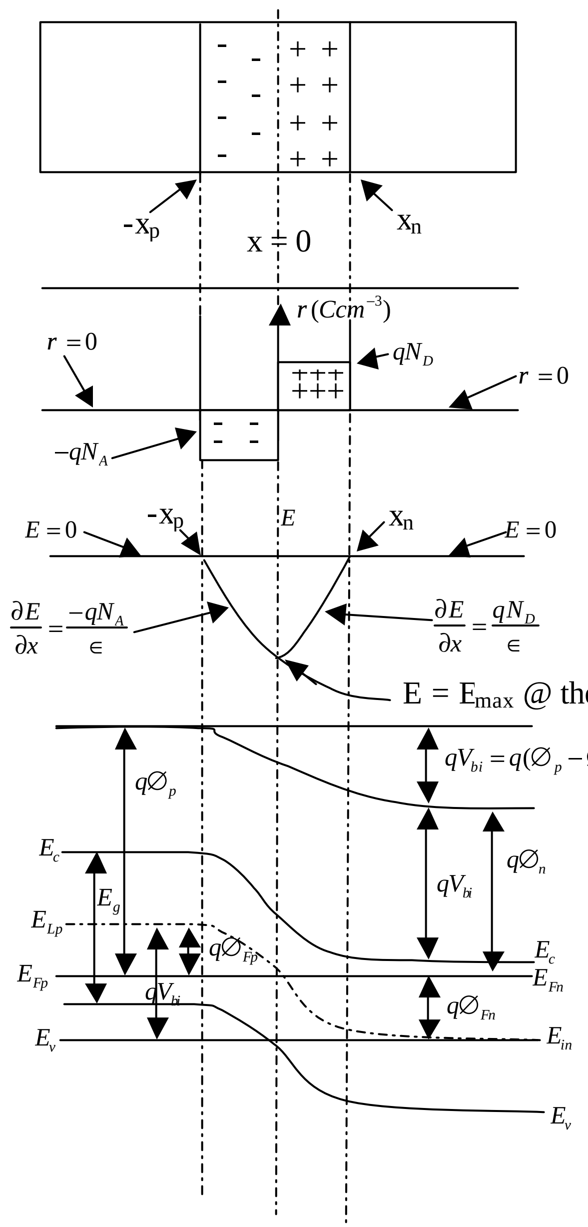
where V = Potential energy(of unit positive charge)

OR $$\frac{\partial^2 E_c}{\partial x^2} = \frac{r}{\in}$$ $E_c = -qV$ = Electron energy (Joules)

or –V = Electron energy (eV)

NOTE	NET CHARGE ⇔ CURVATURE OF THE BANDS NEUTRAL REGIONS ⇒ NO NET CHARGE ⇔ BANDS HAVE NO CURVATURE DO NOT CONFUSE SLOPE WITH CURVATURE Neutral regions can have constant slope or equivalently no curvature

CALCULATING THE RELEVANT PARAMETERS OF A p-n JUNCTION



1. SCHEMATIC OF THE JUNCTION

2. CHARGE PROFILE

3. ELECTRIC FIELD

4. BAND DIAGRAM

NOTE: $qV_{bi} = q\varnothing_{Fp} + q\varnothing_{Fn}$

In the analysis depicted in the foru diagrams above, we assume

(i) that the doping density is constant in the p-region at N_A and in the n-region at N_D and that the change is abrupt at x = 0, the junction.

(ii) the depletion region has only ionized charges $-qN_A$ (C_{cm}^{-3}) in the p-region and $+qN_D$ (Ccm^{-3}) in the n-region. [Mobile charges in the depletion region are neglected, i.e. n, p << N_A^- and N_D^+]

(iii) The transition from the depletion region to the neutral region is abrupt at ($-x_p$) in the p-region and ($+x_n$) in the n-region.

Calculation of the built-in voltage

From the band diagram it is clear that the total band bending is caused by the work function difference of the two materials. If you follow the vacuum level (which is always reflects the electrostatic potential energy variation and hence follows the conduction band in our homojunction), you see that the band bending is the difference of the work function of the p-type material $q\varnothing_p$, and the n-type material $q\varnothing_n$.

$$qV_{bi} \sim q\varnothing_p - q\varnothing_n$$

The built-in potential is therefore the internal potential energy required to cancel the diffusive flow of carriers across the junction and should be exactly equal to the original electro-chemical potential which caused the diffusion in the first place. THIS IS REASSURING.

To calculate V_{bi} from parameters such as doping let us follow the intrinsic level from the p-side, E_{ip}, to the n-side, E_{in}. Again the total band bending of the intrinsic level is the built in potential

$$\text{or } E_{ip} - E_{in} = qV_{bi}$$

$$\text{or } (E_{ip} - E_F) - (E_{in} - E_F) = qV_{bi}$$

Defining $E_{ip} - E_{Fp}$ as $q\varnothing_{Fp}$ [Note in equilibrium
And $E_{Fn} - E_{in}$ as $q\varnothing_{Fn}$ $E_{Fn} = E_{Fp} = E_F$]

We can rewrite qV_{bi} as $q\varnothing_{Fn} + q\varnothing_{Fp} = qV_{bi}$

From Fermi-Dirac Statistics:

$$p_{Po} = n_i\, e\frac{E_i - E_F}{RT} \quad \text{or} \quad n_i\, e\frac{E_{ip} - E_{Fp}}{kT} = n_i\, e^{\frac{q\varnothing}{kT}}$$

similarly $n_{no} = n_i\, e\dfrac{E_i - E_F}{RT}$

Assume full ionization

$p_{Po} = N_A$ and $n_{no} = N_D$

$$\therefore\ N_A = n_i\ e\frac{q\varnothing_{Fp}}{kT} \qquad \& \qquad N_D = n_i\ e\frac{q\varnothing_{Fn}}{kT}$$

or $$q\varnothing_{Fp} = kT\ln\frac{N_A}{n_i} \qquad \& \qquad q\varnothing_{Fn} = kT\ln\frac{N_D}{ni}$$

$$\therefore\ q\varnothing_{Fp} + q\varnothing_{Fn} = kT\left[\ln\frac{N_A}{n_i} + \ln\frac{N_D}{n_i}\right]$$

or $$qV_{bi} = kT\ln\frac{N_A N_D}{n_i^2}$$

$$\boxed{V_{bi} = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2}}$$

<u>Calculating Depletion Region Widths</u>

From the electric field diagram, Figure 3

$$E_{\max} = \frac{-qN_A}{\in}x_p$$

$$\left|E_{\max}\right| = \frac{qN_A}{\in}\cdot x_p$$

The magnitude of the area under the electric field versus distance curve (shaded area in figure 3) is by definition $\left|\int_{-xp}^{xn} E\cdot dx\right|$ = Voltage difference between $-x_p$ and $+x_n$ = V_{bi}

or $$V_{bi} = \frac{1}{2}\cdot W\cdot\left|E_{\max}\right|$$

$$\left[\frac{1}{2}\cdot Base\cdot Height\right]$$

$$V_{bi} = \frac{1}{2}\cdot(x_n + x_p)\cdot\frac{qN_A}{\in}x_p$$

We now invoke charge neutrality. Since the original semiconductors were charge neutral the combined system has to also be charge neutral (since we have not created charges). Now since the regions beyond the depletion region taken as a whole has to be charge neutral. <u>OR</u> all the positive charges in the depletion region have to balance all the negative charges.

If the area of the junction is A cm^2 then the number of positive charges within the depletion region from x = 0 to x = x_n is

$qN_D \cdot x_n \cdot A$ = Coulombs

(qN_D: Coulombs cm^{-3}; x_n: cm^{-3}; A: cm^2)

Similarly, all the negative charges contained in the region between x = $-x_p$ and x = o is

$qN_A\cdot x_p\cdot A = Coulombs$

charge neutrality therefore requires

$qN_A x_p \cdot A = qN_D x_n \cdot A$ or $\boxed{N_A x_p = N_D x_n}$ IMPORTANT

To calculate w, x_n and x_p we use the above relation in the equation for V_{bi} below

$$V_{bi} = \frac{1}{2} \cdot (x_n + x_p) \cdot E_{max}$$

p-side	n-side
$V_{bi} = \frac{1}{2}(x_n + x_p)\frac{qN_A}{\in} x_p$	$V_{bi} = \frac{1}{2}(x_n + x_p)\frac{qN_D}{\in} x_n$
Substituting for x_n or $V_{bi} = \frac{1}{2}\left(\frac{N_A + N_D}{N_D} + x_p\right)\frac{qN_A}{\in} x_p$	$V_{bi} = \frac{1}{2}\left(\frac{N_D + N_D}{N_A} + x_n\right)\frac{qN_D}{\in} x_n$
or $\frac{2\in}{qN_A} V_{bi} = \left(\frac{N_A + N_D}{N_D}\right) x_p{}^2$	$\frac{2\in}{qN_D} V_{bi} = \left(\frac{N_A + N_D}{N_A}\right) x_n{}^2$
NOTE: in this analysis $\lvert E_{max}\rvert$ was calculated as $\frac{qN_A}{\in} x_p$ from the p-side and $\frac{qN_D}{\in} x_n$ fom the n-side	
$x_p = \sqrt{\frac{2\in}{q} \cdot \frac{N_D}{N_A} \cdot \frac{1}{(N_A + N_D)} \cdot V_{bi}}$	$x_n = \sqrt{\frac{2\in}{q} \cdot \frac{N_A}{N_D} \cdot \frac{1}{(N_A + N_D)} \cdot V_{bi}}$

$$W = x_n + x_p$$

$$= \sqrt{\frac{2\in}{q} \cdot V_{bi}} \cdot \left[\frac{\sqrt{N_D}}{\sqrt{N_A} \cdot \sqrt{N_A + N_D}} + \frac{\sqrt{N_A}}{\sqrt{N_D} \cdot \sqrt{N_A + N_D}}\right]$$

$$= \sqrt{\frac{2\in}{q} \cdot V_{bi}} \cdot \left[\frac{N_A + N_D}{\sqrt{N_A N_D} \cdot \sqrt{N_A + N_D}}\right]$$

$$W = \sqrt{\frac{2\in}{q} \cdot \frac{N_A + N_D}{N_A N_D} \cdot V_{bi}}$$

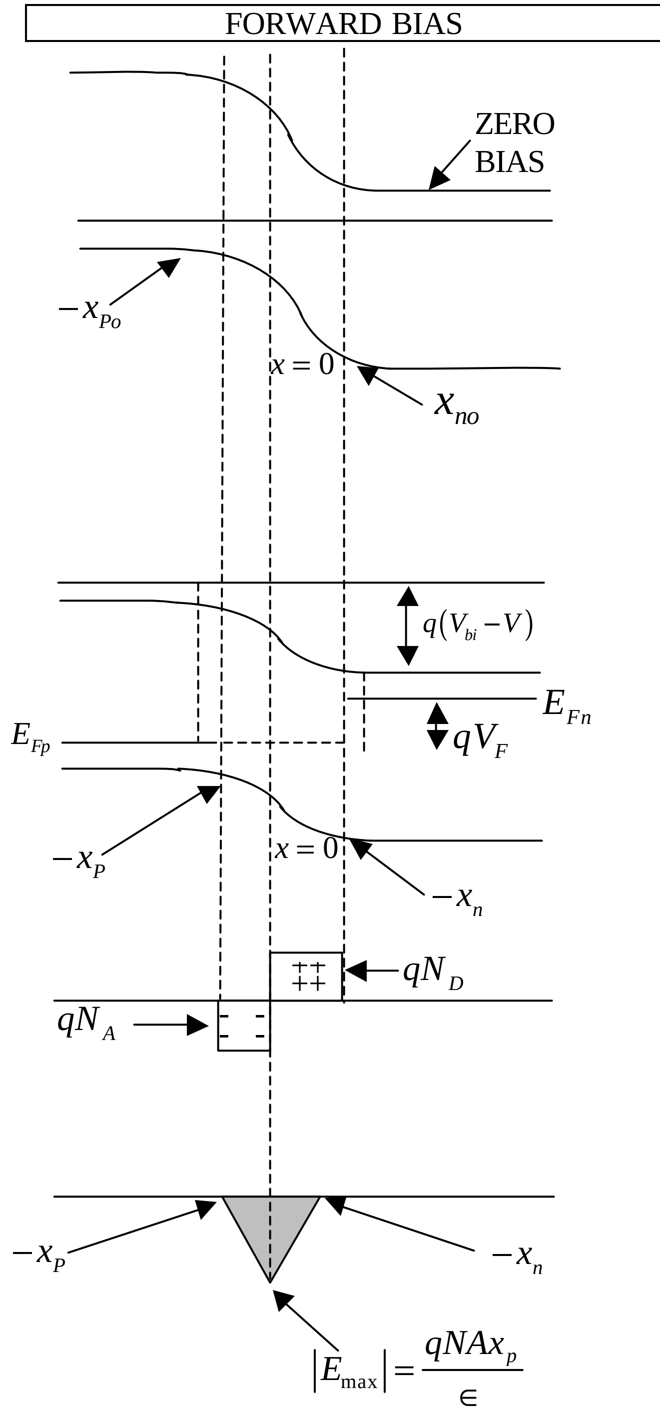
IN EQUILIBRIUM $\boxed{W(0) = \sqrt{\frac{2\in}{q} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D}\right) \cdot V_{bi}}}$ IMPORTANT

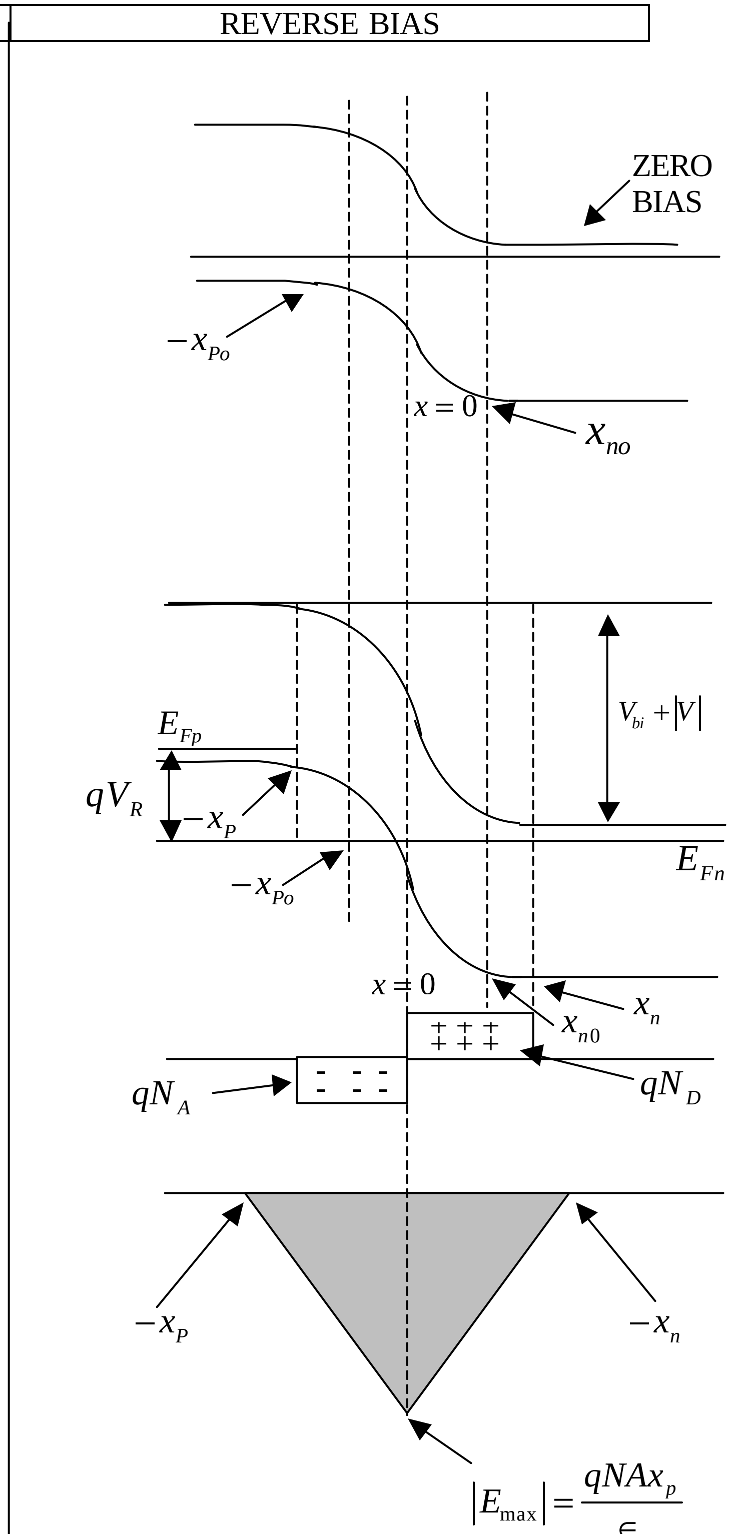
In general

$$W(v) = \sqrt{\frac{2\in}{q} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_{bi} \pm \lvert V\rvert)}$$

V is the applied bias $\begin{cases} +\lvert V\rvert & \text{in reverse bias (w expands)} \\ & \end{cases}$

$-|V|$ in forward bias (w shrinks)





FEATURES:

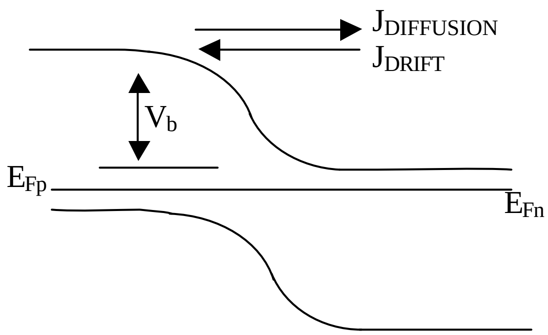
1. Total Band bending is now V_{bi}-V
2. The shaded area $\left|\int E \cdot dx\right| = V_{bi} - |V|$
3. The edges of the depletion region move towards the junction or W decreases.
4. $E_{Fn} - E_{Fp} = V_F$, the electrochemical potentials separate by an amount equal to the potential difference applied, V_F.

FEATURES:

1. Total Band bending is now $V_{bi} + |V|$
2. The shaded area $\left|\int E \cdot dx\right| = V_{bi} + |V|$
3. Depletion region expands.
4. $E_{Fp} - E_{Fn} = |V_R| =$ amount of potential difference, (V_R).

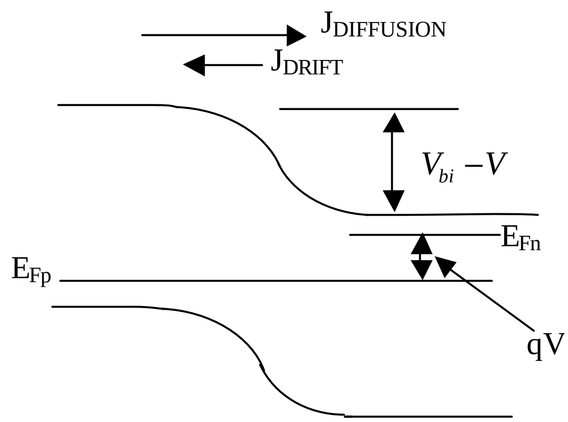
Current flow in p-n junctions VERY IMPORTANT

A. Forward Bias



Zero Bias

$$J^{p}_{DIFFUSION} + J^{p}_{DRIFT} = 0$$
$$J^{n}_{DIFFUSION} + J^{n}_{DRIFT} = 0$$

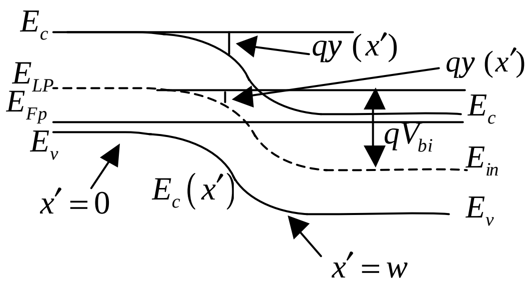


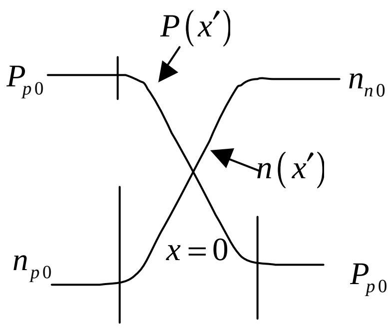
Forward Bias

The depletion width is reduced which increases the diffusion current. The drift current remains the same. The net current is due to the imbalance of drift and diffusion

Under forward bias electrons from the n-region and holes from the p-region cross the junction and diffuse as minority carriers in the p and n-regions respectively. To understand how excess minority carriers are injected and diffuse one has to understand the LAW OF JUNCTION which now follows.

Consider the two situations shown below one at zero bias and the other under forward bias.





$x' = 0$ is @ $x = -x_p$	$x' = W$ is @ x_n

Note that E_i is a function of x' and is E_{ip} in the bulk p and E_{in} in the bulk n.

Note that $p(x')$ is always given by

$$p(x') = n_i e^{\frac{E_i(x') - E_{Fp}(x')}{kT}}$$

Where $E_i(x)$ and $E_{Fp}(x)$ are the intrinsic fermi-level and the fermi level for holes at any place x.

Since we are at zero bias and at equilibrium E_{Fp} is not a function of x'.

$$\therefore\ p(x') = n_i e^{\frac{E_i(x') - E_F}{kT}} = n_i e^{\frac{E_{ip} - E_F - E_{ip} - E_F}{kT}}$$

$$= n_i e^{\frac{E_{ip} - E_F}{kT}} \cdot e^{-\left(\frac{E_{ip} - E_i(x')}{kT}\right)}$$

or $p(x') = (p_{P0}) \cdot e^{-\frac{qy(x')}{kT}}$

where $qy(x') \equiv \frac{E_{ip} - E_i(x')}{kT}$

LAW OF THE JUNCTION

or $p(x')$ decreases exponentially with the local band bending

Note that the edge of the depletion region on the n-side $x = x_r$ or $x' = W$ the hole concentration is given by

$$p(x' = W) = p_{P0} \cdot e^{-\frac{qy\,(x'=W)}{kT}}$$

The total band bending at $(x' = W)$ is V_{bi}

$$\therefore p(x' = W) = p_{p0} e^{-\frac{qV_{bi}}{kT}}$$

We know that $p(x' = W)$ is the hole concentration in the n-type semiconductor or n_{p0}

So, $n_{p0} = p_{p0} e^{-\frac{qV_{bi}}{kT}}$

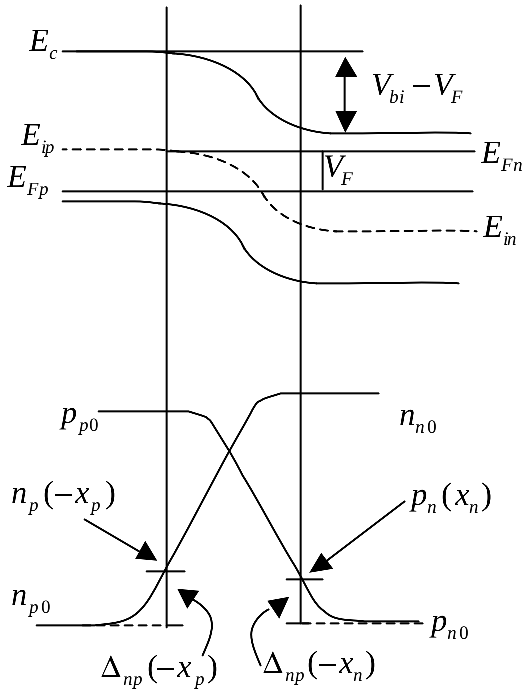
Let us verify that what we derived does not contradict what we learned in the past.

$$n_{p0} = \frac{n_i^2}{n_{n0}} = p_p e^{-\frac{qV_{bi}}{kT}}$$

$$\therefore e^{-\frac{qV_{bi}}{kT}} = \frac{n_i^2}{p_{p0} n_{n0}}$$

$\therefore V_{bi} = \frac{kT}{q} \ln \frac{n_{n0}\, p_{p0}}{n_i^2}$ or with full ionization

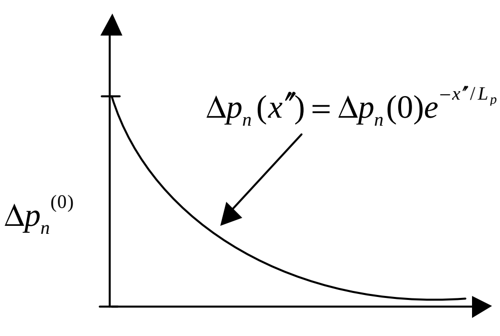
$$\boxed{V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \text{ SAME AS BEFORE}}$$



Change the coordinate system so that x_n in x'' coordinate is 0.

Consider only the n-region.

THEN IN THE n-region



$x = x_n$

$x' = w$

$x'' = 0$

On application of a forward bias the electron and hole concentrations continue to follow the relation that $p_n(x') = p_{p0} e^{-\frac{qy(x')}{kT}}$ the law of the junction.

The difference from the zero bias case is that at the edge of the junction of $x' = W$
$y(W) = V_{bi} - V_F$, not V_{bi} as in the zero bias case.

$$\therefore\ p_n(W) = p_{p0} e^{-q\frac{(V_{bi} - V_F)}{kT}}$$

or $p_n(W) = p_{p0} e^{-\frac{qV_{bi}}{kT}} \cdot e^{\frac{qV}{kT}}$

$$\boxed{p_n(0) = P_{n0} e^{\frac{qV_F}{kT}}}$$

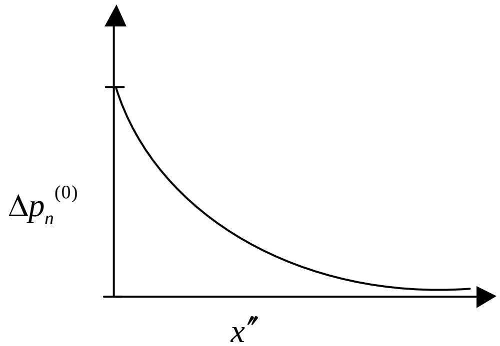
THE MINORITY CARRIER CONCENTRATION IS RAISED FROM ITS ZERO BIAS qv VALUE BY THE FACTOR $e^{\frac{qV_F}{kT}}$.

<u>IMPORTANT</u>

Similarly the minority carrier concentrations at the edge of the depletion region on the p-side

$n_p(-x_p) = n_{p0} e^{\frac{qV_F}{kT}}$

What happens to these excess carriers? They diffuse away from the edge of the depletion region to the bulk. The profile that governs the diffusion is set by the recombination rate of the minority carriers in the bulk. The situation is analyzed by the continuity equation.



At any point x'' the continuity equation states $\frac{1}{q}\nabla \cdot J_p^{(x)} = G - R$ Using, $J_p^{(x)} = J_p^{diff}(x)$

by neglecting drift currents (which will be explained later) we get $-D_p \frac{d^2 p_n^{(x'')}}{d_x^2} = -\frac{\Delta p_n^{(x'')}}{t_p}$

Note that $$\frac{d^2}{d_x^2} p_n(x'') = \frac{d^2}{d_x^2}\left(p_n(x'')^{-p_{n0}} \right) \text{ since } \frac{d^2}{d_x^2} p_{n0} = 0$$

$$\therefore \frac{d^2}{d_x^2} p_n(x'') = \frac{d^2}{d_x^2} \Delta p_n(x'')$$

$$D_p \frac{d^2 \Delta p_n(x'')}{d_x^2} - \frac{\Delta p_n(x'')}{t_p} = 0$$

We know that far away from the junction the <u>excess</u> hole concentration has to be zero since excess holes have to eventually recombine.

$\therefore \Delta p_n(x'') = c_1 e^{+\frac{x''}{L_p}} + c_2 e^{-\frac{x''}{L_p}}$

where $L_p = \sqrt{D_p t_p} =$ Diffusion length of the holes

which can be proven to be the average distance a hole diffuses before it recombines with an electron.

Also, $c_1 \equiv 0$ as $\Delta p_n \to 0$ as $x'' \to \infty$

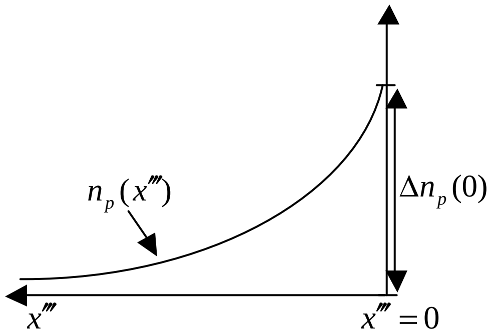
$$\therefore \Delta p_n(x'') = c_2 e^{-\frac{x''}{L_p}}$$

At $x'' = 0$ $\Delta p_n(0) = p_n(0) - p_{n0} = p_{n0} e^{\frac{qV_F}{kT}} - p_n$

Or $\boxed{\Delta p_n^{(0)} = p_{n0}\left(e^{\frac{qV_F}{kT}} - 1 \right)}$ IMPORTANT

$$\boxed{\Delta p_n(x'') = \Delta p_n(0) e^{-\frac{x''}{L_p}}} \qquad \text{-(2)}$$

This exponential relationship applies to the minority electrons as well where



$$\Delta n_p(x''') = \Delta n_p(0) e^{-\frac{x'''}{L_n}}$$

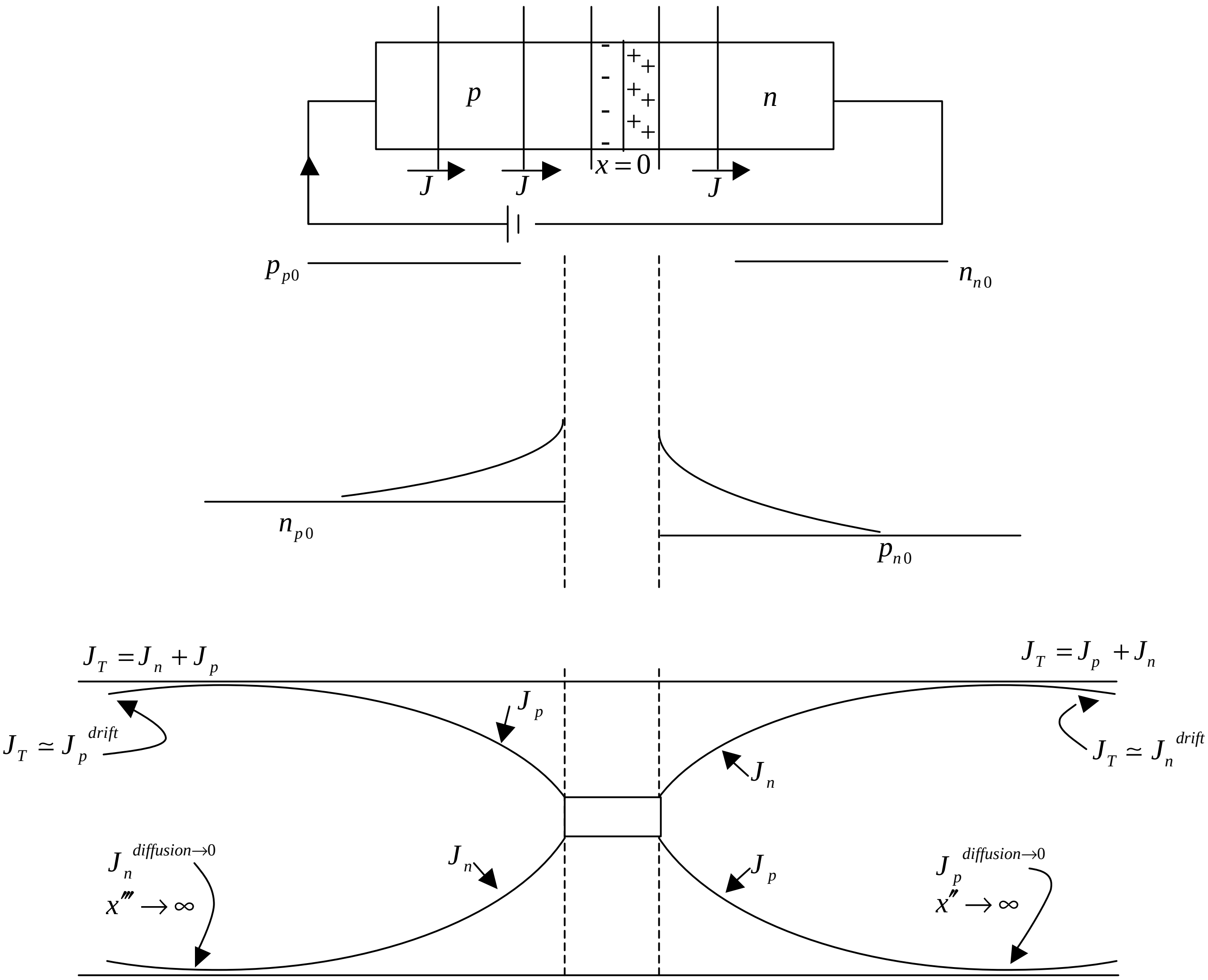
$$\Delta n_p(0) = n_{p0}\left(e^{\frac{qV_F}{kT}} - 1\right)$$

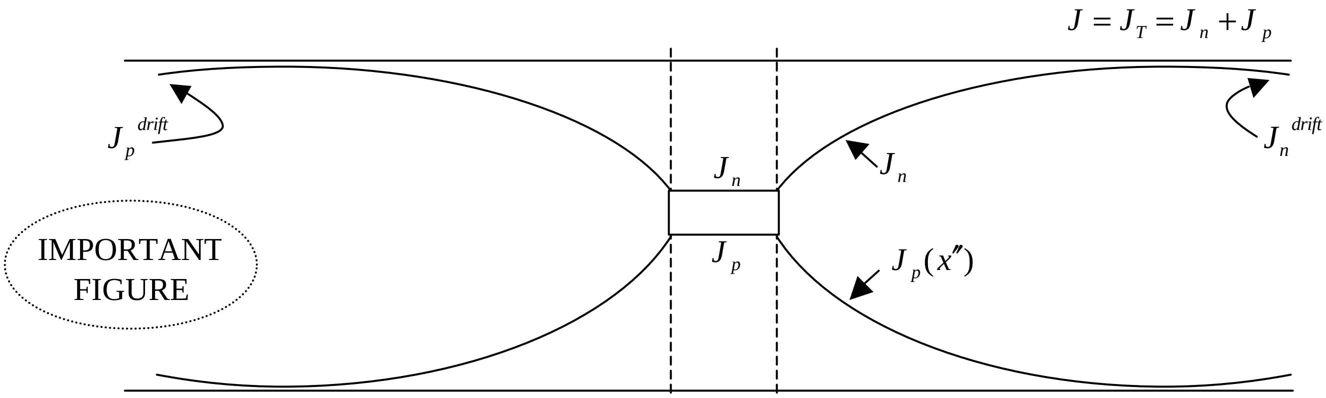
(New Coordinate System)

DERIVATION OF THE DIODE EQUATION UNDER FORWARD BIAS

We now have the minority carrier charge profiles [WHICH IS ALWAYS OBTAINED FROM THE SOLUTION OF THE CONTINUITY EQUATION]. From this we can calculate the current across the diode.

Note that the current measured anywhere in the diode has to be the same and equal to the current in the external circuit.





Observation: Far from the junction diffusion current $\rightarrow 0$ as $J_p(x'') = -qD_p \frac{d}{d_x} \Delta p_n(x'')$

$$= -qD_p \frac{d}{d_x}\left[\Delta p_n(0) e^{-\frac{x''}{L_p}} \right] = +q \frac{D_p}{L_p} \cdot \left[\Delta p_n(0) e^{-\frac{x''}{L_p}} \right]$$

$$\boxed{J_p^{diff}(x'') = q \frac{D_p}{L_p} \Delta p_n(x'')} \quad \text{-(3)}$$

Since $\Delta p_n(x'') \rightarrow 0$ as $x'' \rightarrow \infty$

$$J_p^{diff}(x'') \rightarrow 0 \text{ as } x'' \rightarrow \infty$$

Since J_T is always constant and $J_T = J_n + J_p$ in the region far from the junction

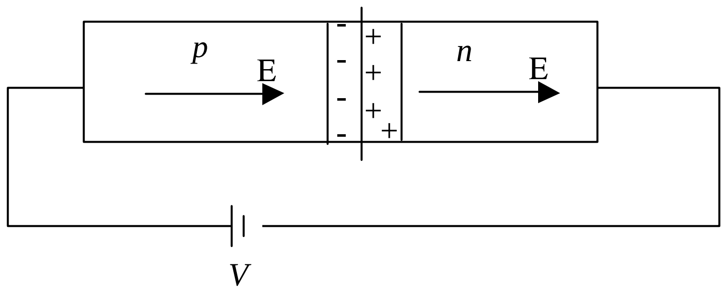
$$J_T = J_n^{drift} + \cancel{J_n^{diff}} + J_p^{drift} + \cancel{J_p^{diff}}$$

No slope in electron profile $n_{no} \neq f(x)$ (for J_n^{diff})

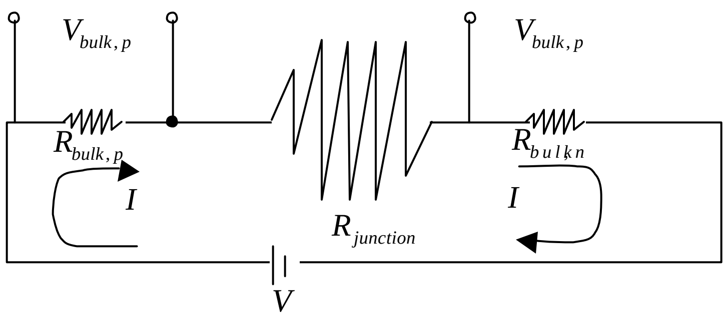
No slope in hole profile $p_{no} \neq f(x)$ (for J_p^{diff})

So what about J_p^{drift} or MINORITY CARRIER DRIFT CURRENTS?

We assumed the regions beyond the depletion regions were neutral. However, the application of a voltage across the diode must result in a field in these regions as shown below.



This can be readily understood if you break the diode into three regions, the bulk p, and the bulk n and depletion region. Since the bulk regions are highly doped they are conductive and hence have a small resistance. The depletion region, which is devoid of carriers, may be considered a large resistance. SCHEMATICALLY, we can consider the diode to be as shown below.



It can be readily seen that the voltage drop in the bulk regions are much smaller than the drop across the junction

$$\therefore\ J_T = J_n^{drift} + J_p^{drift}$$

$$= qm_n \cdot n_{n0} \cdot E + qm_p \cdot p_{n0} \cdot E \text{ (in the n-region)}$$

$$= J_n^{drift}\left[1 + \frac{m_p}{m_n} \cdot \frac{p_{n0}}{n_{n0}}\right]$$

$J_T \cong J_n^{drift}$ far from the junction as $J_p^{drift} << J_n^{drift}$ by the ratio $\frac{m_p}{m_n} \cdot \frac{p_{n0}}{n_{n0}}$

Since we are considering low level injection where $\frac{p_{n0}}{n_{n0}} \stackrel{\Delta}{\equiv} << 1$

<u>MINORITY CARRIER DRIFT CURRENTS CAN ALWAYS BE NEGLECTED</u>

<u>Far from the junction</u> the current is carried by majority carrier drift OR $J_p = J_p^{drift}$ in the p bulk and $J_T = J_n^{drift}$ in the n bulk. As is apparent from the figure as you get closer to the junction, because minority carrier diffusion is significant and $J_T = J_n + J_p$ is always true, J_n is always $J_T - J_p^{diff}$ in the n region. (J_n is always $J_n^{drift} + J_n^{diffusion}$ but it is not necessary at the moment to evaluate each component separately).

HERE COMES AN IMPORTANT ASSUMPTION:
We assume no recombination or generation in the depletion region. This is valid because the depletion region width, W, is commonly $<< L_n$ and also it will be proven later that recombination only occurs at the junction (x = 0) because of the carrier concentration profiles and hence is limited in extent. The latter is the correct reason so you have to accept it.

If this is true then the continuity equation dictates that $\frac{1}{q}\nabla \cdot J_p = G - R = 0 = \frac{1}{q}\nabla \cdot J_n$ OR BOTH J_n and J_p are constant across the junction (as shown).

So IF WE KNEW J_n and J_p in the depletion region then we could add them to get the total current

$J_T = J_n + J_p$ (everywhere) $= J_n + J_p$ (depletion region for convenience)

But we know J_p at the edge of the depletion region is only J_p^{diff} (on the n-side because the minority drift is negligible).

$\therefore J_p$ (depletion region) Assumption of no G-R in the depletion region $J_p^{diffusion}(x'' = 0)$

OR $J_p = q\frac{D_p}{L_p}\Delta p_n(0) \leftarrow$ from (2), (3)

$$J_p = q\frac{D_p}{L_p} p_{n0}\left(e^{\frac{qV_F}{kT}} - 1\right) \text{ - (4a)}$$

Similarly $$J_n = q\frac{D_n}{L_n} n_{p0}\left(e^{\frac{qV_F}{kT}} - 1\right) \text{ - (4b)}$$

$$J_T = J_n + J_p$$

$$J_T = q\left[D_p = \frac{p_{n0}}{L_p} + D_n\frac{n_{p0}}{L_n}\right]\left(e^{\frac{qV_F}{kT}} - 1\right)$$

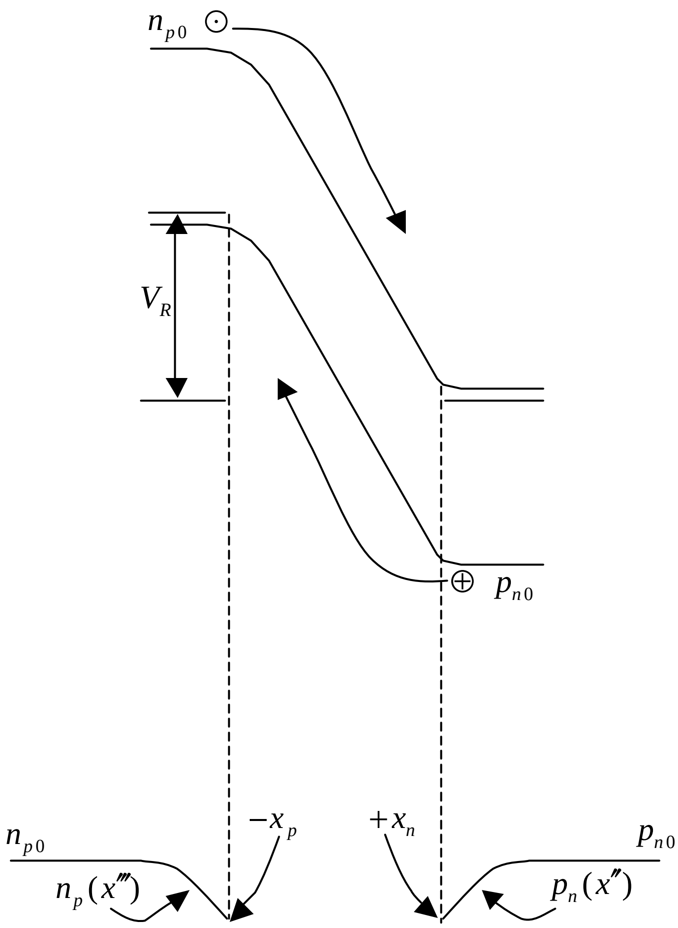
IMPORTANT

Note: The assumption of no G-R in the depletion region allowed us to sum the minority diffusion currents at the edges of the junction to get the total current. This does not mean that only diffusion currents matter. Current is always carried by carrier drift and diffusion in the device. The assumption allowed us to get the correct expression without having to calculate the electric field in the structure (a very hard problem).

So
$$\left.\begin{aligned} J_T &= J_s\left(e^{\frac{qV}{kT}} - 1\right) \\ J_s &= q\left[D_p\frac{p_{n0}}{L_p} + D_n\frac{n_{p0}}{L_n}\right] \end{aligned}\right\} \text{ - (b)}$$

$I_T = J_T \cdot A$ (A = Diode Area)

REVERSE BIAS CHARACTERISTICS



The case for reverse bias is very different.

Here the application of bias increases barriers. The only carriers that can flow are minority carriers that are aided by the electric field in the depletion region.

HERE the minority carriers are electrons injected from the p-region to the n-region (OPPOSITE TO THE FORWARD BIAS CASE).

First Observation:
Since we are only dealing with minority carrier currents we know that minority carrier drift can be neglected. Hence only minority carrier diffusion is relevant. To calculate diffusion currents we need to know the charge profile. Charge profiles are obtained by solving the continuity equation (in this case equivalently the diffusion equation as drift is negligible).
We assume that the large electric field in the reverse-biased p-n junction sweeps minority carriers away from the edge of the junction.

Assume $n_p\left(-x_p\right)=0$ SCHOCKLEY BOUNDARY CONDITIONS - (7)
And $p_n\left(+x_n\right)=0$

We also know that the minority carrier concentration in the bulk is n_{p0} (p-type) and p_{n0} (n-type) respectively. Therefore, the shape of the curve will be qualitatively as shown, reducing from the bulk value to zero at the depletion region edge.
Consider the flow of minority holes. The charge distribution is obtained by solving

$$D_p \frac{d^2 p_n}{dx''^2} + G_{th} - R = 0 \quad \text{-(8)}$$

assuming that the only energy source is thermal.

Then $D_p \frac{d^2 p_n}{dx''^2} + \frac{p_{n0} - p_n}{t_p} = 0$ -(9)

Note that this term is a generation term because $p_n < p_{n0}$ for all x''. This is natural because both generation and recombination are mechanisms by which the system returns to its equilibrium value. When the minority carrier concentration is above the equilibrium minority carrier value then recombination dominates and when the minority carrier concentration is less than that at equilibrium then generation dominates. The net generation rate is (analogous to the recombination rate).

$G_{th} - R = \frac{p_{n0} - p_n}{t_p}$ (ASSUMING t_p the generation time constant = recombination G-R = 0 which is????)

Note: when $p_n = p_{n0}$ true in equilibrium.

Again using $\Delta p_n(x'') = p_{n0} - p_n(x'')$ we get $D_p \frac{d^2 \Delta p_n(x'')}{dx''^2} + \frac{\Delta p_n(x'')}{t_p} = 0$

Again $\Delta p_n(x'') = C_1 e^{+\frac{x''}{L_p}} + C_2 e^{-\frac{x''}{L_p}}$

$C_1 = 0$ (for physical reasons)

At $x'' = \infty$ $\Delta p_n \to 0$

At $x'' = 0$ $\Delta p_n = p_{n0} - p_n(0) = p_{n0}$

$\therefore \Delta p_n(x'') = p_{n0} e^{\frac{-x''}{L_p}}$

$p_{n0} - p_n(x'') = p_{n0} e^{\frac{-x''}{L_p}}$

OR $\boxed{p_n(x'') = p_{n0}\left(1 - e^{\frac{-x''}{L_p}}\right)}$ -(10)

$\therefore$ The flux of holes entering the depletion region is $J_p(x'' = 0) = qD_p \frac{dp_n}{dx''}$ $(x'' = 0)$

$J_p = q\frac{D_p}{L_p} \cdot p_{n0}$ Similarly $J_n = q\frac{D_p}{L_p} n_{p0}$

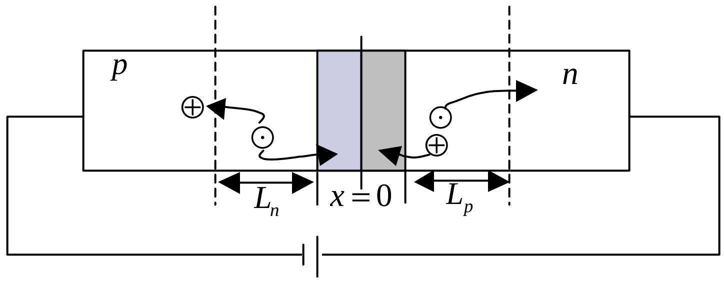
Assuming no generation in the depletion region the net current flowing is

$J_s = q\left[D_p \frac{p_{n0}}{L_p} + D_n \frac{n_{p0}}{L_n}\right]$ -(11)

This is remarkable because we get the same answer if we took the forward bias equation (valid only in forward bias) and arbitrarily allowed V to be large and negative (for reverse bias)

i.e. $J = J_s\left(e^{\frac{qV}{kT}} - 1\right)$ If V is large and negative $J_R = -J_s$ which is the answer we derived in 11.

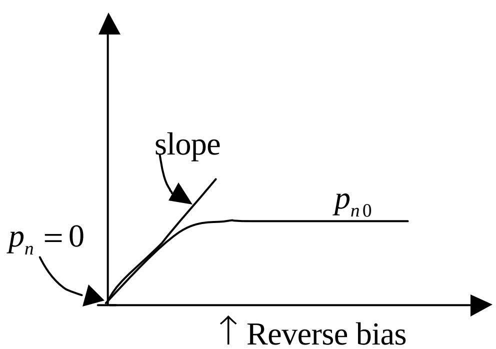
In summary,



The equation (11) can be understood as follows. Concentration on the p-region. Any minority carrier electrons generated within a diffusion length of the n, depletion edge can diffuse to the edge of the junction and be swept away. Minority electrons generated well beyond a length L_n will recombine with holes resulting in the equilibrium concentration, n_{p0}. Similarly holes generated within, L_p, a diffusion length, of the depletion region edge will be swept into the depletion region.

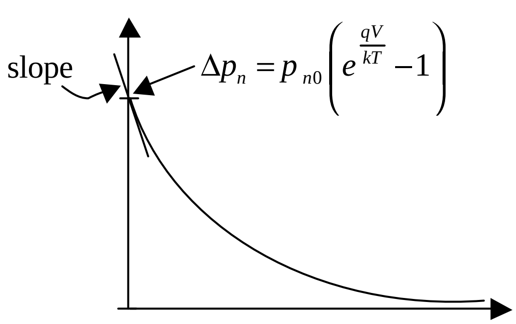
IMPORTANT OBSERVATION:

Look at the first term in equation 11. The slope of the minority carrier profile at the depletion edge = $\frac{p_{n0}}{L_p} = \frac{(\text{Difference from Bulk Value})}{L_p}$ This is always true when recombination and generation dominate.

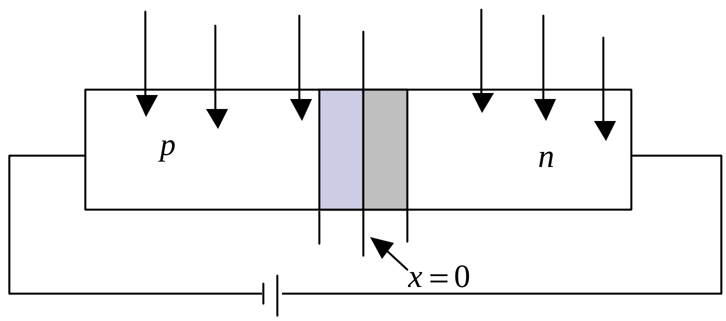


Recall that even in forward bias (shown below) the slope of the carrier profile is again

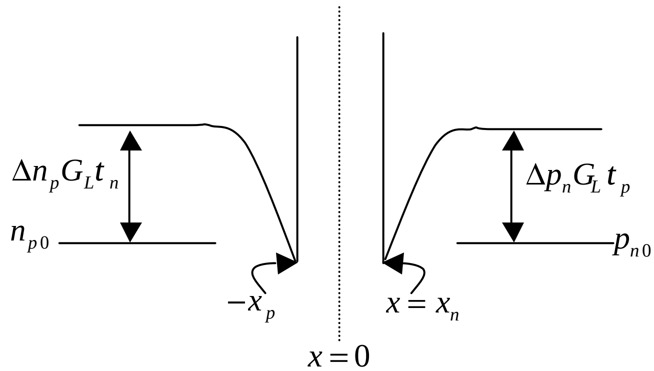
$$\frac{\text{Difference From Bulk Value}}{L_p} = \frac{\Delta p_n(0)}{L_p}$$



In the event that there is light shinning on the p-n junction as shown below



then the charge profile is perturbed in the following manner



where far in the bulk region an excess minority carrier concentration is generated where $\Delta n_p = G_L t_n$ and $\Delta p_n = G_L t_p$

The new equation to be solved for reverse saturation current differs from equation 8 in that a light generation term is added.

$$D_p \frac{d^2 p}{dx^2} + G_{th} - R + G_L = 0 \qquad \text{or} \qquad D_p \frac{d^2 p}{dx''^2} + \frac{p_{n0} - p_n}{t_p} + G_L = 0$$

only difference net thermal generation

with boundary conditions similar to before $p_n(\infty) = p_{n0} + t_p G_L$ and $p_n(0) = 0$

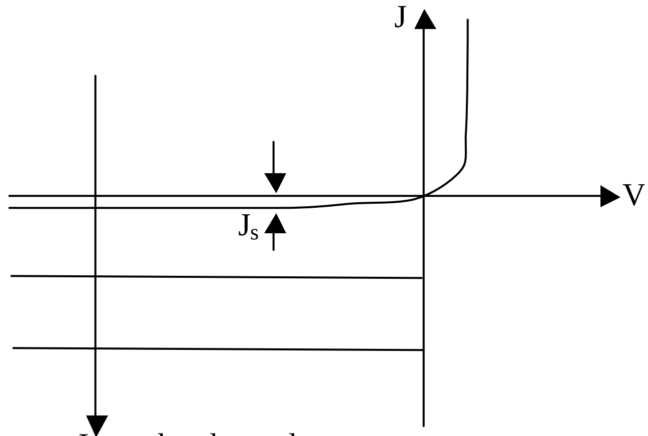
we get $p_{n0}(x'') = \underbrace{\left(p_{n0} + t_p G_L\right)}_{p_{bulk}} \left(1 - e^{\frac{-x}{L_p}}\right)$

The slope of the charge profile at the edge of the depletion region is $\frac{\partial p_n(0)}{\partial x''} = \frac{p_{n0} + t_p G_L}{L_p}$

$\therefore J_p = qD_p \cdot \left(\frac{p_{n0} + t_p G_L}{L_p}\right)$ Similarly, $J_n(x''' = 0) = q\frac{D_n}{L_n}\left(G_L t_n + n_{p0}\right)$

$$J_{\mathrm{Re}verse} = q\left[D_n \cdot \frac{n_{pbulk}}{L_n} + \frac{D_{pnbulk}}{L_p}\right]$$

By changing the slope of the minority profile at the edge of the junction I can control the reverse current across a diode.



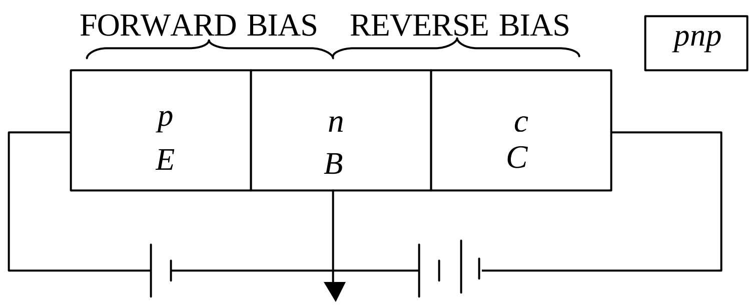
J_R can be charged by changing the slope of minority carrier profile.

Controlling and monitoring the current flowing across a reverse bias diode forms the bases of a large numb er of devices.

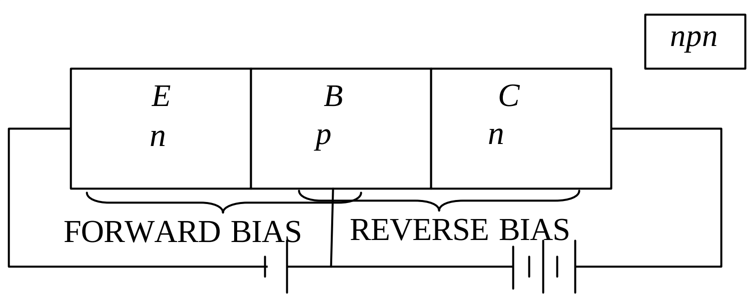
METHOD OF CONTROLLING THE MINORITY CARRIER SLOPE	DEVICE
1. TEMPERATURE: Recall that the slope is (for the p-material) given by $\frac{n_{p0}}{L_n}$ $J_{s,n} = qD_n \frac{n_{p0}}{L_n}$ But $n_{p0} = \frac{n_i^2}{p_{p0}}$ Assuming full ionization $p_{p0} \neq f^{(T)}$ and is always $\approx N_A$ $\therefore n_{p0} = \frac{n_i^2}{N_A} = \frac{N_c N_c e^{-\frac{Eg}{kT}}}{N_A}$ $\therefore J_{s,n}(T) = q \cdot \frac{D_n(T)}{L_n(T)} \cdot N_c(T) N_v(T) e^{-\frac{Eg}{kT}}$	THERMOMETER Read J_s and extract T
EXPONENTIAL DEPENDENCE ON T (the other terms have weaker dependence).	

2. Change n_{p0} to $n_p = n_{p0} + G_L t_n$ as described. The generation rate is dependent on the input photon flux. $\therefore J_{s,n} = q \frac{D_n}{L_n}\left(n_{p0} + G_L t_n\right)$ Measure $J_{s,n}$ Determine G_L and input photon flux	PHOTODETECTOR (used in optical communications)
3. Change the minority carrier concentration by an electrical minority carrier injector i.e. a p-n junction.	TRANSISTOR

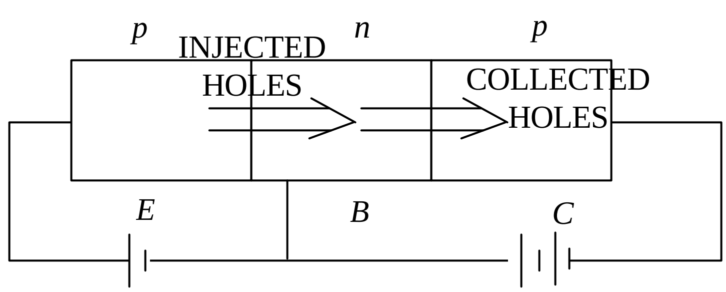
A TRANSISTOR, short for TRANSfer ResISTOR, is the basic amplifying element in electronics. The basis of its amplification and its structure is shown below.



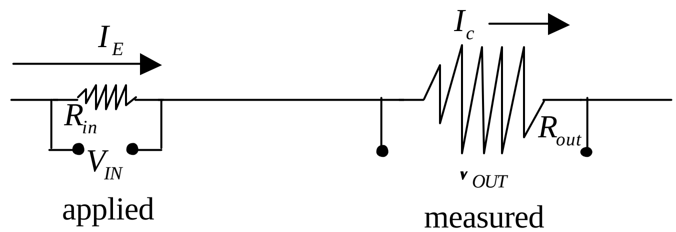
A transistor consists of a forward bias junction in close proximity to a reverse bias p-n junction so that carriers



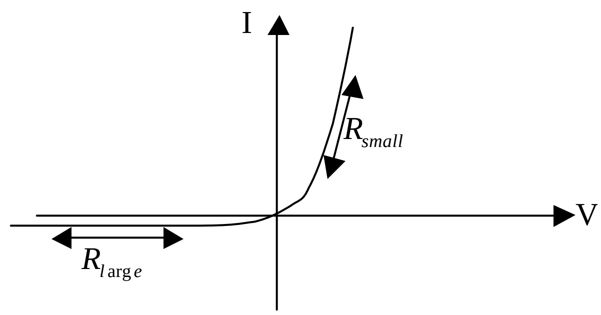
injected from forward bias junction (from the emitter labeled E) can travel through the intermediate layer (called the BASE and labeled B) and across the reverse biased junction into the COLLECTOR, labeled C.
Schematically for the case of the emitter being a p material and the base being n-type, (a pnp transistor) the DOMINANT CURRENT FLOW (to be modified later is shown below).



The holes injected from the p-type emitter contribute to the EMITTER CURRENT, I_E, and the holes collected contribute to the collector current, I_c. These currents in the diagram above are equal (AN APPROXIMATION TO BE MODIFIED LATER). This situation is equivalent to



The forward bias junction across which the input voltage is applied has a low resistance.



The forward bias resistance of a diode can be readily calculated from the I-V characteristics

$$I = I_s\left(e^{\frac{qV}{kT}} - 1\right) \simeq I_s e^{\frac{qV}{kT}}$$

$$\therefore R = \frac{\partial V}{\partial I} = \left(\frac{\partial I}{\partial V}\right)^{-1} = \left(\frac{q}{kT}\cdot I_s e^{\frac{qV}{kT}}\right) \text{ OR } R = \left(\frac{q}{kT}\cdot I\right)^{-1} \text{ OR } \boxed{R = \frac{kT}{qI}} \text{ IMPORTANT}$$

As I increases R decreases.

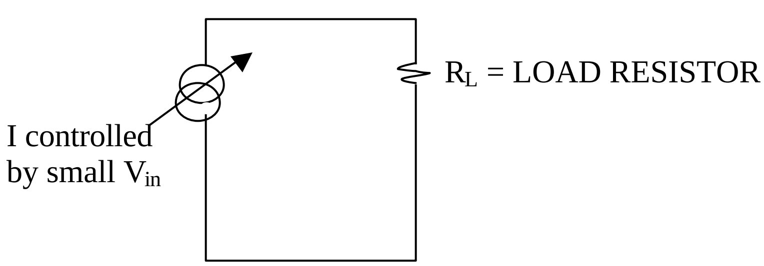
Note that @ room temperature at a current of 25 mA $R = \frac{25mV}{25mA} = 1\Omega$ FOR ANY DIODE

Therefore in a transistor a SMALL INPUT VOLTAGE can generate a large current, I_E, because of the small R_{in}. This same current flows across a large resistance (as the reverse bias resistance of the collector junction is large) as I_c.

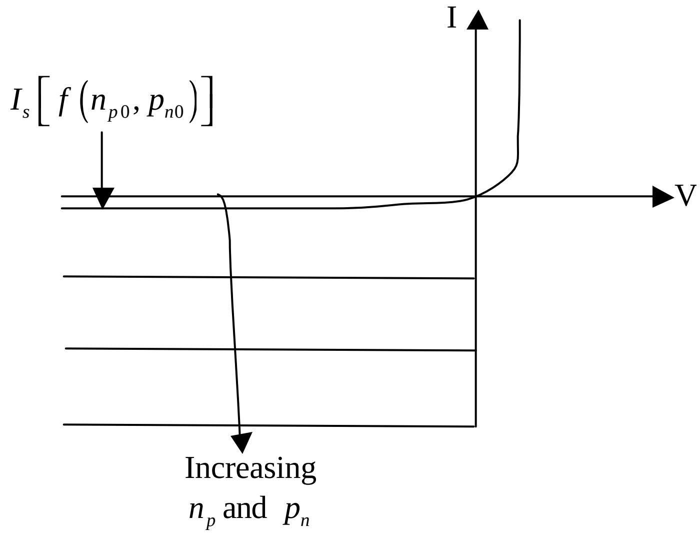
$\therefore$ The measured voltage across the output resistance, R_{out}, is $I_c \cdot R_{out}$. The input voltage is $I_E \cdot R_{in}$. $\therefore$ The voltage gain of device, A_v, is

IMPORTANT $\boxed{A_v = \frac{V_{out}}{V_{in}} = \frac{I_c R_{out}}{I_E R_{in}} \simeq \frac{R_{out}}{R_{in}}}$

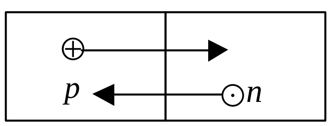
<u>In general</u>, if you can control a current source with a small voltage then you can achieve voltage amplification by passing the current through a large load resistor.



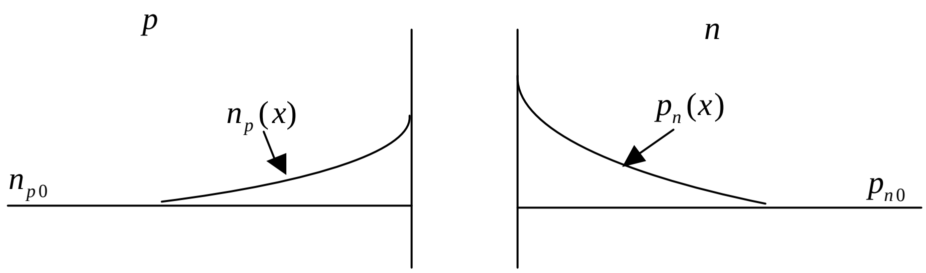
Recall that we could modulate the reverse bias current by changing the minority carrier concentration flux injected into the junction. The diagram is reproduced below.



A transistor achieves modulating the minority concentration by injecting minority carriers using a forward biased p-n junction. Recall that a p-n junction injects holes from a



p-region into the n-region (raising) the minority carrier concentration from p_{n0} to a new value $p_n(x)$



The same applies for the n-region. Let us now consider a p-n-p transistor. It consists of a p-n junction that injects minority carriers into another but now reverse-biased p-n junction.