1. **WE WANT** $I_E = I_c$ so that the fundamental basis of transistor action, that of a current controlled by a small voltage flowing across a large resistor to generate a large voltage is maintained.

This is not true because,

a. **Emitter Injection efficiency**, $\alpha = \frac{I_{E\text{p}}}{I_{E\text{i}}}$ is composed to two current components, $I_{E\text{p}}$ and $I_{E\text{n}}$. $I_{E\text{p}}$ is desirable because it is the component that injects minority carriers into the B-C junction and emerges as $I_c$. $I_{E\text{n}}$ is the electron current injected into the emitter from the base. This is a parasitic current as it does not contribute to collector current, $I_c$. Therefore we define a figure of merit (which we will calculate shortly) called **EMITTER INJECTION EFFICIENCY**, $\alpha$, where

$$\alpha = \frac{I_{E\text{p}}}{I_E} = \frac{I_{E\text{p}}}{I_{E\text{p}} + I_{E\text{n}}}$$

The maximum (and desirable) value of $\alpha$ is 1 and is achieved when $I_{E\text{n}} \to 0$.

b. **BASE TRANSPORT FACTOR**, $\alpha_T = \frac{I_{E\text{p}}}{I_E}$ Of the hole flux emitted from the emitter, a certain fraction is lost due to recombination in the n-type base. However, if the width of the neutral base, $W_B$ is made much smaller than the average distance a minority carrier travels before recombining, $L_p$, then the probability of recombination is reduced.
So what does the charge profile in the base region look like? USE THE CONTINUITY EQUATION. In the absence of electric field this reduces to the diffusion equation

\[ D_p \frac{d^2 p_n}{dx^2} = \text{Net recombination} \equiv 0 \text{ if } W_B << L_p \]

Use a coordinate system shown in the figure above where the neutral base commences at \( x = 0 \) and ends @ \( x = W_B \). The solution of

\[ D_p \frac{d^2 p_n}{dx^2} = 0 \]

is the same as \( D_p \frac{d^2 \Delta p_n(x)}{dx^2} = 0 \) and is \( \Delta p_n(0) \left[ 1 - \frac{x}{W_B} \right] \).

This is a linear decrease in the charge profile with boundary conditions that \( \Delta p_n @ x = 0 \) is \( \Delta p_n(0) \). Therefore the charge profile in the bipolar transistor can be pictured as

\[
\begin{align*}
\Delta n_p(x') &= \Delta n_p(0)e^{-\frac{x'}{L_n}} \\
n_p(0) &= e^{-\frac{x'}{L_n}} \\
p_n(x) &= \Delta p_n(0) \left[ 1 - \frac{x}{W_B} \right] \\
p_n(x) &= n_p0 \left( 1 - e^{-\frac{x'}{L_n}} \right)
\end{align*}
\]

The salient features are

(i) An exponential drop-off in the injected minority profile as the emitter thickness is much larger than a diffusion length for electrons

\[
\Delta n_p(x') = \Delta n_p(0)e^{-\frac{x'}{L_n}} \text{ [like a conventional diode]}
\]

(ii) A linear decrease in the minority (hole concentration in the base)

\[
\Delta p_n(x) = \Delta p_n(0) \left[ 1 - \frac{x}{W_B} \right] \text{ because we neglected recombination.}
\]

(iii) A traditional decrease in the minority carrier concentration \( n_p(x) \) in the collector as determined by reverse biased conditions \( n_p(x^*) = n_p0 \left( 1 - e^{-\frac{x^*}{L_n}} \right) \)

NOW WE CAN CALCULATE ALL THE CURRENTS BECAUSE WE KNOW THE CHARGE PROFILES.

Recall that we want \( I_c = I_E \) which requires \( \alpha = 1 \) and \( \alpha_T = 1 \). In general the current gain \( \alpha \) is defined as \( \alpha = \alpha_T \cdot \alpha \) and \( I_c = \alpha I_E \) WE WANT \( \alpha = 1 \)
CALCULATING $\alpha$  \((A = 1 cm^2)\)

$$\alpha = \frac{I_{Ep}}{I_{Ep} + I_{EN}} = \frac{1}{1 + \frac{I_{EN}}{I_{Ep}}}$$

$$I_{Ep} = qD_p \frac{dp_n}{dx} \bigg|_{x=0} : I_{EN} = qD_n \frac{dn_p}{dx} \bigg|_{x=0}$$

$$I_{Ep} = qD_p \frac{\Delta p_n(0)}{W_B} : I_{En} = qD_n \frac{\Delta n_p(0)}{L_n}$$

OR $$I_{Ep} = qD_p \frac{p_n 0}{W_B} \left( e^{\frac{qV_{EB}}{kT}} - 1 \right) : I_{En} = qD_n \left( e^{\frac{qV_{En}}{kT}} - 1 \right)$$

$$\therefore \frac{I_{En}}{I_{Ep}} = \frac{D_n}{D_p} \cdot n_{p0} 0 \cdot W_B \cdot L_n$$

Using the Law of Mass Action

$$n_{p0} = \frac{n_i^2}{N_{AE}} \text{ & } p_{n0} = \frac{n_i^2}{N_{DB}}$$

where \(N_{AE}\) = Acceptor concentration in the p-type emitter

& \(N_{DB}\) = Donor concentration in the base

$$\therefore \frac{I_{En}}{I_{Ep}} = \frac{D_n}{D_p} \cdot \frac{N_{DB}}{N_{AE}} \cdot \frac{W_B}{L_n}$$

For $\alpha \to 1$ we need $\frac{I_{En}}{I_{Ep}} \to 0$

$$\therefore \text{ We Need}$$

\[
\begin{array}{c|c}
\frac{W_B}{L_n} & \text{for } \alpha \to 1 \\
\frac{N_{AE}}{N_{DB}} & \gg 1 \\
\end{array}
\]

Calculating BASE TRANSPORT FACTOR, $\alpha_T$.

$$\alpha_T = \frac{I_{cp}}{I_{Ep}} = \frac{\text{collected hole current}}{\text{injected hole current}}$$

Also $\alpha_T = \frac{I_{Ep} - I_{Br}}{I_{Ep}}$

where $I_{Br}$ is the hole current lost due to recombination in the base.
We assumed for calculating the hole profile in the base the recombination was zero. This gave us a linear profile. In actuality, the finite (through very small) recombination that does occur (as \( \frac{W_B}{L_p} \ll 1 \) but not 0) causes the hole profile to sag as shown below because holes are lost to recombination.

**Bottom line:** the loss of holes perturbs the profile a bit but it is essentially linear therefore validating our assumption.

Or Neglecting recombination is fine to calculate the collector current as the slope of the profile (which determines \( I_{cp} \)) is not perturbed substantially.

\[
i.e. \quad I_{cp} = I_{Ep} = qD_p \frac{\Delta p_n(0)}{W_B}
\]

But to calculate \( I_{Br} \), the recombination current arises from the re-supply of electrons in the base lost to recombination with the holes. The rate at which holes recombine is given by \( \frac{Q_p}{\tau_p} \).

\[
\Delta p_n(0)
\]

\[
Q_p \quad W_B \quad \text{Base}
\]

Where \( Q_p \) is the stored charge of holes in the base and \( \tau_p \) is the lifetime of the minority holes.

\( Q_p \) can be calculated as

\[
Q_p = \frac{1}{2} \Delta p_n(0) \cdot W_B \quad \text{Area of the charge triangle}
\]
We now define current gain, \( \beta \), to be \( \beta = \frac{I_c}{I_B} = \frac{I_{cp}}{I_{En} + I_{Br}} \) neglecting \( I_{En} \) (by properly designing \( \delta \to 1 \)) gives us \( \beta = \frac{I_{cp}}{I_{Br}} \). But \( I_{cp} \equiv I_{Ep} = qD_p \frac{\Delta p_n(0)}{W_B} \)

\[
\beta = \frac{I_{cp}}{I_{Ep}} = \left( qD_p \frac{\Delta p_n(0)}{W_B} \right) \div \left( \frac{1}{2} \Delta p_n(0) \frac{W_B}{\tau_p} \right)
\]

\[
\beta = 2 \frac{D_p \tau_p}{W_B} \cdot \frac{I_p^2}{W_B^2} \text{ as } D_p \tau_p = L_p
\]

for \( \delta \to 1 \)

The relationship between \( \beta \) and \( \alpha \)

\[ \beta = \text{COMMON-EMITTER CURRENT GAIN} \]

\[ \alpha = \text{COMMON-BASE CURRENT GAIN} \]

![Diagram of transistor configuration]

**COMMON-BASE**

\[ V_{in} = \text{Input voltage between Emitter and Base} \]

\[ V_{out} = \text{Output voltage measured between Collector and Base} \]

**BASE** is the common terminal

\[ I_{in} = I_E ; \quad I_{out} = I_c \quad I_c = \alpha I_E \]

**Note:**

\[ \alpha = \alpha_r \cdot \delta \]

If \( \delta \to 1 \)

then \( \alpha = \alpha_r \)

**Common Emitter Configuration**

Here \( V_{in} \) is applied between the base and emitter and the output voltage measured between collector and emitter. The emitter is the common terminal. The input current, \( I_{in} = I_B \) and the output current, \( I_{out} = I_c \)

\[
\therefore \frac{I_{out}}{I_{in}} = \frac{I_c}{I_B} = \beta
\]
In general

\[ I_E - I_B = I_c \] (Kirchoff's Law)

using \( I_E = \frac{I_c}{\alpha} \) & \( I_B = \frac{I_c}{\beta} \)

\[ \frac{I_c}{\alpha} = \frac{I_c}{\beta} = I_c \]

\[ \frac{1}{\alpha} = \frac{1}{\beta} = 1 \]

OR \( \beta = \frac{\alpha}{1-\alpha} \) Typically: \( \beta > 10, \alpha > 0.9 \)

Therefore if \( \delta = 1 \) then since \( \alpha = \alpha_r \) we get from \( \beta = \frac{\alpha}{1-\alpha} \), \( \alpha = \frac{\beta}{\beta + 1} \equiv \alpha_r \)

\[ \therefore \alpha_r = \frac{2L_p^2}{W_B^2} = \frac{2L_p^2}{1 + \frac{2L_p^2}{W_B^2}} \]

as \( W_B \to 0 \alpha_r \to 1 \) as expected because recombination \( \to 0 \)

**SUMMARY** **DESIGNING GOOD BIPOLAR TRANSISTORS**

\( \delta \to 1 \Rightarrow \frac{N_{p}}{N_{p_{BB}}} \gg 1 \)

\[ \frac{W_B}{L_p} \ll 1 \]

\( \alpha \to 1 \Rightarrow \frac{W_B}{L_p} \ll 1 \)

DECREASE THE BASE THICKNESS, INCREASE THE BASE MINORITY CARRIER LIFETIME, INCREASE EMITTER DOPING.