p-n junctions

A p-n junction is a metallurgical and electrical junction between p and n materials. When the materials are the same the result is a HOMOJUNCTION and if they are dissimilar then it is termed a HETEROJUNCTION.

Junction Formation:

1. Majority carriers diffuse [holes from p to n and electrons from n → p]

2. Bare ionized dopants are exposed on either side of the junction. Positively charged donors on the n-side and negatively charged acceptors on the p-side.

3. The dopant ions are contained in a region of reduced carrier concentration (as the mobile majority charges have diffused as stated in (1)). This region is therefore called the depletion region.

4. The process of diffusion continues until the depletion region expands to a width, \( W(0) \), such that the electric field in the depletion region \( E_{\text{depl}} \) is large enough to repel the diffusing carriers. More precisely,

\[
J_{\text{diffusion}} = J_{\text{drift}}
\]

(once equilibrium has been established)
5. The driving force for carrier motion is the ELECTRO-CHEMICAL POTENTIAL DIFFERENCE that exists between the two semiconductors in the bulk prior to junction formation.

In band diagram terms here are the before and after pictures:

**BEFORE: THE TWO MATERIALS ARE SEPARATE**

**Definitions:**

a) $q_x$ = Electron affinity in units of energy. (use eV or Joules)

b) $E_{Fp}$ = Fermi Level in the p-type material or electro-chemical potential of the p-type material.

c) $E_{Fn}$ = Fermi Level in the n-type material or electro-chemical potential of the n-type material.

**NOTE: THIS IS THE ELECTRO-CHEMICAL POTENTIAL OF ELECTRONS IN BOTH CASES**

d) $q\varnothing_p$ and $q\varnothing_n$ are the work functions of the two materials (p and n) respectively.

e) Note that the work function difference between the two materials $q(\varnothing_p - \varnothing_n)$ is the difference between the electro-chemical potentials of the bulk materials $E_{Fn}$ and $E_{Fp}$.
AFTER: The materials are brought together to form a junction. The fermi levels $E_{Fn}$ and $E_{Fp}$ now equalize or $E_{Fn} = E_{Fp} = E_F$ (IN EQUILIBRIUM)

a) Assume the p-material is kept at a constant potential (say ground).

b) The p-material has to increase its electro-chemical potential of electrons (upward motion of the bands) until the fermi levels line up as shown in the diagram below where the effect is simulated using two beakers of water in equilibrium with different amounts of water in each beaker.

c) The lowering of the electron energy of the n-type semiconductor is accompanied by the creation of the depletion region.
d) The depletion region has net charge and hence the bands have curvature following Gauss’ Law:

\[ \frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon} \quad E = \text{Electric Field} \]

OR \[ \frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\varepsilon} \quad E = -\frac{\partial V}{\partial x} \]

where \( V \) = Potential energy (of unit positive charge)

OR \[ \frac{\partial^2 E_c}{\partial x^2} = \frac{\rho}{\varepsilon} \quad E_c = -qV = \text{Electron energy (Joules)} \]

or \( -V \) = Electron energy (eV)

<table>
<thead>
<tr>
<th>NOTE</th>
<th>NET CHARGE ⇔ CURVATURE OF THE BANDS</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>NEUTRAL REGIONS ⇒ NO NET CHARGE ⇔ BANDS HAVE NO CURVATURE</td>
</tr>
<tr>
<td></td>
<td>DO NOT CONFUSE SLOPE WITH CURVATURE</td>
</tr>
<tr>
<td></td>
<td>Neutral regions can have constant slope or equivalently no curvature</td>
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</tbody>
</table>
CALCULATING THE RELEVANT PARAMETERS OF A p-n JUNCTION

1. SCHEMATIC OF THE JUNCTION

2. CHARGE PROFILE

3. ELECTRIC FIELD

4. BAND DIAGRAM

NOTE: $qV_{bi} = q\Phi_{fp} + q\Phi_{fn}$
In the analysis depicted in the four diagrams above, we assume
(i) that the doping density is constant in the p-region at $N_A$ and in the n-region at $N_D$ and that the change is abrupt at $x = 0$, the junction.
(ii) the depletion region has only ionized charges $-qN_A \text{ (C cm}^{-3})$ in the p-region and $+qN_D \text{ (C cm}^{-3})$ in the n-region. [Mobile charges in the depletion region are neglected, i.e. $n$, $p \ll N_A^-$ and $N_D^+$]
(iii) The transition from the depletion region to the neutral region is abrupt at $(x_p)$ in the p-region and $(x_n)$ in the n-region.

Calculation of the built-in voltage

From the band diagram it is clear that the total band bending is caused by the work function difference of the two materials. If you follow the vacuum level (which is always reflects the electrostatic potential energy variation and hence follows the conduction band in our homojunction), you see that the band bending is the difference of the work function of the p-type material $q\phi_p$, and the n-type material $q\phi_n$.

$$qV_{bi} \sim q\phi_p - q\phi_n$$

The built-in potential is therefore the internal potential energy required to cancel the diffusive flow of carriers across the junction and should be exactly equal to the original electro-chemical potential which caused the diffusion in the first place. THIS IS REASSURING.

To calculate $V_{bi}$ from parameters such as doping let us follow the intrinsic level from the p-side, $E_{ip}$, to the n-side, $E_{in}$. Again the total band bending of the intrinsic level is the built in potential

$$E_{ip} - E_{in} = qV_{bi}$$

or

$$\left(E_{ip} - E_F\right) - \left(E_{in} - E_F\right) = qV_{bi}$$

Defining $E_{ip} - E_F$ as $q\phi_F$ [Note in equilibrium
And $E_{Fn} - E_{in}$ as $q\phi_F$ $E_{Fn} = E_F = E_F$

We can rewrite $qV_{bi}$ as $q\phi_{Fn} + q\phi_{Fp} = qV_{bi}$

From Fermi-Dirac Statistics:

$$p_{po} = n_i e^{\frac{E_i - E_F}{RT}} \quad \text{or} \quad n_i e^{\frac{E_{ip} - E_{Fp}}{kT}} = n_i e^{q\phi_F}$$

similarly $n_{no} = n_i e^{\frac{E_i - E_F}{RT}}$

Assume full ionization
\[ p_{po} = N_A \text{ and } n_{no} = N_D \]

\[ \therefore N_A = n_i e^{\frac{q \mathcal{D}_{fp}}{kT}} \quad \& \quad N_D = n_i e^{\frac{q \mathcal{D}_{fn}}{kT}} \]

or \[ q \mathcal{D}_{fp} = kT \ln \frac{N_A}{n_i} \quad \& \quad q \mathcal{D}_{fn} = kT \ln \frac{N_D}{n_i} \]

\[ \therefore q \mathcal{D}_{fp} + q \mathcal{D}_{fn} = kT \left[ \ln \frac{N_A}{n_i} + \ln \frac{N_D}{n_i} \right] \]

or \[ qV_{bi} = kT \ln \frac{N_A N_D}{n_i^2} \]

\[ V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \]

### Calculating Depletion Region Widths

From the electric field diagram, Figure 3

\[ E_{max} = -\frac{qN_A}{\epsilon} x_p \]

\[ |E_{max}| = \frac{qN_A}{\epsilon} \cdot x_p \]

The magnitude of the area under the electric field versus distance curve (shaded area in figure 3) is by definition \[ \int_{-x_p}^{x_n} E \cdot dx = \text{Voltage difference between } -x_p \text{ and } +x_n = V_{bi} \]

or \[ V_{bi} = \frac{1}{2} W |E_{max}| \]

\[ V_{bi} = \frac{1}{2} \left( x_n + x_p \right) \cdot \frac{qN_A}{\epsilon} x_p \]

We now invoke charge neutrality. Since the original semiconductors were charge neutral the combined system has to also be charge neutral (since we have not created charges). Now since the regions beyond the depletion region taken as a whole has to be charge neutral. OR all the positive charges in the depletion region have to balance all the negative charges. If the area of the junction is \( A \text{ cm}^2 \) then the number of positive charges within the depletion region from \( x = 0 \) to \( x = x_n \) is

\[ qN_D \cdot x_n \cdot A = \text{Coulombs} \]

Similarly, all the negative charges contained in the region between \( x = -x_p \) and \( x = 0 \) is \[ qN_A \cdot x_p \cdot A = \text{Coulombs} \]
charge neutrality therefore requires
\[ qN_Ax_p \cdot A = qN_Dx_n \cdot A \quad \text{or} \quad \frac{N_Ax_p}{N_Dx_n} \]

To calculate \( w \), \( x_n \) and \( x_p \) we use the above relation in the equation for \( V_{bi} \) below
\[ V_{bi} = \frac{1}{2} \cdot (x_n + x_p) \cdot E_{\text{max}} \]

### Table: \( p \)-side and \( n \)-side

<table>
<thead>
<tr>
<th>( p )-side</th>
<th>( n )-side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{bi} = \frac{1}{2} (x_n + x_p) \cdot \frac{qN_A}{\varepsilon} x_p )</td>
<td>( V_{bi} = \frac{1}{2} (x_n + x_p) \cdot \frac{qN_D}{\varepsilon} x_n )</td>
</tr>
<tr>
<td>( \text{Substituting for} \ x_n )</td>
<td>( \text{Substituting for} \ x_n )</td>
</tr>
<tr>
<td>( \text{or} \ V_{bi} = \frac{1}{2} \left( \frac{N_A + N_D}{N_D} + x_p \right) \cdot \frac{qN_A}{\varepsilon} x_p )</td>
<td>( \text{or} \ V_{bi} = \frac{1}{2} \left( \frac{N_D + N_D}{N_A} + x_n \right) \cdot \frac{qN_D}{\varepsilon} x_n )</td>
</tr>
<tr>
<td>( \frac{2\varepsilon}{qN_A} V_{bi} = \left( \frac{N_A + N_D}{N_D} \right) x_p^2 )</td>
<td>( \frac{2\varepsilon}{qN_D} V_{bi} = \left( \frac{N_A + N_D}{N_A} \right) x_n^2 )</td>
</tr>
</tbody>
</table>

**NOTE**: in this analysis \( |E_{\text{max}}| \) was calculated as \( \frac{qN_A}{\varepsilon} x_p \) from the \( p \)-side and \( \frac{qN_D}{\varepsilon} x_n \) from the \( n \)-side

\[ x_p = \sqrt[22]{\frac{2\varepsilon}{q} \cdot N_D \cdot \frac{1}{N_A + N_D} \cdot V_{bi}} \]
\[ x_n = \sqrt[22]{\frac{2\varepsilon}{q} \cdot N_D \cdot \frac{1}{N_A + N_D} \cdot V_{bi}} \]

\[ W = x_n + x_p \]
\[ = \sqrt[22]{\frac{2\varepsilon}{q} \cdot V_{bi} \cdot \left[ \sqrt{\frac{N_D}{N_A + N_D}} + \frac{\sqrt{N_A}}{\sqrt{N_A + N_D}} \right]} \]
\[ = \sqrt[22]{\frac{2\varepsilon}{q} \cdot V_{bi} \cdot \left[ \frac{N_A}{N_A N_D} \cdot \frac{N_D}{N_A + N_D} \right]} \]
\[ W = \sqrt[22]{\frac{2\varepsilon}{q} \cdot \frac{N_A + N_D}{N_A N_D} \cdot V_{bi}} \]

**IN EQUILIBRIUM** \[ W(0) = \sqrt[22]{\frac{2\varepsilon}{q} \cdot \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \cdot V_{bi}} \]

**IMPORTANT**

In general
\[ W(v) = \sqrt[22]{\frac{2\varepsilon}{q} \cdot \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \cdot (V_{bi} \pm |V|)} \]

\( V \) is the applied bias \( + |V| \) in reverse bias (\( w \) expands)
- $|V|$ in forward bias (w shrinks)

<table>
<thead>
<tr>
<th>FORWARD BIAS</th>
<th>REVERSE BIAS</th>
</tr>
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<tbody>
<tr>
<td>$-x_{po}$</td>
<td>$-x_{po}$</td>
</tr>
<tr>
<td>$x = 0$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>$x_{no}$</td>
<td>$x_{no}$</td>
</tr>
<tr>
<td>$q(V_{bi} - V)$</td>
<td>$V_{bi} +</td>
</tr>
<tr>
<td>$qV_F$</td>
<td>$E_{Fn}$</td>
</tr>
<tr>
<td>$x = 0$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>$-x_p$</td>
<td>$-x_p$</td>
</tr>
<tr>
<td>$-x_n$</td>
<td>$-x_{po}$</td>
</tr>
<tr>
<td>$qN_A$</td>
<td>$qN_A$</td>
</tr>
<tr>
<td>$qN_D$</td>
<td>$qN_D$</td>
</tr>
<tr>
<td>$E_{max} = \frac{qNA_x_p}{\epsilon}$</td>
<td>$E_{max} = \frac{qNA_x_p}{\epsilon}$</td>
</tr>
</tbody>
</table>

FEATURES:
1. Total Band bending is now $V_{bi} - V$
2. The shaded area $\int E \cdot dx = V_{bi} - |V|$ |
3. The edges of the depletion region move towards the junction or W decreases.
4. $E_{Fn} - E_{FP} = V_F$, the electrochemical potentials separate by an amount equal to the potential difference applied, $V_F$.

FEATURES:
1. Total Band bending is now $V_{bi} + |V|
2. The shaded area $\int E \cdot dx = V_{bi} + |V|
3. Depletion region expands.
4. $E_{FP} - E_{Fn} = |V_R|$ = amount of potential difference, ($V_R$).
Under forward bias electrons from the n-region and holes from the p-region cross the junction and diffuse as minority carriers in the p and n-regions respectively. To understand how excess minority carriers are injected and diffuse one has to understand the LAW OF JUNCTION which now follows.
Consider the two situations shown below one at zero bias and the other under forward bias.

Note that $E_i$ is a function of $x'$ and is $E_{ip}$ in the bulk $p$ and $E_{in}$ in the bulk $n$.

Note that $p(x')$ is always given by

$$p(x') = n_i e \frac{E_i(x') - E_{fp}(x')}{kT}$$

Where $E_i(x)$ and $E_{fp}(x)$ are the intrinsic fermi-level and the fermi level for holes at any place $x$.

Since we are at zero bias and at equilibrium $E_{fp}$ is not a function of $x'$.

$$\therefore p(x') = n_i e \frac{E_i(x') - E_F}{kT} = n_i e \frac{E_{ip} - E_F - E_{ip} - E_F}{kT}$$

$$= n_i e \frac{E_{ip} - E_F}{kT} \cdot e^{-\frac{E_{ip} - E_i(x')}{kT}}$$

or

$$p(x') = (p_{p0}) e^{-\frac{q\psi(x')}{kT}}$$

LAW OF THE JUNCTION

where $q\psi(x') \equiv \frac{E_{ip} - E_i(x')}{kT}$

or $p(x')$ decreases exponentially with the local band bending
Note that the edge of the depletion region on the n-side \( x = x_r \) or \( x' = W \) the hole concentration is given by
\[
p(x' = W) = p_{p0} e^{q\psi(x' = W)} \frac{kT}{kT}
\]
The total band bending at \( (x' = W) \) is \( V_{bi} \)
\[
\therefore \ p(x' = W) = p_{p0} e^{-qV_{bi}} \frac{kT}{kT}
\]
We know that \( p(x' = W) \) is the hole concentration in the n-type semiconductor or \( n_{p0} \)
So, \( n_{p0} = p_{p0} e^{-qV_{bi}} \frac{kT}{kT} \)

Let us verify that what we derived does not contradict what we learned in the past.
\[
n_{p0} = \frac{n_i^2}{n_{n0}} = p_{p} e^{-qV_{bi}} \frac{kT}{kT}
\]
\[
\therefore e^{-qV_{bi}} \frac{kT}{kT} = \frac{n_i^2}{p_{p0} n_{n0}}
\]
\[
\therefore V_{bi} = \frac{kT}{q} \ln \frac{n_{n0} p_{p0}}{n_i^2} \text{ or with full ionization}
\]
\[
V_{bi} = \frac{kT}{q} \ln \frac{N_A N_P}{n_i^2} \text{ SAME AS BEFORE}
\]

12. **TRANSPORT IN QUASI-NEUTRAL REGIONS**

Some basic observations:
\[
J_n = -q\mu_n n \nabla \psi + q D_n \nabla n \\
J_p = -q\mu_p p \nabla \psi - q D_p \nabla p
\]
Flow Equation
\[
\Rightarrow J = J_n + J_p = (q\mu_n n + q\mu_p p)(-\nabla \psi) - q(D_p \nabla p - D_n \nabla n)
\]
In quasi-neutral regions \( \nabla p = \nabla n \) and since \( D_p \) and \( D_n \) are usually, the diffusion currents for majority and minority carriers tend to cancel.
\[
\Rightarrow \text{In quasi-neutral regions, } J = \sigma E
\]
where \( \sigma = q\mu_n n + q\mu_p p \) \{Conductivity units (ohm cm)\(^{-1}\)}

Since \( \mu_n \) and \( \mu_p \) are often comparable, \( \sigma \) is dominated by majority carriers.
We will examine below precisely when it is OK to neglect altogether the drift of the minority carriers. For the moment, the main point is that \( J = \sigma E \) is a good starting point for estimating behavior.
Poisson’s Equation: \[ \nabla^2 \psi = -\frac{\rho}{\varepsilon} \]

But \( \rho \) is small \( \Rightarrow \psi \) is a linear function of position \( \Rightarrow \mathcal{E} \) is constant except for whatever perturbing \( \mathcal{E} \)–fields are set up by small, non-zero charge densities.

We treat the one-dimensional case with two assumptions:

- Uniform doping \( \left( \frac{\partial n_o}{\partial x} = \frac{\partial p_o}{\partial x} = 0 \right) \)
- A single recombination time, i.e. \( U = \frac{n-n_o}{\tau} = \frac{p-p_o}{\tau} \) \( \tau \) constant

Obviously, more complex cases will exist, for example, with ion implanted or diffused doping profiles, but the key points are best illustrated with this simple case:

\[
\begin{align*}
J_n &= qn\mu_n nE + qD_n \frac{dn}{dx} \\
\frac{dJ_n}{dx} &= q \frac{dn}{dt} - (G_L - U) \\
J_p &= qp\mu_p nE - qD_p \frac{dp}{dx} \\
\frac{dJ_p}{dx} &= -q \frac{dp}{dt} + (G_L - U)
\end{align*}
\]

Substitute for \( J_n \) and \( J_p \) in \( \frac{\partial J_n}{\partial x} \) and \( \frac{\partial J_p}{\partial x} \).

\[
\begin{align*}
\frac{dn}{dt} &= G_L - \left( \frac{n-n_o}{\tau} \right) + \mu_n \mathcal{E} \frac{dn}{dx} + D_n \frac{\partial^2 n}{\partial x^2} + \mu_n n \frac{\partial \mathcal{E}}{\partial x} \\
\frac{dp}{dt} &= G_L - \left( \frac{p-p_o}{\tau} \right) - \mu_p \mathcal{E} \frac{dp}{dx} + D_p \frac{\partial^2 p}{\partial x^2} - \mu_p p \frac{\partial \mathcal{E}}{\partial x}
\end{align*}
\]

Note: \( \frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\varepsilon} \), which is small, but we can cancel it out altogether by multiplying \( n \) equation by \( \mu_p p \) and \( p \) equation by \( \mu_n n \) and adding

We further use the fact that \( n-n_o = p-p_o = n' \) in quasi-neutral material and \( \frac{\partial}{\partial t} \) or \( \frac{\partial}{\partial x} \) of \( p_o \) and \( n_o \) are zero.

This leads to

\[
(\mu_n n + \mu_p p) \frac{dn'}{dt} = (\mu_n n + \mu_p p) \left( G_L - \frac{n'}{\tau} \right) + \mu_p \mu_n (p-n) \mathcal{E} \frac{dn'}{dx} + (\mu_n D_p n + \mu_p D_n p) \frac{\partial^2 n'}{\partial x^2}
\]

Or, defining so-called AMBIPOLAR QUANTITIES:
\[
\frac{\partial n}{\partial t} = G_L - \frac{\mu_A}{\tau} - \frac{n_p'}{\mu_A} \frac{\partial n'}{\partial x} + D_A \frac{\partial^2 n'}{\partial x^2}
\]

AMBIPOlar DIFFUSION EQUATION

\[
\mu_A = \frac{\mu_n \mu_p (n - p)}{\mu_n n + \mu_p p}; \quad D_A = \frac{D_n D_p (n + p)}{D_n n + D_p p}
\]

Note: 1) For \( n \gg p \), \( n \)-type material in low level injection

\[
\mu_A \rightarrow \mu_p, \quad D_A \rightarrow D_p \quad \text{THE MINORITY CARRIER DOMINATES}
\]

\[
\text{THE TRANSPORT IN QUASI-NEUTRAL MATERIAL}!!
\]

2) For high-level injection, \( \mu_A \) gets small, diffusion of both carriers is important, and the full non-linear ambipolar equation must be handled (usually numerically).

The idea is this: Excess minority carriers are neutralized (almost) by excess minority carriers. The small non-uniform \( \mathcal{E} \) fields needed to keep the excess majority carriers around are set up by the small \( \frac{\partial \mathcal{E}}{\partial x} \) term in the majority carrier flow equation.

The ambipolar diffusion equation assumes 1) \( p - p_o = n - n_o = n' \) and

2) \( \mu_n, \mu_p \) not functions of position and

3) \( \mu = n' / \tau, \quad \tau \) well behaved

and yields the behavior of excess carriers \( n' \) (or \( p' \)) in terms of appropriate boundary conditions on the partial differential equation.

Furthermore, the drift term \( \mu_A \mathcal{E} \frac{\partial n'}{\partial x} \) becomes important relative to the diffusion term \( D_A \frac{\partial^2 n'}{\partial x^2} \) only when

\[
\mathcal{E} \sim \frac{D_A}{\mu_A} \frac{1}{n'} \frac{\partial n'}{\partial x} \sim \frac{kT / q}{L} \frac{(n + p)}{n - p}
\]

when \( L \) is a characteristic length for \( n' \)

\( kT / q \) is 25 \( mv \) \( \quad L \sim \mu m \) typically

\( \Rightarrow \) when \( \mathcal{E} \) is few hundred \( v / cm \), can

\( \Rightarrow \) get minority carrier drift effects
However, for \( n_o = 10^{15} \text{ cm}^{-3} \) \( \mu_n \sim 500 \text{ cm}^2 / \text{v sec} \)

\[
J_n \text{ (drift)} = q \mu_n n_0 E = \left(1.6 \times 10^{-19}\right)(500)(10^{15})(100) \sim A/\text{cm}^2
\]
which is a small current density.
For \( n_o = 10^{18} \), \( J \) would be large, \( \sim kA/\text{cm}^2 \)
Before \( E \) was big enough to affect \( n' \) equation.

This leads to a strategy for handling quasi-neutral regions:

1) Assume \( E \) – field is negligible, \( \mu_A = \mu_{\text{minority}}, D_A = D_{\text{minority}} \)
2) Solve minority carrier diffusion equation (i.e. the ambipolar equation) with suitable boundary conditions.
3) Use quasi-neutrality to find majority carrier diffusion, and use boundary condition on \( J \) to find total \( E \).
4) Check to see that \( E \) was negligible, otherwise, re-solve with \( E \frac{\partial n'}{\partial x} \) term in place (harder!).

Types of boundary conditions

1) Specify \( n' \) at a boundary (Dirichlet).
2) Specify the minority carrier diffusion current at a boundary \( \frac{\partial n'}{\partial x} \) (Newman).
3) Can relate \( n' \) to \( \frac{\partial n'}{\partial x} \) at the boundary (Mixed)

Recombination velocity is used for mixed conditions
At \( x_o \), the mixed conditions would be

\[
-D_p \frac{dp'}{dx} = s_p \frac{p'(x_o)}{x_o}
\]
Excess hole concentration at \( x_o \)

Flux of excess hole to \( x_o \)

Surface recombination velocity
Surface Recombination Velocity

\[ S_p \rightarrow 0 \Rightarrow \text{reflecting surface (no slope of } p' \text{ at boundary)} \]
\[ S_p \rightarrow \infty \Rightarrow \text{absorbing surface } \Rightarrow p'(x_o) \rightarrow 0 \]

In practical cases \( S_p \) is limited to \( \sim 10^7 \text{ cm/sec} \)
On application of a forward bias the electron and hole concentrations continue to follow the relation that \( p_n(x') = p_{p0}e^{-\frac{q\psi(x')}{kT}} \) the law of the junction.

The difference from the zero bias case is that at the edge of the junction of \( x' = W \) \( \psi(W) = V_{bi} - V_F \), not \( V_{bi} \) as in the zero bias case.

\[
\therefore \quad p_n(W) = p_{p0}e^{-\frac{q(V_{bi} - V_F)}{kT}}
\]

or \( p_n(W) = p_{p0}e^{-\frac{qV}{kT}}e^{\frac{qV}{kT}} \)

\[
p_n(0) = P_{p0}e^{\frac{qV}{kT}}
\]

THE MINORITY CARRIER CONCENTRATION IS RAISED FROM ITS ZERO BIAS \( qV \) VALUE BY THE FACTOR \( e^{\frac{qV}{kT}} \).

IMPORTANT

\[
\Delta p_n(x') = \Delta p_n(0)e^{-\frac{qV}{kT}}
\]

\[
x = x_p
\]

\[
x' = W
\]

\[
x'' = 0
\]
Similarly the minority carrier concentrations at the edge of the depletion region on the p-side

\[ n_p(x_p) = n_{p0} e^{\frac{qV_F}{kT}} \]

What happens to these excess carriers? They diffuse away from the edge of the depletion region to the bulk. The profile that governs the diffusion is set by the recombination rate of the minority carriers in the bulk. The situation is analyzed by the continuity equation.

At any point \( x' \) the continuity equation states

\[ \frac{1}{q} \nabla \cdot J_p^{(x)} = G - R \]

Using, \( J_p^{(x)} = J_{p \text{ diff}}^{(x)} \)

by neglecting drift currents (which will be explained later) we get

\[ -D_p \frac{d^2 p_n^{(x')}}{d x'^2} = - \frac{\Delta p_n^{(x')}}{\tau_p} \]

Note that

\[ \frac{d^2}{d x'^2} p_n(x') = \frac{d^2}{d x'^2} \left( p_n(x') - p_{n0} \right) \quad \text{since} \quad \frac{d^2}{d x'^2} p_{n0} = 0 \]

\[ \therefore \frac{d^2}{d x'^2} p_n(x') = \frac{d^2}{d x'^2} \Delta p_n(x') \]

\[ D_p \frac{d^2 \Delta p_n(x')}{d x'^2} - \frac{\Delta p_n(x')}{\tau_p} = 0 \]

We know that far away from the junction the excess hole concentration has to be zero since excess holes have to eventually recombine.

\[ \therefore \Delta p_n(x') = c_1 e^{-\frac{x'}{L_p}} + c_2 e^{-\frac{x'}{L_p}} \]

where \( L_p = \sqrt{D_p \tau_p} \) = Diffusion length of the holes

which can be proven to be the average distance a hole diffuses before it recombines with an electron.

Also, \( c_1 \equiv 0 \) as \( \Delta p_n \to 0 \) as \( x' \to \infty \)

\[ \therefore \Delta p_n(x') = c_2 e^{-\frac{x'}{L_p}} \]

At \( x' = 0 \) \( \Delta p_n(0) = p_n(0) - p_{n0} = p_{n0} e^{\frac{qV_F}{kT}} - p_n \)

Or \( \Delta p_n^{(0)} = p_{n0} \left( e^{\frac{qV_F}{kT}} - 1 \right) \)

IMPORTANT

\[ \Delta p_n^{(0)} = \Delta p_n(0) e^{-\frac{x'}{L_p}} \]

\[-(2)\]
This exponential relationship applies to the minority electrons as well where

\[
\Delta n_p(x) = \Delta n_p(0)e^{\frac{\Delta x}{L_p}}
\]

\[
\Delta n_p(0) = n_{p0}\left(e^{\frac{\Phi_p}{kT}} - 1\right)
\]

(New Coordinate System)
DERIVATION OF THE DIODE EQUATION UNDER FORWARD BIAS

We now have the minority carrier charge profiles [WHICH IS ALWAYS OBTAINED FROM THE SOLUTION OF THE CONTINUITY EQUATION]. From this we can calculate the current across the diode. Note that the current measured anywhere in the diode has to be the same and equal to the current in the external circuit.
Observation: Far from the junction diffusion current \( \to 0 \) as \( J_p(x) = -qD_p \frac{d}{dx} \Delta p_n(x) \)

\[
J_p \text{ diff } (x^*) = q \frac{D_p}{L_p} \Delta p_n(x^*)
\] -(3)

Since \( \Delta p_n(x^*) \to 0 \) as \( x^* \to \infty \)

\( J_p \text{ diff } (x^*) \to 0 \) as \( x^* \to \infty \)

Since \( J_T \) is always constant and \( J_T = J_n + J_p \) in the region far from the junction

So what about \( J_p \) drift or MINORITY CARRIER DRIFT CURRENTS?
We assumed the regions beyond the depletion regions were neutral. However, the application of a voltage across the diode must result in a field in these regions as shown below.

![Diode Schematic](image)

This can be readily understood if you break the diode into three regions, the bulk p, and the bulk n and depletion region. Since the bulk regions are highly doped they are conductive and hence have a small resistance. The depletion region, which is devoid of carriers, may be considered a large resistance. SCHEMATICALLY, we can consider the diode to be as shown below.

![Diode Schematic](image)

It can be readily seen that the voltage drop in the bulk regions are much smaller than the drop across the junction

$$J_T = J_n^{drift} + J_p^{drift}$$

$$= q\mu_n \cdot n_{n0} \cdot E + q\mu_p \cdot p_{n0} \cdot E \text{ (in the n-region)}$$

$$= J_n^{drift} \left[ 1 + \frac{\mu_p}{\mu_n} \cdot \frac{p_{n0}}{n_{n0}} \right]$$

$$J_T \equiv J_n^{drift} \text{ far from the junction as } J_p^{drift} \ll J_n^{drift} \text{ by the ratio } \frac{\mu_p}{\mu_n} \cdot \frac{p_{n0}}{n_{n0}}$$

Since we are considering low level injection where $\frac{P_{n0}}{n_{n0}} \equiv \ll 1$

MINORITY CARRIER DRIFT CURRENTS CAN ALWAYS BE NEGLECTED

Far from the junction the current is carried by majority carrier drift OR $J_p = J_p^{drift}$ in the p bulk and $J_n^{drift}$ in the n bulk. As is apparent from the figure as you get closer to the junction, because minority carrier diffusion is significant and $J_T = J_n + J_p$ is always true, $J_n$ is always $J_T - J_p^{diff}$ in the n region. ($J_n$ is always $J_n^{drift} + J_n^{diffusion}$ but it is not necessary at the moment to evaluate each component separately).
HERE COMES AN IMPORTANT ASSUMPTION:
We assume no recombination or generation in the depletion region. This is valid because the depletion region width, \( W \), is commonly \( \ll L_n \) and also it will be proven later that recombination only occurs at the junction (\( x = 0 \)) because of the carrier concentration profiles and hence is limited in extent. The latter is the correct reason so you have to accept it.

If this is true then the continuity equation dictates that

\[
\frac{1}{q} \nabla \cdot J_p = G - R = 0 = \frac{1}{q} \nabla \cdot J_n \quad \text{OR BOTH} \quad J_n \quad \text{and} \quad J_p
\]

\( J_p \) are constant across the junction (as shown).

So IF WE KNEW \( J_n \) and \( J_p \) in the depletion region then we could add them to get the total current

\[
J_T = J_n + J_p \quad \text{(everywhere)} = J_n + J_p \quad \text{(depletion region for convenience)}
\]

But we know \( J_p \) at the edge of the depletion region is only \( J_p \) (on the n-side because the minority drift is negligible).

\[
\therefore J_p \quad \text{(depletion region) Assumption} \quad \text{of no G-R in the depletion region} \quad J_p \quad \text{diffusion} \quad (x" = 0)
\]

OR \( J_p = q \frac{D_p}{L_p} \Delta p_n(0) \quad \text{← from (2), (3)} \)

\[
J_p = q \frac{D_p}{L_p} p_n(0) \left( \frac{q V_F}{kT} e^{\frac{q V_F}{kT}} - 1 \right) \quad \text{(4a)}
\]

Similarly \( J_n = q \frac{D_n}{L_n} n_p(0) \left( \frac{q V_F}{kT} e^{\frac{q V_F}{kT}} - 1 \right) \quad \text{(4b)}

\[
J_T = J_n + J_p
\]

\[
J_T = q \left[ \frac{D_p}{L_p} p_n(0) + \frac{D_n}{L_n} n_p(0) \right] \left( \frac{q V_F}{kT} e^{\frac{q V_F}{kT}} - 1 \right)
\]

IMPORTANT

Note: The assumption of no G-R in the depletion region allowed us to sum the minority diffusion currents at the edges of the junction to get the total current. This does not mean that only diffusion currents matter. Current is always carried by carrier drift and diffusion in the device. The assumption allowed us to get the correct expression without having to calculate the electric field in the structure (a very hard problem).

So

\[
J_T = J_n \left( \frac{q V_F}{e^{\frac{q V_F}{kT}}} - 1 \right)
\]

\[
J_n = q \left[ \frac{D_p}{L_p} p_n(0) + \frac{D_n}{L_n} n_p(0) \right] \left( \frac{q V_F}{e^{\frac{q V_F}{kT}}} - 1 \right) \quad \text{(b)}
\]

\[
I_T = J_T \cdot A \quad \text{(A = Diode Area)}
\]
RECOMBINATION IN $p-n$ JUNCTIONS

We learnt in the SRH analysis that under the approximation of

1. $\sigma = \sigma_n = \sigma_p$, and
2. $E_i = E_i$, and
3. $e_n e_p \sigma_n \sigma_p$ unperturbed in non-equilibrium

We get

$$U = \frac{1}{\tau} \frac{pn - n_i^2}{n + p + 2n_i}$$

where $\tau = \frac{1}{\sigma V_i N_t}$ and $pn - n_i^2$ = Driving force for recombination

and $n + p + 2n_i$ = Resistance to recombination

This applies to any semiconductor with or without band bending. Note that the values of $p$ & $n$ are functions of band bending, photon flux, etc.

- **NOTE** that $U$ is maximized for a certain level of perturbation when the denominator is minimized.

Apply this analysis to a $p-n$ junction and you obtain Sah-Noyce-Shockley or the SNS analysis.
Under a forward bias of $V_f$, the product of $n_p$ is a constant across the depletion layer and is

$$np = n_i^2 \exp \left( \frac{qV_f}{Kt} \right).$$

This is easily seen by recognizing that

$$n(x) = n_{n_0} e^{-\frac{q\psi(x)}{kT}}$$

$$p(x) = p_{p_0} e^{-\frac{q\psi(x)}{kT}}$$

and that $n(x), p(x) = n_n \cdot p_p e$

$$\psi(x) + \psi'(x) = V_{bi} - V_f$$

$$\therefore n(x), p(x) = n_{n_0} p_{p_0} e^{\frac{-qV_{bi}}{kT} + \frac{qV_f}{kT}}$$

$$= n_{n_0}, p_{p_0} e^{\frac{qV_f}{kT}}$$
or \( n(x) - p(x) = n_i^2 e^{\frac{qV_f}{kT}} \)

Under steady state bias of \( V_f \) the term \( (n + p + 2n_i) \) is minimized when \( n = p \). The value of \( x \) where this occurs is chosen to be zero. **THIS IS THE MAXIMUM RECOMBINATION PLANE.**

\[
\text{or } n(0), p(0) = n^2(0) = p^2(0) = n_i^2 e^{\frac{qV_f}{kT}}
\]

\[
\text{or } n(0) = p(0) = n_i e^{\frac{q f}{kT} \frac{q f}{kT} 2}
\]

Q: Is the maximum recombination plane coincident with the metallurgical junction plane?

If we move away from the maximum recombination plane, the electron and hole concentrations change proportionally to the term \( \exp\left(\pm \frac{q\psi(x)}{kT}\right) \)

\[
\text{Towards } p \quad \text{Towards } n
\]

\[
\psi
\]

\[
\psi
\]

\[
x = 0
\]

MAXIMUM RECOMBINATION PLANE

Where \( n(x) = n(0) e^{\frac{q\psi}{kT}} \) and \( p(x) = p(0) e^{\frac{q\psi}{kT}} \)

Assuming the distance \( x \) is small so that we can assume the electric field constant for purposes of the analysis to be \( E = E(0) \).
Then \( \psi = \mathcal{E}(\pm x) = \pm (\mathcal{E}, x) \)

\[
\therefore \quad U = \frac{1}{\tau} \frac{np - n_i^2}{n + p + 2n_i} \quad \text{or} \quad U = \frac{1}{\tau} \frac{qV_f}{n_i e^{\frac{kT}{e}} - n_i^2} = \frac{n_i e^{\frac{kT}{e}}}{n(0)e^{\frac{kT}{e}} + p(0)e^{\frac{kT}{e}}} + 2n_i
\]

Neglecting \( n_i^2 \) in the numerator and \( 2n_i \) in the denominator we get

\[
U = \frac{1}{\tau} \frac{n_i e^{\frac{kT}{e}}}{2 \cosh \frac{qEX}{kT}}
\]

To calculate the total recombination current we need to integrate over the volume of the depletion region. Since the recombination rate curve is highly peaked about \( x = 0 \), the maximum recombination plane the, following approximations remain valid.

1) Linearizing the potential \( \psi = \pm \mathcal{E}x \) since only small values of \( x \) contribute to the integral,

\[
\therefore \quad J_{SNS} = q \int U(x) \, dx
\]

AND 2) \( \int_{-\infty}^{\infty} \rightarrow \int_{w}^{\infty} \) as contributions drop off exponentially
\[
\frac{qn_i}{2\tau} \exp \left( \frac{qV_f}{2kT} \right) \int \frac{dx}{\cosh \left( \frac{qE(x=0) \cdot x}{kT} \right)}
\]

But \[
\int_{-\infty}^{\infty} \frac{dx}{qE(x) \cosh \left( \frac{qE(0) \cdot x}{kT} \right)} = \frac{\pi kT}{qE(0) \cdot x}
\]

\[
\therefore \quad J_{SNS} = \frac{qn_i \cdot \pi kT}{2\tau \cdot qE(0)} \cdot \exp \left( \frac{q \cdot V_f}{kT \cdot 2} \right)
\]

- NOTE: that the factor of 2 comes from \( x = 0 \) being defined as the plane where \( \frac{1}{2} \cdot V_f \) is dropped from either side of the junction; the maximum recombination plane.

So when you see recombination currents written as \( J = J_o \cdot \exp \left( \frac{q \cdot V}{kT \cdot n} \right) \) where \( n \) is referred to as an ideality factor think of it as a voltage partition factor.
First Observation:
Since we are only dealing with minority carrier currents we know that minority carrier drift can be neglected. Hence only minority carrier diffusion is relevant. To calculate diffusion currents we need to know the charge profile. Charge profiles are obtained by solving the continuity equation (in this case equivalently the diffusion equation as drift is negligible).
We assume that the large electric field in the reverse-biased p-n junction sweeps minority carriers away from the edge of the junction.

\[ n'_p(x) = -x_p \]  
\[ p'{}_n(x) = +x_n \]

We also know that the minority carrier concentration in the bulk is \( n_{p0} \) (p-type) and \( p_{n0} \) (n-type) respectively. Therefore, the shape of the curve will be qualitatively as shown, reducing from the bulk value to zero at the depletion region edge.
Consider the flow of minority holes. The charge distribution is obtained by solving
\[ D_p \frac{d^2p_n}{dx^2} + G_{lh} - R = 0 \]  
assuming that the only energy source is thermal.

The case for reverse bias is very different.
Here the application of bias increases barriers. The only carriers that can flow are minority carriers that are aided by the electric field in the depletion region.

HERE the minority carriers are electrons injected from the p-region to the n-region (OPPOSITE TO THE FORWARD BIAS CASE).
Then $D_p \frac{d^2 p_n}{dx^2} + \frac{p_{n0} - p_n}{\tau_p} = 0$ \hspace{1cm} (9)

Note that this term is a generation term because $p_n < p_{n0}$ for all $x^*$. This is natural because both generation and recombination are mechanisms by which the system returns to its equilibrium value. When the minority carrier concentration is above the equilibrium minority carrier value then recombination dominates and when the minority carrier concentration is less than that at equilibrium then generation dominates. The net generation rate is (analogous to the recombination rate).

\[ G_{th} - R = \frac{p_{n0} - p_n}{\tau_p} \hspace{1cm} \text{(ASSUMING } \tau_p \text{ the generation time constant = recombination G-R = 0)} \]

which is???

Note: when $p_n = p_{n0}$ true in equilibrium.

Again using $\Delta p_n (x^*) = p_{n0} - p_n (x^*)$ we get $D_p \frac{d^2 \Delta p_n (x^*)}{dx^2} + \frac{\Delta p_n (x^*)}{\tau_p} = 0$

Again $\Delta p_n (x^*) = C_1 e^{x^*/\tau_p} + C_2 e^{-x^*/\tau_p}$

$C_1 = 0$ (for physical reasons)

At $x^* = \infty$ $\Delta p_n \rightarrow 0$

At $x^* = 0$ $\Delta p_n = p_{n0} - p_n (0) = p_{n0}$

$\therefore \Delta p_n (x^*) = p_{n0} e^{-x^*/\tau_p}$

$p_{n0} - p_n (x^*) = p_{n0} e^{-x^*/\tau_p}$

OR $\boxed{p_n (x^*) = p_{n0} \left( 1 - e^{-x^*/\tau_p} \right)}$ \hspace{1cm} (10)

$\therefore$ The flux of holes entering the depletion region is $J_p (x^* = 0) = q D_p \frac{dp_n}{dx} \hspace{1cm} (x^* = 0)$

$J_p = q \frac{D_p}{L_p} p_{n0}$ Similarly $J_n = q \frac{D_n}{L_n} n_{p0}$

Assuming no generation in the depletion region the net current flowing is

$J_s = q \left[ D_p \frac{p_{n0}}{L_p} + D_n \frac{n_{p0}}{L_n} \right]$ \hspace{1cm} (11)

This is remarkable because we get the same answer if we took the forward bias equation (valid only in forward bias) and arbitrarily allowed $V$ to be large and negative (for reverse bias)

i.e. $J = J_s \left( \frac{qV}{kT} - 1 \right)$ If $V$ is large and negative $J_p = -J_s$ which is the answer we derived in 11.
In summary,

The equation (11) can be understood as follows. Concentration on the p-region. Any minority carrier electrons generated within a diffusion length of the n, depletion edge can diffuse to the edge of the junction and be swept away. Minority electrons generated well beyond a length $L_n$ will recombine with holes resulting in the equilibrium concentration, $n_{p0}$. Similarly holes generated within, $L_p$, a diffusion length, of the depletion region edge will be swept into the depletion region.

IMPORTANT OBSERVATION:
Look at the first term in equation 11. The slope of the minority carrier profile at the depletion edge = $\frac{p_n^0}{L_p} = \frac{\text{Difference from Bulk Value}}{L_p}$

This is always true when recombination and generation dominate.

Recall that even in forward bias (shown below) the slope of the carrier profile is again

$$\frac{\text{Difference From Bulk Value}}{L_p} = \frac{\Delta p_n(0)}{L_p}$$

$$\Delta p_n = p_n^0 \left( \frac{qV}{kT} - 1 \right)$$
In the event that there is light shining on the p-n junction as shown below

then the charge profile is perturbed in the following manner

where far in the bulk region an excess minority carrier concentration is generated where

\[ \Delta n_p = G_L \tau_n \]  
\[ \Delta p_n = G_L \tau_p \]

The new equation to be solved for reverse saturation current differs from equation 8 in that a light generation term is added.

\[
D_p \frac{d^2 p}{dx^2} + G_{th} - R + G_L = 0 \quad \text{or} \quad D_p \frac{d^2 p}{dx^2} + \frac{p_{n0} - p_n}{\tau_p} + G_L = 0
\]

only difference

\[ \text{net thermal generation} \]

with boundary conditions similar to before \( p_n(\infty) = p_{n0} + \tau_p G_L \) and \( p_n(0) = 0 \)

we get \( p_{n0}(x^*) = \left( \frac{p_{n0} + \tau_p G_L}{P_{bulk}} \right) \left( 1 - e^{-L_p} \right) \)

The slope of the charge profile at the edge of the depletion region is

\[
\frac{\partial p_n(0)}{\partial x^*} = \frac{p_{n0} + \tau_p G_L}{L_p}
\]

\[
\therefore J_p = qD_p \left( \frac{p_{n0} + \tau_p G_L}{L_p} \right) \quad \text{Similarly, } J_n(x^* = 0) = q \frac{D_n}{L_n} \left( G_L \tau_n + n_{p0} \right)
\]

\[
J_{\text{Reverse}} = q \left[ D_n \cdot \frac{n_{\text{bulk}}}{L_n} + \frac{D_{p\text{bulk}}}{L_p} \right]
\]
By changing the slope of the minority profile at the edge of the junction I can control the reverse current across a diode.

Controlling and monitoring the current flowing across a reverse bias diode forms the bases of a large number of devices.

$J$ can be charged by changing the slope of minority carrier profile.
METHOD OF CONTROLLING THE MINORITY CARRIER SLOPE

1. TEMPERATURE: Recall that the slope is (for the p-material) given by \( \frac{n_{p0}}{L_n} \)

\[ J_{s,n} = qD_n \frac{n_{p0}}{L_n} \]

But \( n_{p0} = \frac{n_i^2}{p_{p0}} \)

Assuming full ionization \( p_{p0} \neq f(T) \) and is always \( \approx N_A \)

\[ \therefore n_{p0} = \frac{n_i^2}{N_A} = \frac{N_c N_i e^{\frac{E_g}{kT}}}{N_A} \]

\[ \therefore J_{s,n}(T) = q \cdot \frac{D_n(T)}{L_n(T)} N_c(T) N_i(T) e^{\frac{E_g}{kT}} \]

EXPONENTIAL DEPENDENCE ON T (the other terms have weaker dependence).

2. Change \( n_{p0} \) to \( n_p = n_{p0} + G_L \tau_n \) as described. The generation rate is dependent on the input photon flux.

\[ \therefore J_{s,n} = q \frac{D_n}{L_n} \left( n_{p0} + G_L \tau_n \right) \]

Measure \( J_{s,n} \) Determine \( G_L \) and input photon flux

3. Change the minority carrier concentration by an electrical minority carrier injector i.e. a p-n junction.

DEVICE

THERMOMETER
Read \( J_s \) and extract T

PHOTODETECTOR
(used in optical communications)

TRANSISTOR

A TRANSISTOR, short for TRANSfer ResISTOR, is the basic amplifying element in electronics. The basis of its amplification and its structure is shown below.
A transistor consists of a forward bias junction in close proximity to a reverse bias p-n junction so that carriers injected from forward bias junction (from the emitter labeled E) can travel through the intermediate layer (called the BASE and labeled B) and across the reverse biased junction into the COLLECTOR, labeled C.

Schematically for the case of the emitter being a p material and the base being n-type, (a pnp transistor) the DOMINANT CURRENT FLOW (to be modified later is shown below).

The holes injected from the p-type emitter contribute to the EMITTER CURRENT, \( I_E \), and the holes collected contribute to the collector current, \( I_c \). These currents in the diagram above are equal (AN APPROXIMATION TO BE MODIFIED LATER). This situation is equivalent to

The forward bias junction across which the input voltage is applied has a low resistance.

The forward bias resistance of a diode can be readily calculated from the I-V characteristics
\[ I = I_e \left( \frac{qV}{kT} - 1 \right) = I_e e^{\frac{qV}{kT}} \]

\[ \therefore R = \frac{\partial V}{\partial I} = \left( \frac{\partial I}{\partial V} \right)^{-1} = \left( \frac{q}{kT} \cdot I_e e^{\frac{qV}{kT}} \right) \]

OR \[ R = \left( \frac{q}{kT} \cdot I \right)^{-1} \]

OR \[ R = \frac{kT}{qI} \]

IMPORTANT

As I increases \( R \) decreases.

Note that @ room temperature at a current of 25 mA \( R = \frac{25mV}{25mA} = 1 \Omega \) FOR ANY DIODE

Therefore in a transistor a SMALL INPUT VOLTAGE can generate a large current, \( I_e \), because of the small \( R_{in} \). This same current flows across a large resistance (as the reverse bias resistance of the collector junction is large) as \( I_c \).

\[ \therefore \text{The measured voltage across the output resistance, } R_{out}, \text{ is } I_c \cdot R_{out}. \text{ The input voltage is } I_E \cdot R_{in}. \therefore \]

The voltage gain of device, \( A_v \), is

\[ A_v = \frac{V_{out}}{V_{in}} = \frac{I_c R_{out}}{I_E R_{in}} = \frac{R_{out}}{R_{in}} \]

IMPORTANT

In general, if you can control a current source with a small voltage then you can achieve voltage amplification by passing the current through a large load resistor.

Recall that we could modulate the reverse bias current by changing the minority carrier concentration flux injected into the junction. The diagram is reproduced below.
A transistor achieves modulating the minority concentration by injecting minority carriers using a forward biased p-n junction. Recall that a p-n junction injects holes from a p-region into the n-region (raising) the minority carrier concentration from $p_{n0}$ to a new value $p_n(x)$. The same applies for the n-region. Let us now consider a p-n-p transistor. It consists of a p-n junction that injects minority carriers into another but now reverse-biased p-n junction.