Consider a core of high μ material with an air gap. The equivalent magnetic circuit is:

\[ \Phi = \frac{NI}{R_c + R_g} \]

Self-inductance is:

\[ L = \frac{N^2 I}{I(R_c + R_g)} = \frac{N^2}{R_c + R_g} = \frac{N^2}{R_t} \]

We know the electrical energy required to establish a current \( I \) in an inductor is \( U_m = \frac{1}{2} LI^2 \). We designate this as \( U_m \) to emphasize the energy is stored in the magnetic field. But where is the energy? Let's rewrite the expression:

\[ U_m = \frac{1}{2} LI^2 = \frac{1}{2} \frac{N^2 I^2}{R_t} = \frac{1}{2} \frac{i^2 R_t}{R_t} = \frac{1}{2} \Phi^2 R_t + \frac{1}{2} \Phi^2 R_t \]

\[ = \frac{1}{2} \frac{B^2 A_c}{\mu_c} + \frac{1}{2} \frac{B^2 A_g}{\mu_o} \]

**Energy in Core**  
**Energy in Gap**

We can think of the total energy, \( U_m \), as being divided between the core and the gap. But since \( \mu_c = \mu_r \mu_o \gg \mu_o \), most of the energy is in the air gap. This is purely an interpretation of the equation, but it turns out the same analysis holds for magnetic fields in free space. Recalling the energy in the gap:

\[ U_m = \frac{1}{2} \frac{B^2 A_g}{\mu_o} = \frac{1}{2} \frac{B^2 A_g}{\mu_o} = \frac{1}{2} B \cdot \frac{B}{\mu_o} \cdot A \cdot g = \frac{1}{2} BH \cdot \text{Volume} \]

If \( B + H \) are not constant across \( A \), \( U_m = \frac{1}{2} \int B \cdot H \, dV \). This is the
general expression for energy in a magnetic field.

In electrical circuit analysis we assume \( L \) is constant & vary \( V_m \)
by varying \( I \). Let's instead try to vary \( V_m \) by varying \( L \).

(Ref. Inman Section 7.3.4)

Power delivered by the current source is

\[
P_s = VI = \frac{dN^2}{dt} \cdot I = \frac{d(I^2)}{dt} \cdot I \quad (L = \frac{N^2}{I})
\]

if \( I \) is constant \( P_s = \frac{1}{2} \frac{dL}{dt} \)

\( dV_s \) is the incremental energy supplied by the source in affecting
a change in inductance \( dL \). But the change in magnetic
energy is only half that:

\[

\frac{dV_m}{dt} = \frac{1}{2} \frac{dL}{dt} \Rightarrow dV_m = \frac{1}{2} I^2 dL = \frac{1}{2} dV_s
\]

The rest of \( dV_s \) must perform work in changing \( L \), through
(namely) linear or rotary motion.

\[
dw = Fdx \quad \text{or} \quad dw = T \theta, \quad \text{and} \quad dV_s = dV_m + dw \quad \text{(conservation of energy)}
\]

or \( I^2 dL = \frac{1}{2} I^2 dL + dw \), so \( dw = \frac{1}{2} I^2 dL = Fdx \) or \( T \theta \)

for rotary motion \( T = \frac{1}{2} \frac{dL}{d\theta} \)
Variable Reluctance Motors

Continuing the analysis, \[ T = \frac{I^2}{2} \frac{dl}{d\theta} \] if it constant.

If \( L \) is dominated by air gap reluctance, then
\[ L = \frac{N^2}{R_g} = \frac{N^2 \mu_0 A}{g} \]
and
\[ dl = \frac{N^2 M_0 dA}{g}, \] if the structure allows \( A \) to vary but not \( g \).

Neglecting flux anywhere but the gap, the effective area is
\[ A = w r \theta \]
and
\[ dA = w r d\theta \]

So
\[ T = \frac{I^2}{2} \frac{dl}{d\theta} = \frac{I^2}{2} \frac{N^2 \mu_0 w r d\theta}{g} \]

\[ T = \frac{I^2}{2} \frac{N^2 \mu_0 w r}{g} \]

\[ dW = T d\theta \]

The torque is in the direction to increase inductance. This is the behavior of a variable reluctance motor. As drawn, this structure will not produce sustained rotation. Multiple windings are required to continue applying torque as the rotor rotates.

Applying 3-phase currents

The winding pairs \( A-A', B-B', C-C' \) create a rotating \( B \) field.
Motors

The basic function of any motor is to convert electrical energy to mechanical energy:

\[ P_e = V I \Rightarrow P_m = T \omega \]

Since only real electrical power can become real mechanical power, we can anticipate that the behavior of a loaded motor is not purely inductive, even though the electrical connection is to a winding (3).

Two principles of physics govern motor operation:

1. A conductor carrying a current in an externally established field experiences a force \( F = B I L \) (from \( F = q V \times B \)).

2. An EMF is induced in a conductor moving in a magnetic field \( V_{\text{EMF}} = B L V \).

Without going into the details of motor design, we can picture that a current-carrying conductor positioned at some radius \( R \) on a rotating shaft will create torque \( T = F R = B I L R \) in the presence of a \( B \) field. As the shaft rotates, an EMF, the "back EMF," is generated in the conductor.

Two coefficients capture these relations.

For a given motor: \( T = I K_T \), \( V_{\text{EMF}} = K_E \omega \).

A simple equivalent circuit is:

\[ \begin{align*}
V_m & \quad R_w & \quad L_w & \quad V_{\text{EMF}} = K_E \omega \\
+ & \quad R_w & \quad L_w & \quad +
\end{align*} \]

where:
\( R_w = \) Winding Res.
\( L_w = \) Winding Inductance
From which the general observations follow - motor 
speed follows applied voltage, motor torque follows current. 
Real power is defined by $I^2$ flowing into voltage source $V_{in}$. 

3-Phase motors: Three phase motors use three windips, driven 
120° out of phase from each other, to produce a rotating B 
field of constant magnitude. The more perfectly sinusoidal the 

Two basic types: induction & synchronous. Synchronous 
motors the shaft rotates at the same velocity as the B field 
of the stator. Requires a rotor field; permanent magnet or winding. 
Induction motors have a non-driven winding or squirrel cage 
which acts like the secondary of a transformer. Current only 
flows (is induced) if there is a difference in rotation rates. 
Slip frequency (not just slip in phase). Model: 

\[ s = \frac{\omega_s - \omega_r}{\omega_r} \]