Buck Converter - Discontinuous Current Mode

In the previous analysis the circuits were assumed (or designed) to operate such that the current never drops to zero in the inductor ($I_{Lm} > 0$). But this mode is only guaranteed for a range of load resistances $R_{min} < R < R_{max}$.

If $R = R_{max}$ (which would be the case if the load were removed, for example) then the circuit will enter a "discontinuous" current mode where the inductor current falls to zero and remains there until the next charging cycle.

This is sometimes called the "complete energy transfer" mode because all the energy stored in the inductor during the charging phase (switch "on") is transferred to the load.

Note that during the "dead-time", switch $Q_1$ is off and $D_1$ is reverse biased so no current can flow in $L$. All the load current is supplied by the capacitor during this period.

\[ V_{in} \quad I \quad C \quad \frac{dI}{dt} \quad V \quad I \quad R \]

Equivalent during dead-time period.
Let's look at the discontinuous mode analytically for the ideal case (no loss, ideal diode).

The inductor voltage waveform is

\[ V_L(t) = \begin{cases} 0 & \text{for } 0 \leq t < T_r \\ (V_m - V_{out}) & \text{for } T_r \leq t < T_{on} \\ V_{out} & \text{for } T_{on} \leq t < T \\ 0 & \text{for } T \leq t \leq T + T_r \\ \end{cases} \]

Volt-second balance:

\[ (V_m - V_{out}) T_{on} = V_{out} T_r \]  \( (12) \)

or \[ V_m T_{on} = V_{out} (T_r + T_{on}) \]

From the inductor current waveform shown earlier, the total charge delivered to the load during one cycle is

\[ \Delta Q = \frac{1}{2} i_{\text{max}} (T_{on} + T_r) \]  \( (13) \)

and the peak current is

\[ i_{\text{max}} = \frac{T_r}{T_{on}} \frac{V_{out}}{L} = \frac{V_{out}}{L} \]  \( (14) \)

The average inductor current over one cycle is equal to the load current so

\[ \langle i_L \rangle = \frac{\Delta Q}{T} = \frac{V_{out}}{R} \]

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\[ V_{out} = \frac{R}{2T} i_{\text{max}} \frac{V_m T_{on}}{V_{out}} = \frac{R}{2T} \frac{T_{on}(V_m - V_{out})}{V_{out}} \]

Use \( T_{on} = DT \) and \( f_s = \frac{1}{T} \).
\[ V_{\text{out}} = \frac{R}{2L_fS} \frac{D}{V_{\text{out}}} \left( V_{\text{in}} - V_{\text{out}} \right) \]

Note this could be derived somewhat more easily from \( P_{\text{in}} = P_{\text{out}} \)

This is a quadratic. Let's

\[ K = \frac{2L_fS}{R} \quad x = \frac{V_{\text{in}}}{V_{\text{out}}} \quad (x > 1 \text{ for buck}) \]

so

\[ 1 = \frac{D^2}{K} x \left( x-1 \right) \Rightarrow x^2 - x - \frac{K}{D^2} = 0 \]

The solution is:

\[ \frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \sqrt{1 + \frac{4K}{D^2}} \]  \( \text{(15)} \)

Note that \( K \) includes \( R \) so the result is now load-dependent, unlike the continuous mode. But \( V_{\text{out}} \) is still controlled by the duty-cycle.

Looking back at (5) we can express the condition for continuous mode as:

\[ \frac{1}{R} \geq \frac{(1-D)}{2L_fS} \Rightarrow \quad K \geq \frac{1}{(1-D)} \]

So the boundary between continuous and discontinuous modes is defined by:

\[ K = K_{\text{crit}} = \frac{1}{1-D} \]

\[ K > K_{\text{crit}} \text{ is continuous} \]

\[ K < K_{\text{crit}} \text{ is discontinuous} \]
Since $K$ is controlled by the load resistance, this is just a condition on the load:

$$R_{\text{cut}} = \frac{2f_s L}{K_{\text{cut}}} = \frac{2f_s L}{1-D} \quad (16)$$

If $R > R_{\text{cut}}$: discontinuous
If $R < R_{\text{cut}}$: continuous

Most supplies will need to function properly in a no-load situation, so the discontinuous mode will be encountered at least occasionally. The controller must be designed for that possibility.

It is possible to design the buck converter to operate in the discontinuous mode for all loads up to a max current $(I_{\text{load}})_{\text{max}}$, which occurs at some minimum load resistance $R_{\text{min}}$.

$$R_{\text{min}} \geq R_{\text{cut}} = \frac{2f_s L}{1-D}$$

So
$$L \leq \frac{R_{\text{min}}}{2f_s} (1-D)$$

Right at the boundary, $D = \frac{V_{\text{out}}}{V_{\text{in}}}$. So

$$L \leq \frac{R_{\text{min}}}{2f_s} \left(1 - \frac{V_{\text{out}}}{V_{\text{in}}} \right) \quad (17)$$

Worst case at $(V_{\text{in}})_{\text{min}}$:

This suggests using a small inductor, but (14) reminds us that the peak current will scale inversely with $L$. 
Usually we want to keep the peak current below some maximum \((I_d)_{\text{max}}\) which is the maximum current in the transistor, so this constrains \(L\) to

\[
\frac{i_{\text{max}}}{(V_{\text{in}} - V_{\text{out}})} \leq \frac{(I_d)_{\text{max}}}{L}
\]

\[
G \quad L \geq \frac{\text{T}_{\text{o}}(V_{\text{in}} - V_{\text{out}})}{(I_d)_{\text{max}}}
\]

The worst-case \(\text{T}_{\text{o}}\) is at the boundary where

\[
\text{T}_{o} = D \cdot T = \frac{V_{\text{out}}}{V_{\text{in}}} T = \frac{V_{\text{out}}}{V_{\text{in}} f_s}
\]

So

\[
L \geq \frac{V_{\text{out}} (1 - V_{\text{out}}/V_{\text{in}})}{(I_d)_{\text{max}} f_s}
\]  \(\text{(18)}\)

Both (17) and (18) must be satisfied simultaneously which can only happen when

\[
\frac{P_{\text{min}}}{2 f_s (1 - V_{\text{out}}/V_{\text{in}})} \geq \frac{V_{\text{out}}}{(I_d)_{\text{max}}} (1 - V_{\text{out}}/V_{\text{in}})
\]

\[
P_{\text{min}} = \frac{V_{\text{out}}}{(I_d)_{\text{max}}}
\]

So

\[
(I_d)_{\text{max}} \geq 2 (I_{\text{out}})_{\text{max}}
\]

The transistor technology must be chosen to accommodate peak currents of at least twice the maximum load current.