Returning to DC-DC converter topologies - As a reminder, here are the first two we looked at:

\[ \frac{V_o}{V_i} = D \]

Question: is it possible to buck with a boost converter or boost with a buck? i.e., can the input voltage range extend on either side of the output? No. In the limit, both reduce to a short from input to output. \( \frac{V_o}{V_i} = 1 \). When would this buck-boost capability be desirable?

An automotive system needing to deliver 12V from a 9-16V supply. Could you cascade a buck & boost?

\[ \frac{V_o}{V_i} = \frac{D}{1-D} \]

But the center capacitor provides unnecessary energy storage. Eliminate it & merge the inductors:
The inductor terminal connected to ground changes from the negative to the positive reference terminal. What if we kept the negative terminal always at ground (reverse + _ of inductor). The second switch + diode can be removed, but the current in the capacitor is in the direction to charge $V_o$ negative. This is an inverting converter.

\[ V_c = V_v \]

Using the principle of volt-second balance

\[ \frac{V_iDT}{L} = -V_0 \frac{(1-D)T}{L} \]

\[ \Rightarrow \frac{V_0}{V_i} = \frac{-D}{1-D} \]

When $D = \frac{1}{2}$, $\frac{V_0}{V_i} = -1$

$0 < D < \frac{1}{2}, |V_i| < |V_0| ; D > \frac{1}{2}, |V_i| > |V_0|$
inductor waveform, \( V_c \) constant, continuous \( i_L \)

\[
\begin{array}{c}
V_L \\
V_i \\
V_c \\
\downarrow \\
\Delta i_L
\end{array}
\]

A second balance is a way of saying \( \langle \Delta V \rangle \) is constant: \( \Delta i_L^+ = \Delta i_L^- \).

A second way of looking at this is that \( \langle V_c \rangle = 0 \):

\[
\frac{1}{T} (V_i DT + V_c (1-D)T) = 0 \quad \Rightarrow \quad V_c = -\frac{D}{1-D}
\]

\( \Delta i \) is back-biased. Notice: there is no direct connection from input to output.

Energy is stored in \( L \) during \( D \) and transferred to the load during \( 1-D \).

\( i_D \) is in equal to \( i_L \) when the switch is off. Since \( \langle i_c \rangle = 0 \) in the steady state, \( \langle i_D \rangle = \langle i_{D0} \rangle \) (load current), \( \langle i_{D0} \rangle = \frac{V_c}{R} \)

with \( V_c < 0 \), \( \langle i_D \rangle > 0 \), as it must be. By inspection, \( \langle i_D \rangle = (1-D) \langle i_L \rangle \)

so \( \langle i_L \rangle = -\frac{1}{(1-D)} \frac{V_c}{R} = \frac{DV_i}{(1-D)^2 R} \)

\[
\langle i_D \rangle = \frac{DV_i}{(1-D)^2 R}
\]

\[
\Delta i_{\text{min}} = \langle i_L \rangle - \frac{\Delta i_L}{2} = \frac{DV_i}{(1-D)^2 R} - \frac{V_c DT}{2L}
\]

\[
L_{\text{min}} = (1-D)^2 \frac{TR}{2} \quad \text{if} \quad \Delta i_{\text{min}} = 0
\]
The capacitor supplies the load current during the off-time of the switch.

\[ \Delta V_c = \frac{1}{2} \int_{t_1}^{t_2} i_c dt = \frac{I_c \Delta t}{C} \text{ if } i_c \text{ constant during } \Delta t \]

\[ i_0 + i_c + i_0 = 0, \quad i_c = -(i_D + i_0) \quad < \Delta i_c > = 0 \text{ in S.S.} \]

\[ \Delta V = \Delta Q = \frac{(V_i - V_o) \Delta T}{C} = \frac{V_o D}{f R C} \]

\[ \Delta V_{	ext{esr}} = I_{\text{max}} R_C \quad V_{\text{ds max}} = V_i - V_o = V_i + |V_o| = V_{\text{ch max}} \]

**Discontinuous Current Mode:**

Particularly simple in this case using ideal power transfer

\[ P_o = P_i \]

\[ \frac{V_i^2}{R} = \frac{1}{2} L_0 I_{\text{max}}^2 f \quad \Rightarrow \quad \frac{V_o^2}{R} = \frac{1}{2} L \left( \frac{1}{L} V_i \Delta T \right)^2 \]

\[ \frac{V_o}{V_i} = \sqrt{\frac{D^2 T R}{2 L}} \]

Notice \( \frac{V_o}{V_i} \) depends on \( R \) as usual in DCM.

Energy stored in inductor during \( 0 < t < D T \) is completely transferred during \( DT < t < T \).
SEPIC Converter

The buck-boost we analyzed previously has a disadvantage, which is that it's inverting. How about a noninverting B-B?

\[ \frac{V_o}{V_i} = \frac{V_o}{1-D} + V_b \]

Imagine this brute-force solution - a boost with a battery to drop the voltage.

Then if \( V_b = V_i \), \( V_o = \frac{V_i}{1-D} \)

This is a stupid design, but it does what we want. Replace the battery with a capacitor. That will drop the DC voltage, but \( \langle i_c \rangle \) must \( = 0 \). Since \( \langle i_c \rangle \neq 0 \), we need something else to provide the DC load current.

Add an inductor.

By quick analysis, if \( \langle v_c \rangle = 0 \), \( \langle v_e \rangle = V_i \), assume \( V_i \) constant.

So make \( \frac{V_o}{V_i} \), we'll once again use \( \langle i_e \rangle = 0 \) to solve \( V_e \).
\begin{align*}
\Delta I_1 &= \frac{V_1 \cdot DT}{L_1} \\
\Delta I_L &= \frac{V_1 \cdot DT}{L_L} = \frac{V_1 \cdot DT}{L_2} \\
\Delta I_2 &= \frac{V_1 \cdot (V_0 + V_0) (1 - D) T}{L} = \frac{V_0 (1 - D) T}{L_1} \\
\Delta I_L &= -\frac{V_0 (1 - D) T}{L_2}
\end{align*}

Balance \( \Delta I_1 \): \( \frac{V_1 \cdot DT}{L_1} = \frac{V_0 (1 - D) T}{L_1} \Rightarrow \frac{V_0}{V_1} = 1 - D
\]

Notice \( V_{L_1} = V_1 + \Delta I_1 \), \( \Delta I_1 = \Delta I_L \). Same change in flux \((V_1 \cdot \frac{dI}{dt})\) results in same change in current. What does this sound like?

A coupled inductor with a 1:1 inductance ratio, i.e., if \( L_1 = L_2 \)

we can wind both on the same core.

Remember how coupled inductors work:

\begin{align*}
V_1 &= L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} \\
M &= \frac{L_{1} L_{2}}{L_{1} + L_{2}}
\end{align*}

If \( L_1 = L_2 = L \) and \( k = 1 \)

\( M = L \)

\( V_1 = 2L \frac{dI}{dt} \)

The inductance of a coupled inductor is \( \frac{1}{2} \) of the inductance of a discrete inductor.
Let's look at the waveforms:

\[ V_L = V_L, V_0 \]

\[ \langle i_{L1} \rangle = \langle i_{L0} \rangle \ast \]

\[ \langle i_{L2} \rangle = \langle i_{L0} \rangle \]

\[ \langle i_{C1} \rangle = 0 \]

\[ \langle i_{C2} \rangle = 0 \]

\[ \langle i_d \rangle = 0 \]

In regard to efficiency, we have copper loss in two inductor windings + ESR loss in two capacitors. This is not the lowest-loss approach.

*Notice that \( i_{L1} = i_{L1} \) is continuous, which can be an advantage for low-emission, low-interference applications.*
Cuk Converter

named for Slobodan Cuk of Scripps, later Caltech.

\[
\begin{align*}
V_{L1} & = V_i - V_o \\
\text{switch closed: } V_{L1} & = V_i - \Delta i_{L1} = \frac{V_i D T}{L_1} \\
V_{L2} & = V_o + (V_i - V_o) = \frac{V_i D T}{L_2} \\
\text{switch open: } V_{L1} & = V_i - (V_i - V_o) = \frac{V_o (1-D) T}{L_1} \\
V_{L2} & = \frac{V_o (1-D) T}{L_2} \\
\text{V.i balance: } \frac{V_i D T}{L_1} & = -\frac{V_o (1-D) T}{L_1} \\
\frac{V_o}{V_i} & = \frac{-D}{1-D}
\end{align*}
\]

advantages: both input and output current are continuous.

back-boost