DESIGN TIPS

The Sepic Converter

The most basic converter that we looked at last month is the buck converter. It is so named because it always steps down, or bucks, the input voltage. The output of the converter is given by:

\[ V_o = \frac{1}{D} V_i \]

Interchange the input and the output of the buck converter, and you get the second basic converter – the boost. The boost always steps up, hence its name. The output voltage is always higher than the input voltage, and is given by:

\[ V_o = \frac{1}{1-D} V_i \]

What if you have an application where you need to both step up and step down, depending on the input and output voltage? You could use two cascaded converters – a buck and a boost. Unfortunately, this requires two separate controllers and switches. It is, however, a good solution in many cases.

The buck-boost converter has the desired step up and step down functions:

\[ V_o = \frac{1}{D} - \frac{1}{1-D} V_i \]

The output is inverted. A flyback converter (isolated buck-boost) requires a transformer instead of just an inductor, adding to the complexity of the development.

One converter that provides the needed input-to-output gain is the Sepic (single-ended primary inductor converter) converter. A Sepic converter is shown in Fig. 1. It has become popular in recent years in battery-powered systems that must step up or down depending upon the charge level of the battery.

In this issue, we’ll go to the other end of the spectrum, and look at a converter that is far more complex, yet is often used by engineers who are unaware of the difficulties that follow.

By Dr. Ray Ridley, Ridley Engineering

Fig. 1. The Sepic converter can both step up and step down the input voltage, while maintaining the same polarity and the same ground reference for the input and output.

Fig. 2. When the switch is turned on, the input inductor is charged from the source, and the second inductor is charged from the first capacitor. No energy is supplied to the load capacitor during this time. Inductor current and capacitor voltage polarities are marked in this figure.

Fig. 3. With the switch off, both inductors provide current to the load capacitor.

In the last issue, we talked about the simplest of all converters, the buck converter, and showed how its control transfer functions could be extraordinarily complex. In this issue, we’ll go to the other end of the spectrum, and look at a converter that is far more complex, yet is often used by engineers who are unaware of the difficulties that follow.

By Dr. Ray Ridley, Ridley Engineering
**DESIGN TIPS**

**The PWM Switch Model in the Sepic Converter**

The best way to analyze both the AC and DC characteristics of the Sepic converter is by using the PWM switch model, developed by Dr. Vatché Vorpérian in 1986. Some minor circuit manipulations are first needed to reveal the location of the switch model, and this is shown in Fig. 4.

First, capacitor C1 is moved to the bottom branch of the converter. Then, inductor L2 is pulled over to the left, keeping its ends connected to the same nodes of the circuit. This reveals the PWM switch model of the converter, with its active, passive, and common ports, allowing us to use well-established analysis results for this converter.

For more background on the PWM switch model, the textbook “Fast Analytical Techniques for Electrical and Electronic Circuits” [1] is highly recommended.

**DC Analysis of the Sepic Converter**

Fig. 5 shows the equivalent circuit of the Sepic converter with the DC portion of the PWM switch model in place. The DC model is just a 1:1 transformer. We replace the inductors with short circuits, and the capacitors with open circuits for the DC analysis. You can, if you like, include any parasitic resistances in the model [2], but that’s beyond the scope of this article.

After the circuit is manipulated as shown in the figure, we can write the KVL equation around the outer loop of the converter:

\[ V_i + V_c - \frac{1}{D} V_o = 0 \]

Rearranging gives:

\[ V_i = \frac{1}{D} - \frac{1}{D} V_o \]

And the DC gain is given by:

\[ G_{dc} = \frac{1}{D} \left( \frac{V_i}{V_o} \right) \]

Here we see the ability of the converter to step up or down, with a gain of 1 when D=0.5. Unlike the buck-boost and Cuk converters, the output is not inverted.

**AC Analysis of the Sepic Converter**

You won’t find a complete analysis of the Sepic converter anywhere in printed literature. What you will find are application notes with comments like, “the Sepic is not well-understood.” Despite the lack of documentation for the converter, engineers continue to use it when applicable.

Proper small-signal analysis of the Sepic converter is a difficult analytical task, only made practical by advanced circuit analysis techniques originally developed by Dr. David Middlebrook and continued by Vorpérian [3].

If you’re going to build a Sepic, as a minimum, you need to understand the control characteristics. Fortunately, Vorpérian’s work is now available for this converter, and you can download the complete analysis notes [2].

The simplified analysis of the Sepic converter, derived in detail in [2], ignores parasitic resistances of the inductors and capacitors, and yields the following result for the control-to-output transfer function:

\[ \frac{V_o}{V_i} = \frac{1}{\left[ \frac{C_i}{C_d} + \frac{1}{L_i(C_i + C_d)} \right]} \]

Where

\[ C_i = \frac{R}{w_0 \left( \frac{L_i}{L_i + L_d} \right) + \frac{1}{C_i C_d}} \]

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DESIGN TIPS

Power Systems Design Europe     November 2006

There are several possibilities. First, the dynamic and step load requirements on the system may be very benign, with no reason to design a loop with high bandwidth. This allows the loop gain to be reduced below 0 dB before the extreme phase delay of the second resonance.

Secondly, in many practical cases, the parasitic resistances of the circuit move the RHP zeros to the left half plane, greatly reducing the phase delay. This can also be done with the addition of damping networks to the power stage, a topic beyond the scope of this article.

Thirdly, some engineers do not build a proper Sepic. In some application notes, the two inductors are wound on a single toroidal core, which provides almost unity coupling between the two. In this case, the circuit no longer works as a proper Sepic. Don’t fall into this design trap - the circuit will be far from optimum.

As you can see from these expressions, the “simplified” analysis is anything but simple. Including the parasitic resistances greatly complicates the analysis, but may be necessary for worst-case analysis of the Sepic converter. The analysis of this converter involves the use of the powerful extra element theorem, and Vorpérian’s book on circuit analysis techniques.

In addition to the inevitable fourth-order denominator of the Sepic, the most important features to note in the control transfer function are the terms in the numerator. The first term is a single right-half-plane (RHP) zero. Right-half-plane zeros are a result of converters where the response to an increased duty cycle is to initially decrease the output voltage.

When the power switch is turned on, the first inductor is disconnected from the load, and this directly gives rise to the first-order RHP zero. Notice that the expression only depends on the inductor L1, the load resistor R, and the duty cycle.

The complex RHP zeros arise from the fact that turning on the switch disconnects the second inductor from the load. These zeros will actually move with the values of parasitic resistors in the circuit, so careful analysis of your converter is needed to ensure stability under all conditions.

PSpice Modeling of the Sepic Converter

The analytical solution above does not include all of the parasitic circuit elements. As you will see from [2], there is a prodigious amount of work to be done even without the resistances.

We can also use PSpice to help understand the Sepic better. Fig. 7 shows the circuit model for a specific numerical application of the Sepic, and it includes resistances which will affect the stability of the converter, sometimes in dramatic ways.

The PSpice file listing can be downloaded from [2] so you can reproduce these results to analyze your own Sepic converter.

Fig. 8 shows the result of the PSpice analysis. The two resonant frequencies predicted by the hand analysis can clearly be seen in the transfer function plot. What is remarkable is the extreme amount of phase shift after the second resonance. This is caused by the delay of the second pair of poles, and the additional delay of the complex RHP zeros. The total phase delay through the converter is an astonishing 630 degrees. Controlling this converter at a frequency beyond the second resonance is impossible.

Summary

The Sepic converter definitely has some select applications where it is the topology of choice. How do designers get away with building such a convert-

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Figure 7. Analysis can also be done with PSpice. This figure shows a specific design example for a 15 W converter. Parasitic resistances are included in the PSpice model.

\[ Q_i = R \]

\[ \omega_c = \frac{R}{L_1} \]

\[ \frac{1}{L_1 C_1} \]

\[ \frac{1}{L_2 C_2} \]

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Analysis of the Sepic Converter

by

Dr. Vatché Vorpérián
Notes on the small-signal analysis of the

The isolated sepic converter with synchronous rectifiers

Fig. 1

Fig. 2

To get from Fig 1 to Fig 2 replace the transformer with its equivalent circuit model:

L_p \rightarrow \frac{L_s}{n^2}

reflect V_{in}, L_1, C_1 & Q_1, as follows to the secondary side:

L_f = L_{1n}^2

C_f = C_{1n}^2

V_g = V_{in} n

Q_1 \rightarrow Q_{1-ref} = \frac{V_{DS-ref} = n V_{DS}}{I_D}

I_{DS-ref} = \frac{V_D}{n}
- now redraw by around $C_f$ and slide $Q_2$ next to $Q_{1-ref}$ in the return line like this

This is the PWM switch
Replace the PWM switch with its equivalent circuit model

\[ r_e = r_{c_1} + r_{c_2} \parallel R \]

Set the small-signal sources to obtain the dc model
1. Determination of the conversion ratio and the dc operating point of the PWM switch

\[ I_o = \frac{I_g}{D} - I_g \Rightarrow \frac{P_g}{I_o} = \frac{D}{D'} \]  also \[ \frac{I_c}{I_o} = \frac{1}{D'} \]

As in the buck-boost converter, the current conversion is not affected by the parasitic elements and is given by KCL at point "p":

\[ M = \frac{V_o}{V_g} = \frac{P_g}{I_o} \eta \]
\[ \eta = \frac{P_{out}}{P_{out} + P_{lost}} \]
\[ P_{out} = I_o^2 R \]
\[ P_{lost} = I_o^2 r_{L2} + I_g^2 r_{L1} + I_c^2 r_c \]
\[ \Rightarrow \eta = \frac{1}{1 + \frac{r_{L2}}{R} + \frac{r_{L1}/D'}{R(D')} + \frac{r_c}{R} \left( \frac{I_c}{I_o} \right)^2} \]

\[ M = \frac{D}{D'} \]
\[ \frac{1}{1 + \frac{r_{L2}}{R} + \frac{r_{L1}/D'}{R(D')} + \frac{r_c}{R} \left( \frac{I_c}{I_o} \right)^2} \]

Operating point:
\[ I_c = \frac{I_o}{D'} \]
\[ V_{cp} = V_g + V_o + I_o r_{L2} - I_g r_{L1} \]
\[ V_{cp} = V_g + V_o \left( \frac{1}{D'} + \frac{r_{L2}}{R} \frac{D}{D'} \frac{I_o}{D'} \right) \]
2. Determination of $D(s)$

With $\hat{v}_g$ and $\hat{d}$ set to zero, we obtain the circuit above from which we will determine $D(s)$ for the small signal transfer functions:

\[
\frac{\hat{v}_g}{\hat{v}_o} = M \frac{N_f(s)}{D(s)}
\]

\[
\frac{\hat{v}_o}{\hat{d}} = K_D \frac{N_d(s)}{D(s)}
\]

\[
Y_{in}(s) = \frac{\hat{v}_g}{\hat{v}_o} = G_{in} \frac{N_f(s)}{D(s)}
\]

\[
Z_o(s) = \frac{\hat{v}_o}{\hat{i}_T} = R_o \frac{N_d(s)}{D(s)} ; \quad \hat{i}_T \text{ is test current source connected at the output}
\]
Fast analytical techniques for ELECTRICAL and ELECTRONIC CIRCUITS
by Vatché Vorpérian
Jet Propulsion Laboratory
California Institute of Technology
Cambridge University Press
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Today, the only method of circuit analysis known to most engineers and students is nodal or loop analysis. Although this works well for obtaining numerical solutions, it is almost useless for obtaining analytical solutions in all but the simplest cases.

In this unique book, Vorpérian describes remarkable alternative techniques to solve, almost by inspection, complicated linear circuits in symbolic form and obtains meaningful analytical answers for any transfer function or impedance. Although not intended to replace traditional computer based methods, these techniques provide engineers with a powerful set of tools for tackling circuit design problems. They also have great value in enhancing students understanding of circuit operation. The numerous problems and worked examples in this book make it an ideal textbook for senior/graduate courses, or a reference book.

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• Analyze PWM converters easily using the model of the PWM switch.

Originally developed and taught at institutions and companies around the world by Professor David Middlebrook at Caltech, the extended and new techniques described in this book are an indispensable set of tools for linear electronic circuit analysis and design.

Publisher’s note: Dr. Vatché Vorpérian is one of the rare few researchers who delight in the process of analysis of analog circuits, and in finding simple and elegant solutions to seemingly insurmountable problems. I have observed, on numerous occasions, his ability to derive models and equations overnight. In this latest book for the electrical engineer, he reveals many of his techniques. Much of it is applied to power conversion circuits—which is one of the few remaining disciplines where hand analysis is crucial to the development of circuit topologies and new technologies. It’s a ‘must have’ text for anyone serious about the field of power electronics.

Vatché’s major contributions to our field include the PWM switch model, the ZCS and ZVS quasi-resonant switch model, and the analysis of the series and parallel resonant converter.

Corrections to Previous Articles in SPM

Winter 2002 issue, Designer Series’ Part VII. We published a formula for temperature dependence of the R material. A minus sign was omitted in front of the linear coefficient of the 5th order polynomial. The correct equation should be as follows:

\[ P_{core} = ( - 3.626 \ln f + 28.32 ) f^{1.729} AB^{L-0.00075f+2.8332} g(T) \]

where

\[ g(T) = 1.8387T^5 - 4.194T^4 + 2.0955T^3 + 2.7933T^2 - 4.1658T + 2.6345 \]

was +4.1658T' in the original article.

\[ T' = T / 100 \quad \text{where } T \text{ is in degrees } C. \]

Thanks to Phil Cooke of Analog Devices for finding this error. We apologize for any inconvenience this may have caused.

Designer Series Part III, January 2001. we published an article on loop gain crossover frequency. The flyback converter of Figure 1a had a typographical error - the output voltage of the converter should have been 12 V, not 24 V. If you run the calculation for the RHP zero with 24 V output, you would have a frequency of 13.8 kHz instead of 20 kHz, and the maximum duty cycle would be 0.62.

Thanks to Fred Waechter of Phihong USA for finding this error.
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Since the converter is of fourth order, we have:

\[ D(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 \]  

(1)

For a properly designed fourth-order converter, the denominator consists of two quadratic factors whose resonances are well separated and almost entirely damped by the load. The parasitic resistances have almost no effect on the two resonant frequencies of \( D(s) \) and contribute very little to the damping of the resonances under normal loading conditions. Therefore we can write:

\[ D(s) = \left(1 + \frac{s}{\omega_{o1}Q_1} + \frac{s^2}{\omega_{o1}^2}\right) \left(1 + \frac{s}{\omega_{o2}Q_2} + \frac{s^2}{\omega_{o2}^2}\right) \]  

(2)

Expanding Eq. (2) and comparing to Eq. (1) under the assumption of moderate to high \( Q \) and well-separated resonances, we get

\[ a_1 \approx \frac{1}{\omega_{o1}Q_1} \]  

(3a)

\[ a_2 \approx \frac{1}{\omega_{o1}^2} \]  

(3b)

\[ a_3 \approx \frac{1}{\omega_{o1}Q_1\omega_{o2}} + \frac{1}{\omega_{o2}Q_2\omega_{o1}^2} \]  

(3c)

\[ a_4 \approx \frac{1}{\omega_{o1}^2\omega_{o2}^2} \]  

(3d)

We will determine these coefficients according to Eqs. (4-7) given below and perform all necessary approximations as we go along. The reference circuit is shown on the next page.

\[ a_1 = \frac{L_{J_1}}{R^{(1)}} + \frac{L_{J_2}}{R^{(2)}} + C_{J_2}R^{(3)} + C_{J_1}R^{(4)} \]  

(4)
The order in which the elements, or the ports, are taken in any one term is immaterial. For example:

\[
\frac{L_f}{R(1)} \frac{L_f}{R(1)} C_{f1} R_{(1)}^{(4)} = \frac{L_f}{R(1)} C_{f1} R_{(1)}^{(4)} \frac{L_f}{R(1)} \frac{L_f}{R(1)}
\]
reference circuit indicating normal port conditions

i) Determination of \( a \),

\[
R^{(c)} = r_L + \frac{r_c}{D^2} + \left( \frac{D}{D'} \right)^2 (R + r_{L2}) \approx \left( \frac{D}{D'} \right)^2 R
\]
\[ R^{(2)} = r_{L2} + R + \left( \frac{D}{D'} \right)^2 \left( \frac{r_c}{D^2} + r_{L1} \right) \approx R \]

\[ R^{(3)} = \frac{r_{C2} + R}{1 + \left( r_{L2} + \left( \frac{D}{D'} \right)^2 \left( \frac{r_c}{D^2} + r_{L1} \right) \right)^2} \approx r_{C2} + r_{L2} + \frac{r_c}{D^2} + r_{L1} \left( \frac{D}{D'} \right)^2 \]
\[ R^{(4)} = R_{c_1} + R^{(4')}, \]

According to the EET

\[ R^{(4')} = R_0 \cdot \frac{1 + \frac{R_n}{r_c/D^2 + R(0'/D)^2}}{1 + \frac{R_d}{r_c/D^2 + R(0'/D)^2}}, \]

\[ R_0 = r_{L_1} + r_{L_2}, \quad R_d = r_{L_1} + \left(\frac{D'}{D}\right)^2 r_{L_2} \]

\[ R_n = \frac{r_{L_1} r_{L_2}}{D^2} \]

\[ \therefore R^{(4)} \approx r_{C_1} + r_{L_1} + r_{L_2} \]
Determination of $a_2$

In order to determine $a_2$, we need to determine the following:

$$R_{(1)}^{(2)}, R_{(1)}^{(3)}, R_{(1)}^{(4)}, R_{(2)}^{(3)}, R_{(2)}^{(4)}, \text{ and } R_{(3)}^{(4)}$$

According to the reference circuit below, all of the above can be deduced from $R_{(1)}^{(1)}, R_{(2)}^{(2)}, R_{(3)}^{(3)}$, and $R_{(4)}^{(4)}$ as follows:

$$R_{(1)}^{(n)} = \lim_{L_1 \to \infty} R_{(1)}^{(n)} \quad n = 2, 3, 4$$

$$R_{(2)}^{(n)} = \lim_{L_2 \to \infty} R_{(2)}^{(n)} \quad n = 3, 4$$

$$R_{(3)}^{(4)} = R_{(4)}^{(4)} \bigg|_{R \to c_2 \parallel R}$$

Reference circuit indicating normal port conditions
Now in its 18th year of development, POWER 4-5-6 Plus is the most powerful design, analysis, and simulation software available for power supply development. The latest version includes topology selection, detailed magnetics design, capacitor selection, and feedback design in addition to voltage-mode or current-mode control.

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And now, a special version of the software is available for AP300 Frequency Response Analyzer users that predicts the response of your power supply and compares it with data collected from the AP300. They communicate with each other to show measurements overlaid on theoretical curves.
we now have

\[ R_{(1)}^{(2)} \to \infty \]

\[ R_{(1)}^{(3)} = \frac{D}{R} + R \approx R \]

\[ R_{(2)}^{(3)} = \frac{D}{R} + R \approx R \]

\[ R_{(1)}^{(4)} = \frac{D}{R} + \frac{\frac{D}{R} \left( \frac{D'}{D} \right)^2 \frac{D}{R}^2}{D^2} \approx R \left( \frac{D'}{D} \right)^2 \]

\[ R_{(2)}^{(4)} = \frac{D}{R} + \frac{\frac{D}{R} \left( \frac{D'}{D} \right)^2 \frac{D}{R}^2}{D^2} + R \approx R \]

\[ R_{(3)}^{(4)} = \frac{D}{R} + \frac{R_0}{1 + \frac{\frac{D}{R}^2 + R C_2 \left( \frac{D'}{D} \right)^2 \frac{D}{R}^2}{R_0}} \]

where \( R_0, R_d \) and \( R_0 \) are the same as before given for \( R^{(4)} \).

Hence, ignoring parasitic elements, we have

\[ a_2 = \frac{L f_2}{R \left( \frac{D'}{D} \right)^2} \]

\[ + \frac{L f_2}{R \left( \frac{D'}{D} \right)^2} + C f_2 R + \frac{L f_1}{R \left( \frac{D'}{D} \right)^2} C f_1 \left( \frac{D'}{D} \right)^2 \]

\[ + \frac{L f_2}{R} C f_1 R + \frac{L f_2}{R} C f_2 R + C f_2 \left( \frac{D}{R} \right)^2 \]

\[ a_1 = \frac{L f_1}{R} \left( C f_2 \left( \frac{D}{R} \right)^2 + C f_1 \right) \]

\[ + \frac{L f_2}{R} \left( C f_1 + C f_2 \right) \]

\[ \omega_{01} = \frac{1}{\sqrt{a_2}} = \frac{1}{\sqrt{\frac{L f_1}{R} \left( C f_2 \left( \frac{D}{R} \right)^2 + C f_1 \right) + \frac{L f_2}{R} \left( C f_1 + C f_2 \right)}} \]
Determination of $a_3$

In order to determine $a_3$, we need to determine the following

$$R_{(21)}^{(3)} = \lim_{t_2 \to \infty} R_{(2)}^{(3)} = C_2 + R \approx R$$

$$R_{(21)}^{(4)} = \lim_{t_2 \to \infty} R_{(2)}^{(4)} \to \infty$$

$$R_{(31)}^{(3)} = \lim_{t_2 \to \infty} R_{(3)}^{(4)} = C_1 + \frac{C_2 + C_1 || R + C_2 || R D^2}{D^2}$$

$$R_{(32)}^{(4)} = \lim_{t_2 \to \infty} R_{(3)}^{(4)} = C_1 + \frac{C_2 || R + C_1 + C_2 || R}{D^2}$$

$$a_3 = \frac{L_{f_1}}{(D')^2 R} \frac{L_{f_2}}{C_{f_2}} + \frac{L_{f_1}}{(D')^2 R} \frac{L_{f_2}}{C_{f_1}} C_{f_1}$$

The indeterminacy can be removed by changing the order in which the ports are taken

$$\frac{L_{f_1}}{R_{(21)}^{(3)}} \frac{L_{f_2}}{R_{(21)}^{(4)}} = \frac{L_{f_1}}{C_{f_1}} R_{(21)}^{(4)} \frac{L_{f_2}}{C_{f_1}} R_{(21)}^{(3)} \frac{L_{f_2}}{R_{(41)}^{(4)}}$$

1 2 4 1 4 2
$R_{(41)}^{(2)}$ is determined from Fig. (a) above which can be transformed to Fig. (b) by reflecting $R$ through $(0',0)^2$. It follows from Fig. (b) that

$$R_{(41)}^{(2)} = L_2 + D^2 \left[ \frac{C_1 + \frac{C}{D^2} + R(D')^2}{D} \right]$$

$$= L_2 + C + D^2 C_1 + R D'^2 \simeq R D'^2$$

$$\therefore \quad \alpha_3 \simeq \frac{L_2}{(D')^2 R} \cdot \frac{C_1}{C} \cdot \frac{R(D')^2}{D} \cdot \frac{L_2}{R D'^2}$$

$$\alpha_3 \simeq \frac{L_1 L_2 C_1}{D^2 R}$$
Determination of $\alpha_4$

Since from $\alpha_3$ we already have the expression of the term in which the ports are taken in the order 1 4 2, we will write $\alpha_4$ as

$$\alpha_4 = \frac{L_1}{R^{(1)}} \cdot \frac{C_1}{R^{(2)}} \cdot \frac{L_2}{R^{(3)}} \cdot \frac{C_2}{R^{(24)}}$$

$$\Rightarrow \quad \alpha_4 = \alpha_3 \cdot \frac{C_2}{R^{(24)}} \quad \text{and} \quad R^{(24)} = R_{c_2} + R \approx R$$

$$\Rightarrow \quad \alpha_4 = \frac{L_1 \cdot L_2 \cdot C_1 \cdot C_2}{D' \cdot 2}$$

Since $\alpha_4 = 1/\omega_0^2 \omega_0^2$, we have

$$\omega_0^2 = \frac{D' \cdot 2}{L_1 \cdot L_2 \cdot C_1 \cdot C_2} \left[ \frac{L_1}{C_1} \left( \frac{D'}{D} \right)^2 + C_1 \right] + \frac{L_2}{C_2} \left( C_1 + C_2 \right)$$

$$\omega_0^2 = \sqrt{\frac{1}{L_1 \cdot C_1 \parallel C_2} \frac{D'}{D''}} + \frac{1}{L_1 \cdot C_1 \parallel C_2}$$
Determination of $Q_1$

\[
Q_1 \approx \frac{1}{\omega_0 Q_1} \Rightarrow Q_1 = \frac{1}{\alpha_1 \omega_0},
\]

\[
\alpha_1 = \frac{L_{f_1}}{R} \left( \frac{D}{D'} \right)^2 + \frac{L_{f_2}}{R}
\]

\[
\Rightarrow Q_1 = \frac{R}{\omega_0 \left( \frac{L_{f_1}}{R} \left( \frac{D}{D'} \right)^2 + L_{f_2} \right)}
\]

Determination of $Q_2$

\[
a_3 \approx \frac{1}{\omega_0^2 Q_1 \omega_0^2} + \frac{1}{\omega_0^2 Q_2 \omega_0^2}
\]

\[
= \frac{1}{\omega_0^2 \omega_0^2} \left[ \frac{\omega_0}{Q_1} + \frac{\omega_0}{Q_2} \right]
\]

\[
= \frac{a_4}{Q_1} \left( \frac{\omega_0}{Q_1} + \frac{\omega_0}{Q_2} \right)
\]

\[
\Rightarrow \omega_0 \frac{Q_2}{a_4} = \frac{a_3}{a_4} - \frac{\omega_0}{Q_1} = \frac{1}{\omega_0 \frac{Q_2}{a_4}} - \omega_0 \left( \frac{L_{f_2} + \frac{L_{f_1}}{R} \left( \frac{D}{D'} \right)}{R} \right)
\]

\[
= \frac{\omega_0}{R} \left( \frac{1}{C_{f_2} \omega_0^2} - L_{f_2} - \frac{L_{f_1}}{R} \left( \frac{D}{D'} \right)^2 \right)
\]

\[
= \frac{\omega_0}{R} \left( \frac{L_{f_1}}{R} \left( \frac{D}{D'} \right)^2 + L_{f_2} \left( \frac{D}{D'} \right)^2 + \frac{L_{f_1} C_{f_1}}{C_{f_2}} + \frac{L_{f_2} C_{f_1}}{C_{f_2}} \right)
\]

\[
- L_{f_1} \left( \frac{D}{D'} \right)^2 - L_{f_2}
\]
\[ \frac{\omega_0^2}{Q_2} = \frac{\omega_0^2}{R} \frac{C_f}{C_f^2} (\frac{L_f}{1} + \frac{L_f^2}{2}) \]

\[ Q_2 = \frac{R}{\omega_0 (L_f + L_f^2)} \frac{C_f}{C_f^2} \left( \frac{\omega_0}{\omega_0^2} \right)^2 \]

Putting all the results together, we have:

\[ D(s) = 1 + \alpha_0 s + \alpha_1 s^2 + \alpha_2 s^3 + \alpha_3 s^4 \]

\[ \alpha_0 = \left( \frac{L_f}{D^2} \right)^2 + \frac{L_f^2}{2} \frac{\omega_0^2}{R} \]

\[ \alpha_1 = \frac{L_f}{C_f^2 + \left( \frac{D}{D_2} \right)^2} + \frac{L_f}{C_f^2} \]

\[ \alpha_2 = \frac{L_f L_f}{C_f^2} + \frac{L_f L_f}{C_f^2} \frac{1}{D^2} \]

\[ \alpha_3 = \frac{L_f L_f}{C_f^2} \frac{1}{D^2} \]

\[ D(s) \approx \left( 1 + \frac{s}{\omega_0 Q_1} + \left( \frac{s}{\omega_0} \right)^2 \right) \left( 1 + \frac{s}{\omega_0 Q_2} + \left( \frac{s}{\omega_0} \right)^2 \right) \]

\[ \omega_0 = \frac{1}{\sqrt{L_f \left( \frac{D}{D_2} \right)^2 + L_f (C_f + C_f^2)}} \]

\[ Q_1 = \frac{R}{\omega_0 \left( \frac{D}{D_2} \right)^2 + L_f^2} \]

\[ \omega_0 = \sqrt{L_f \frac{C_f^2}{D^2} + L_f C_f^2} \]
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3. Determination of the line-to-output transfer function

The line-to-output transfer function is of the form

\[ \frac{\hat{V}_d(s)}{\hat{V}_g(s)} = M \frac{N_g(s)}{D(s)} \]

in which \(M\) and \(D(s)\) have already been determined and \(N_g(s)\) corresponds to the null conditions in the response \(\hat{V}_d(s)\) to the excitation \(\hat{V}_g(s)\). The first null is given by the zero of the impedance branch, \(r_c + jL_c\), connected across the response \(\hat{V}_d(s)\) so that the first factor of \(N_g(s)\) is

\[ N_g(s) = (1 + s r_c C_{f2}) N_2(s) \]

The second null condition, \(N_2(s)\), is given by \(i_f(s) = 0\) which we will investigate next.
With \( \hat{i}_f = 0 \), KCL at node "p" yields \( \hat{i}_a = \hat{i}_c = 0 \). With \( \hat{v}_o(s) = 0 \), we see that the voltage drop across the impedance branch \( r_c + 1/sC_f \) is \( \hat{v}_{ap} \)

\[ \hat{v}_{ap} = \hat{i}_i (r_c + 1/sC_f) \]

Since \( \hat{i}_f = 0 \), \( \hat{i}_i \) also flows through the impedance branch \( r_{L_2} + sL_{f_2} \). With \( \hat{v}_o(s) = 0 \) and the voltage drop across \( r_c \) being zero, the voltage drop across the branch \( r_{L_2} + sL_{f_2} \) is \( \hat{v}_{cp} \)

\[ \hat{v}_{cp} = -\hat{i}_i (r_{L_2} + sL_{f_2}) \]

Since \( \hat{v}_{cp} = D \hat{v}_{ap} \), we have

\[ \hat{i}_i (r_c + 1/sC_f) D = -(r_{L_2} + sL_{f_2}) \hat{i}_i \Rightarrow 1 + sC_f (D r_c + r_{L_2}) + s^2 L_{f_2} C_f = 0 \]

\[ \therefore N_2(s) = 1 + sC_f (D r_c + r_{L_2}) + s^2 L_{f_2} C_f \]
4. Determination of the control-to-output transfer function

\[
\frac{\hat{V}_o(s)}{D(s)} = K_D \frac{N_d(s)}{D(s)}
\]

Although \(K_D\) can be determined from the circuit above by letting \(s \to 0\) (inductors short and capacitors open), we can derive it from the expression of the conversion ratio

\[
V_o = N(0) \cdot V_g \Rightarrow K_D = \frac{dV_o}{dD} = V_g \frac{dN}{dD} = V_g \frac{d}{dD} \left( \frac{D}{1-D} \right)
\]

\[
\Rightarrow K_D = \frac{1}{D^2}
\]

\(N_d(s)\) corresponds to the null conditions in the response, \(\hat{V}_i(s)\), to the excitation sources \((V_o/D)d^\cdot\) and \(I_c^\cdot\). The first
The second null condition in the control-to-output transfer function

Null is given by the zero of the impedance connected across \( \hat{v}_0(s) \). This impedance consists of \( (c_z + 1/sC_{f_2}) \) so that the first factor in \( N_1(s) \) is \( 1 + sC_{f_2}c_z \).

\[
N_1(s) = \left(1 + sC_{f_2}c_z\right)N_2(s)
\]

in which \( N_2(s) \) corresponds to the second null condition shown in the circuit above given by the condition \( \hat{i}_f(s) = 0 \).

Referring to the circuit above, we have at ground node 'c', according to KCL,

\[
\hat{i}_c(s) = \hat{i}_1 + sC_{f_2}\hat{i}_2
\]

(1)

With \( \hat{i}_f(s) = 0 \) and KCL at node 'p', we have

\[
I_c = D\hat{i}_c - \hat{i}_c = -D\hat{i}_c
\]

(2)
Equations (1) and (2) given
\[ i_1 + i_2 + \frac{V_c}{D} = 0 \]  \hspace{1cm} (I)

Next, we will assume \( C_i = L_2 = L_2 = C_f = 0 \). The voltage across \( L_f \) is the same as the voltage drop across \( C_f \), and \( L_f \) is which, with \( V_f = 0 \), can be written as:
\[ i_2 sL_f = i_1 (sL_f + \frac{1}{sC_f}) \]  \hspace{1cm} (3)

\[ i_2 s^2L_f C_f - i_1 (1 + s^2L_f C_f) = 0 \]  \hspace{1cm} (II)

Finally, with \( V_f(0) = 0 \), the voltage across port "a-p" of the PWM switch is
\[ \frac{V_D}{D} \hat{D} + i_1 \frac{1}{sC_f} = \hat{V}_{ap} \]  \hspace{1cm} (4)

and the voltage across port "c-p" is
\[ \hat{V}_{cp} = -\hat{i}_1 sL_f \]  \hspace{1cm} (5)

Since \( \hat{V}_{cp} = D \hat{V}_{ap} \), we have
\[ \left[ \frac{V_D}{D} \hat{D} + i_1 \frac{1}{sC_f} \right] D = -\hat{i}_1 sL_f \]  \hspace{1cm} (6)

\[ \therefore \ sC_f \hat{V}_{ap} \hat{D} + i_1 (D + s^2L_f C_f) = 0 \]  \hspace{1cm} (III)
Putting equations (I)-(III) together \( V_D = V_{ap} \)

\[
\begin{align*}
\dot{i}_1 + \dot{i}_2 + \frac{I_c}{D} \dot{d} &= 0 \\
- \dot{i}_2 (1 + s^2 L f^2_{f1} f^2_{f1}) + \dot{i}_2 s^2 L f^2_{f1} f^2_{f1} &= 0 \\
\dot{i}_1 (D + s^2 L f^1 f^1) + s C_f V_{ap} \dot{d} &= 0
\end{align*}
\]

With \( \dot{i}_1, \dot{i}_2 \) and \( \dot{d} \) different from zero, the only way the above can be satisfied is to have their determinant vanish:

\[
\begin{vmatrix}
1 & 1 & \frac{I_c}{D'} \\
- (1 + s^2 L f^2_{f1} f^2_{f1}) & s^2 L f^1 f^1 & 0 \\
D + s^2 L f^1 f^1 & 0 & s C_f V_{ap}
\end{vmatrix} = 0 \tag{7}
\]

or

\[
S^3 L f^2 f^1 f^1 f^1 - \frac{V_{ap}}{I_c} D' \left[ L f^1 f^1 + L f^2 f^1 f^1 \right] S^2 + S L f^1 D - \frac{V_{ap} D'}{I_c} = 0 \tag{8}
\]

\[
\frac{V_{ap}}{I_c} = \frac{V_g + V_o}{I_o / D'} = D' \frac{V_o}{I_o} (1 + \frac{V_g}{V_o}) = D' R \left( 1 + \frac{1}{R} \right)
\]

\[
\therefore \quad \frac{V_{ap}}{I_c} = \frac{D'^2}{D} \cdot R \tag{9}
\]
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Substitution of Eq. (9) in (8) yields the desired result, \( N_2(s) \):

\[
N_2(s) = 1 - s L_1 \left( \frac{D}{D^1} \right)^2 \frac{1}{R} + s^2 C_1 \left( \frac{L_1 + L_2}{R^2} \right) - s^3 \frac{L_2 L_1 C_1}{R} \left( \frac{D}{D^2} \right)^2
\]  

(10)

\( N_2(s) \) factors as follows:

i) Normal load conditions

\[
N_2(s) \approx \left( 1 - s L_1 \left( \frac{D^2}{D^2} \right)^2 \frac{1}{R} \right) \left( 1 - s \frac{C_1 (L_1 + L_2)}{L_1} R \left( \frac{D}{D^1} \right)^2 + s^2 \frac{L_2 C_1}{D} \right)
\]  

(11)

The first factor corresponds to the expected RHP zero due to the pulsating output filter current during \( D T_s \).

ii) No load (Synchronous rectification)

\[
N_2(s) = 1 + s^2 \frac{C_1}{L_1} (L_1 + L_2)
\]  

(12)

In this case the RHP zero behavior disappears as expected.

iii) Very light load (Synchronous rectification)

\[
N_2(s) \approx \left[ 1 - s L_1 \left( \frac{D^3}{D^2} \right)^2 \frac{1}{R} + s^2 C_1 \left( \frac{L_1 + L_2}{R^2} \right) \right] \left( 1 - s \frac{L_2}{R} \frac{D}{D^2} \right)
\]  

(13)
5. Determination of the open-loop output impedance

\[ Z_0(s) = R_0 \frac{N_0(s)}{D(s)} \]

where \( D(s) \) is the same as before and \( R_0 \) is determined from the adjacent circuits.

\[ R_U = R / \left( \left( \frac{L_2}{D} \right)^2 + \left( \frac{L_1 + L_2}{D} \right) \right) \]
Circuit for the determination of the denominator of the output admittance or the numerator of the output impedance.

Since the output impedance is the reciprocal of the output admittance we have

\[ Z_0(s) = R_0 \frac{N_0(s)}{D(s)} \]
\[ Y_0(s) = \frac{1}{Z_0(s)} = C_0 \frac{D(s)}{N_0(s)} \]

where we see that the denominator of \( Y_0(s) \) is \( N_0(s) \) which can be determined by setting the excitation of \( Y_0(s) \) to zero. The excitation of \( Y_0(s) \) is a test voltage source connected at the output which upon setting equal to zero results in the circuit shown above. It follows immediately that

\[ N_0(s) = \lim_{R \to 0} D(s) \]
Before taking the limit, we realize that one of the factors of \( N_0(s) \) must be \( 1 + sC_2C_3f_2 \) because this is the zero of an impedance branch connected directly across the output:

\[
N_0(s) = (1 + sC_2C_3f_2) N_0'(s)
\]

\[
N_0'(s) = \lim_{R \to 0} \lim_{C_2 \to 0} \lim_{C_3 \to 0} D(s)
\]

In taking this new limit, we will first ignore all the parasitic elements and obtain

\[
N_0'(s) \approx 6.5 + b_3 s^3 = 6.5\left(1 + \frac{b_3 s^2}{b_1}\right)
\]

which implies that \( N_0'(s) \) has a zero at the origin and an undamped resonance at \( \omega_{o2} = \sqrt{b_1/b_3} \).

The zero at the origin is consistent with the fact that the low-frequency asymptote of \( Z_0(s) \), \( R_0 \), goes to zero when all the parasitic elements go to zero.

Next we determine \( \omega_{o2} \):

\[
\omega_{o2} = \frac{b_1}{b_3} = \lim_{C_2 \to 0} \lim_{C_3 \to 0} \left(\frac{a_1}{a_3}\right) = \frac{D'}{L_f^2 L_f C_f} \left(\frac{L_f}{L_f + \left(\frac{D'}{D}\right)^2 L_f}\right)
\]
It follows that

\[ \omega_{oz} = \sqrt{\frac{1}{C_1 \frac{L_2}{D^2} \parallel \frac{L_1}{D_0^2}}} \]

This is very consistent with the physical picture, without parasitic elements shown below.
Finally, we include the parasitic elements to determine the zero at very low frequencies and the damping of the resonance at \( \omega_{02} \). Now we have

\[
N_0'(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3
\]
\[
\approx (1 + b_1 s) \left( 1 + \frac{b_2 s}{b_1} + \frac{b_3 s^2}{b_1} \right)
\]
\[
\approx (1 + \frac{s}{s_{\tau_1}}) \left( 1 + \frac{s}{\omega_{02} \tau_{Q2}} + \frac{s^2}{\omega_{02}^2} \right)
\]

\[
b_1 = \lim_{R \to 0} \lim_{C_1 \to 0} \lim_{C_2 \to 0} q_1 = \frac{L_{f_1}}{r_1 + \frac{r_2}{D^2} + r_2 \left( \frac{D'}{D} \right)^2}
\]
\[
+ \frac{L_{f_2}}{r_2 + r_2 \left( \frac{D'}{D} \right)^2 + \frac{r_2}{D^2}}
\]
\[
+ c_f \left[ \frac{c_c + r_1 + r_2}{r_c + r_1 + r_2} \right] \left[ \frac{r_c + D'^2 r_1 + r_2 r_2}{r_c + r_1 + r_2} \right]
\]

\[
\approx \frac{L_{f_1} + \left( \frac{D'}{D} \right)^2 L_{f_2}}{r_1 + \frac{r_2}{D^2} + \left( \frac{D'}{D} \right)^2 r_2}
\]

\[
b_1 \approx \frac{L_{f_1} + \left( \frac{D'}{D} \right)^2 L_{f_2}}{r_1 + \frac{r_2}{D^2} + \left( \frac{D'}{D} \right)^2 r_2}
\]

\[
s_{\tau_1} = \frac{1}{b_1}
\]
\[ b_2 = \lim_{f_2 \to 0} a_2 = C_f \left( \frac{L_f}{L_1} + \frac{L_f}{L_2} \right) \]

\[ \frac{1}{\omega_{oz} Q_2} = \frac{b_2}{b_1} \implies Q_2 = \omega_{oz} \frac{b_1}{b_2 \omega_{oz}^2} \]

Making the necessary substitutions, we get

\[ Q_2 = \frac{\omega_{oz} \frac{L_f}{L_1} \frac{L_f}{L_2}}{r_c + r_{d_1} D^2 + r_{d_2} D^2} \]

Putting all the results together, we have:

\[ N_0(s) = \left( 1 + \frac{s}{s_{Z_2}} \right) \left( 1 + \frac{s}{s_{Z_1}} \right) \left( 1 + \frac{s}{\omega_{oz} Q_2} + \frac{s^2}{\omega_{oz}^2} \right) \]

\[ s_{Z_1} = \frac{r_{d_1} + r_c D^2 + r_{d_2} (D'D)^2}{L_f + (D'D)^2 L_f} \]

\[ s_{Z_2} = \frac{1}{r_c C_f} \]

\[ \omega_{oz} = \sqrt{\frac{1}{C_f \frac{L_f}{D^2} \frac{L_f}{D'} L_f}} \]
6. Determination of the open-loop input admittance

The input admittance is of the form

\[ Y_{in}(s) = G_{in} \frac{N_i(s)}{D(s)} \]

\( D(s) \) is the same as before while \( G_{in} \) is determined from the adjacent circuit

\[ G_{in} = \frac{1}{L_1 + \frac{r_c}{D'} + \left(\frac{D'}{D}\right)^2 (R + r_{L2})} \]
Circuit for the determination of the denominator of the input impedance or the numerator of the input admitance

Since the input admitance is the reciprocal of the input impedance, we have:

\[ Y_{in}(s) = C_{in} \frac{N_i(s)}{D(s)} \]
\[ Z_{in}(s) = R_{in} \frac{D(s)}{N_i(s)} \]

whence we see that \( N_i(s) \) is the denominator of \( Z_{in}(s) \) which can be determined from the circuit by reducing the excitation of \( Z_{in}(s) \) to zero. The excitation of \( Z_{in}(s) \) is a test current source connected at the input which upon reducing to zero results in the circuit shown above whence it follows that

\[ N_i(s) = \lim_{D(s) \to 0} D(s) \]

\[ N_i(s) \to \infty \]
Next we determine the coefficients $a_i$ in the limit $L_1 \to \infty$.

\[
\lim_{L_1 \to \infty} a_1 = 0.
\]

\[
\lim_{L_1 \to \infty} R^{(1)} = \infty
\]

\[
\lim_{L_1 \to \infty} R^{(2)} = \infty
\]

\[
\lim_{L_1 \to \infty} R^{(3)} = C_2 + R
\]

\[
\lim_{L_1 \to \infty} R^{(4)} = C_1 + \lim_{L_1 \to \infty} \left[ \frac{\frac{C_1}{L_1} + \frac{C_2}{L_2}}{1 + \frac{C_1}{L_1} + \frac{C_2}{L_2}} \right]
\]

\[
= C_1 + \frac{C_1}{L_1} \left( 1 + \frac{C_2}{L_2} \right)
\]

\[
= C_1 + \frac{C_1}{L_1} \left( \frac{C_2}{L_2} \right)
\]

\[
= C_1 + \left( \frac{C_2}{L_2} \right)
\]

\[
\lim_{L_1 \to \infty} R^{(4)} = C_1 + \frac{C_1 + C_2 + \frac{C_2}{L_2}}{D^2}
\]

\[
\lim_{L_1 \to \infty} a_1 = C_2 \left( C_2 + R \right) + C_1 \left( \frac{C_1 + C_2 + \frac{C_2}{L_2}}{D^2} \right)
\]

\[
\approx \left[ C_2 + C_1 \left( \frac{D}{D} \right)^2 \right] R
\]
\[
\lim_{L_1 \to \infty} a_2
\]

In this limit only the following two terms survive

\[
\lim_{L_1 \to \infty} a_2 = \lim_{L_1 \to \infty} \left[ \frac{L_{f_2}}{R(2)} C_f R^{(4)}_{(2)} \right] + \lim_{L_1 \to \infty} \left[ C_f C_{f_2} R^{(3)} R^{(4)}_{(3)} \right] = \frac{L_{f_2} C_f}{D^2} + C_f C_{f_2} \left( r_{c_2} + R \right) \left( r_{c_1} + \frac{r_{c_1 + r_{c_2} + D^2 R_{11} C_2}}{D^2} \right) \leq \frac{L_{f_2} C_f}{D^2}
\]

\[
\lim_{L_1 \to \infty} a_3
\]

In this limit only one term survives

\[
\lim_{L_1 \to \infty} a_3 = \lim_{L_1 \to \infty} \left[ \frac{L_{f_2}}{R(2)} C_f R^{(3)}_{(2)} C_f R^{(4)}_{(32)} \right] = \frac{L_{f_2} C_f C_{f_1}}{C_{f_2}} \left( r_{c_2} + R \right) \lim_{L_1 \to \infty} \left[ \frac{R^{(4)}_{B_2}}{R^{(2)}} \right] = \frac{L_{f_2} C_f C_{f_1}}{R} \frac{r_{L_1} / D^2}{r_{c_1} (0.1D)^2} = \frac{L_{f_2} C_f C_{f_1} R}{D^2}
\]
\[ \lim_{L_2 \to \infty} a_4 = 0 \]

Putting all the results together we get

\[ N_i(s) = 1 + SR \left( C_f + C_f \left( \frac{D'}{D} \right)^2 \right) + s^2 \frac{L_f C_f}{D^2} + s^3 \frac{L_f C_f C_f R}{D^2} \]

In order to determine the approximate factors of \( N_i(s) \) analytically, we study the circuit at low and high frequencies.

At low frequencies we have:

\[ L_2 \text{ at low frequencies is a short.} \]

At high frequencies we have:

\[ \zeta = R \left( \frac{D'}{D} \right)^2 \left[ C_f + C_f \left( \frac{D'}{D} \right)^2 \right] = R \left( C_f + C_f \left( \frac{D'}{D} \right)^2 \right) \]
Hence, having identified the low-frequency factor, we investigate the possibility of factoring \( N_i(s) \) as follows:

\[
N_i(s) \approx \left(1 + \frac{S}{S_{2i}}\right) \left(1 + \frac{S}{\omega_i Q_i} + \frac{S^2}{\omega_i^2}\right)
\]

where

\[
S_{2i} = \frac{1}{R \left(C_{f_2} + C_{f_1} \left(\frac{D'}{D}\right)^2\right)}
\]

\[
\omega_i^2 = \frac{C_{f_2} + C_{f_1} \left(\frac{D'}{D}\right)^2}{L_{f_2} C_{f_1} C_{f_2}} = \frac{1}{L_{f_2} C_{f_1} C_{f_2} D^2} \left(\frac{C_{f_1} C_{f_2}}{D^2} + \frac{D'}{D^{2'}}\right)
\]

\[
Q_i = \frac{R}{\omega_i} \frac{C_{f_2} + C_{f_1} \left(\frac{D'}{D}\right)^2}{C_{f_1} L_{f_2}} D^2 = \omega_i R C_{f_2}
\]

This resonance can be verified by examining the circuit without any damping as shown below.
*SEPIC CONVERTER WITH PWM SWITCH MODEL
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SEPIC CONVERTER FROM POWER SYSTEMS DESIGN EUROPE NOVEMBER 2006 ISSUE
Dr. Ray Ridley’s Design Tips

Vin  1  100  AC  0
RL1  1  2   0.001
L1   2  4   10E-05
RL2  4  3   0.001
L2   3  0   10E-05
RC1  100  7  0.003000
C1   7  0   6.80E-04
RC2  5  6   0.001000
C2   6  0   2.20E-03
R   5  0   1
Vc  11  0   AC  1
Rvc  11  0  10MEG
X1  100  5  4  11 PWMCCM

.AC  DEC  100  10Hz  10000Hz
.PRINT AC  VDB(5) VP(5)
.PROBE

*PWM SWITCH MODEL
SWITCH MODEL PARAMETER VALUES: Vap=-Vo/D  Ic = -Io/D'
.SUBCKT PWMCCM 1  2  3  4
E2   7  1  4  0  -41.667
G1   1  2  4  0  -37.5
Fxf  7  2  Vxf  0.6
Exf  9  2  7  2  0.6
Vxf  9  3  0
Rvc  4  0  10MEG

.ENS
.END
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