Abstract—This paper presents an analytic approach to spontaneous emission in resonators with distributed Bragg reflectors (DBR’s). The foundation of our analysis is the hard mirror (or penetration depth) approximation, which we extend to radiation with both angular and frequency distributions. This has allowed us to derive approximate analytic expressions for the divergence angle of the spontaneous emission, the spontaneous emission rate and the spontaneous emission coupling factor in planar DBR resonators. These analytic tools provide insight into the considerable limitations to controlling spontaneous emission with DBR boundaries. We also explore cavity controlled spontaneous emission with the classical tools of intracavity field profiles, the induced EMF Method and millimeter wave experiments—all of which are applied to distributed mirror boundaries.

I. INTRODUCTION

The birth of the quantum theory of radiation brought with it the realization that the spontaneous decay rate of an excited state depends on an atom’s environment as well as its electronic structure. Purcell in 1946 noted that the spontaneous emission rate was in fact sensitive to the placement of reflectors adjacent to a radiating dipole and could be “controlled” with the inclusion of appropriate boundary conditions [1]. The mechanism for this controlled spontaneous emission is that the fields radiated by the atomic dipole must be consistent with the boundary conditions. The same mechanism manifests itself as a sensitivity of the radiation resistance, or rate of energy loss, of antennas to conducting surfaces in their immediate surroundings.

Semiconductor microcavities that seek to optimize device performance—such as threshold current, modulation bandwidth and relative intensity noise [2]–[4]—by ‘controlling’ spontaneous emission typically consist of wavelength scale resonators with high reflectivity mirrors. Vertical cavity semiconductor devices utilize distributed mirrors to obtain high finesse cavities. The theoretical investigation of spontaneous emission in distributed mirror resonators has involved extensive numerical calculation of the angular spectrum of the spontaneous emission [4]–[6]. In this paper, we discuss an analytic approach to spontaneous emission in practical optical microcavities. To this end, we will introduce the concepts of (1) the equivalent hard mirror and (2) the equivalent array of image dipoles. Although the penetration depth (or hard mirror) approximation is often used to simplify calculations, there has been a great deal of confusion stemming from the haphazard application of this approximation. Because the hard mirror approximation is the foundation for our analysis, we start in Section II by deriving the criteria for its application. Section III covers the extension of the hard mirror approximation to radiation with both angular and frequency distributions. In Section IV, we introduce an approximate model for the DBR cavity by considering a simple image array. Using this simplified geometry, we derive an analytic expression for the spontaneous emission rate of an atom in a realistic semiconductor microcavity. Finally, in Section V, we demonstrate a technique utilizing microwave antennas and cavities to experimentally verify our theoretical analysis.

II. BACKGROUND: THE SIMPLE HARD MIRROR

The absence of low-loss metallic mirrors for resonators at optical frequencies has required the implementation of dielectric or semiconductor multilayer coatings—distributed Bragg reflectors (DBR’s). The distributed nature of these mirrors results in their reflectivity exhibiting a complex dependence on the frequency and angle of the incident light [7], [8]—as can be seen from Fig. 1. Therefore, the calculation of optical parameters for DBR structures in vertical cavity lasers has typically required the spatial and temporal Fourier decomposition of the incident mode profile—making the calculation numerically intensive [9], [10].

While numerical solutions are usually sufficient, an analytic approximation allows us to isolate the essential physics of the DBR resonator. We seek to approximate the DBR reflectivity with the simplest possible reflector—the hard mirror. A hard mirror is defined here to mean a reflector with a constant complex reflectivity,

$$r_h = r_c \exp(-i\phi). \quad (1)$$

The hard mirror reflection is independent of both incident angle and frequency. $r_c$ is allowed to be either a positive or negative real number. In addition to specifying $r_h$, we have the freedom to displace the hard mirror by some distance, $L_{pen}$ (Fig. 2). As seen from this distance, the hard mirror has a phase variation that is dependent on both the frequency and the incident angle

$$r'_h(k_z) = r_c \exp(-i\phi) \exp(2ik_zL_{pen}) = r_c \exp(-i\phi') \quad (2)$$

The mechanism for this controlled spontaneous emission is that the fields radiated by the atomic dipole must be consistent with the boundary conditions. The same mechanism manifests itself as a sensitivity of the radiation resistance, or rate of energy loss, of antennas to conducting surfaces in their immediate surroundings.
The magnitude and phase of the reflectivity from a GaAs-AlAs DBR versus (a) the incident angle for s-polarized light at the Bragg frequency and (b) the frequency at normal incidence. The dashed lines show the phase variation of hard mirrors using the penetration depths (a) \( L_D \) and (b) \( L_r \).

where the phase \( (\phi') \) varies linearly with the frequency and with the cosine of the incident angle

\[
k_z = \frac{n_t \omega}{c} \cos \theta_i \approx \frac{n_t \omega}{c} \left( 1 - \frac{\theta_i^2}{2} \right). \quad (3)
\]

The primed variables, \( r'_h \) and \( \phi' \), indicate that the reference plane is a distance \( L_{\text{pen}} \) away from the hard mirror and \( n_t \) is the cavity index. \( L_{\text{pen}} \) is used as a fitting parameter for the phase variation in a real DBR. As seen from the dashed curves in Fig. 1(a) and (b), the DBR resembles the hard mirror only for angles and frequencies well within the band stop, where the power reflectivity, \( r_h^2 \), is approximately constant. \( r_h \) can be either positive or negative depending on the reflected phase shift from the DBR on exact resonance. Since a normally incident plane wave at the Bragg frequency (with a wave vector of \( k_e \)) must have only a 0 or \( \pi \) phase shift upon reflection, the phase of the hard mirror \( \phi \) must compensate for the displacement;

\[
r_h'(k_z) = r_h \exp(-2ik_cL_{\text{pen}}) \exp(2ik_cL_{\text{pen}}). \quad (4)
\]

By far the most common mistake made in implementing the hard mirror approximation is in neglecting the constant phase shift of the hard mirror reflection.

It is also important to realize that the two \( L_{\text{pen}} \)'s used in Fig. 1(a) and (b) were different. In general, the effective distances obtained from the variation of the phase as a function of incidence angle at the Bragg frequency, \( L_D = (1/2k_c)/(\partial^2 \phi'/\partial \theta_i^2) \), and the phase as a function frequency at normal incidence, \( L_r = -(\epsilon/2n_t)(\partial \phi'/\partial \omega) \), will be different [10], [11]. The respective hard mirror reflections are

\[
r'_h(\theta_i, \omega = \omega_e) = r_c \exp(-i(k_c - k_e)2L_D) \quad (5)
\]

and

\[
r'_h(\theta_i = 0, \omega) = r_c \exp(-i(k_c - k)2L_r). \quad (6)
\]

It has been shown that \( L_D \) and \( L_r \) can be calculated analytically and are related by a single multiplicative constant: \( L_D = \gamma L_r \) [10]. The hard mirror approximation replaces the low and high index materials of the DBR with material having the cavity index, \( n_t \), so \( \gamma \) should depend only on the ratios \( n_L/n_t \) and \( n_H/n_t \). In smaller index materials the wavelength is larger relative to the beam spot size, therefore the rate of diffraction varies inversely with the index. Also since \( L_r \) is proportional to the index, \( \gamma \) has quadratic dependence on the material indices. Lastly, because the lowest index material will dominate the DBR diffraction the ratio must have form \( n_t/n_t^2 + n_t^2/n_H^2 \), not \( n_t^2/(n_t^2 + n_H^2) \). The final form for \( \gamma \) is [10]

\[
\gamma = \frac{n_t^2}{2} \left( \frac{1}{n_t^2} + \frac{1}{n_H^2} \right). \quad (7)
\]

Before proceeding further, let us summarize what the two hard mirror approximations have involved. Both approaches fit the phase variation in a DBR with an effective penetration depth. They also assume a reflector with a constant magnitude and phase of reflectivity. Because all of these parameters can be specified analytically, the hard mirror approximation is extremely useful.

We have, however, neglected the vector nature of the electric field in the hard mirror definition. It can be shown that the
TABLE I

THREE DBR’S HAVING DIFFERENT LOW AND HIGH INDICES AND MIRROR PERIODS WITH THE PARAMETERS NECESSARY FOR CONSTRUCTION OF THEIR HARD MIRROR EQUIVALENTS AND MIRROR.

<table>
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<tr>
<th>DBR</th>
<th>( n_h )</th>
<th>( n_l )</th>
<th>( \Delta n )</th>
<th>( \Delta L )</th>
<th>( L_p )</th>
<th>( L_D )</th>
<th>( L_{pen} )</th>
<th>( \gamma )</th>
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TABLE II

THREE DBR’S HAVING DIFFERENT LOW AND HIGH INDICES AND MIRROR PERIODS WITH THE PARAMETERS NECESSARY FOR CONSTRUCTION OF THEIR HARD MIRROR EQUIVALENTS AND MIRROR.

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A. Recovering the Hard Mirror Boundary

For a paraxial beam with a frequency bandwidth narrower than the band stop of the DBR, the reflectivity of the DBR is the product of the two hard mirror responses discussed above[10]:

\[ r_{DBR}(k, k_e) = r_e^{i(k - k_e)2L_c} - i(k - k_e)2L_D. \]  \( \text{(8)} \)

Generally, (8) cannot be reduced to a hard mirror specified by a single complex reflection and a penetration depth, \( L_{pen} \), as in (4). Equation (8) represents a mirror with a frequency dependent phase that has been displaced from the physical mirror boundary by a distance \( L_D \). In this way all of the angular dependence has been removed from the effective hard mirror reflectivity. Comparing the hard mirror reflectivity as seen from a distance of \( L_D \),

\[ r_h'(\omega) = r_e^{-i\phi(\omega)}e^{2L_D k_e}, \]  \( \text{(9a)} \)

with (6), we find that

\[ \phi(\omega) = 2(k_e - k)L_c + 2kL_D. \]  \( \text{(9b)} \)

This frequency dependent reflectivity will be the generalized hard mirror boundary that we shall use in the remainder of this paper.

The frequency dependent reflectivity, while allowing for analytic evaluation, does not have the intuitive appeal of the simple hard mirror described in (4). However, (8) reduces to the simple hard mirror reflectivity for three cases:

1) paraxial beams at the Bragg frequency, where \( k = k_e \) and \( L_{pen} = L_D \)
2) normally incident plane waves with frequencies within the band stop, where \( k = k_e \) and \( L_{pen} = L_c \)
3) DBR’s with \( \gamma = 1 \), where \( L_{pen} = L_c = L_D \)

Case 1) and 2) represent trivial solutions and are the hard mirror approximations that have been explored above. Case 3) represents a simple hard mirror construction which is expected to approximate the phase variation as a function of both frequency and angle.

Fig. 3 shows \( \gamma \) and the fractional bandwidth of a DBR as a function of the low and high indices (normalized to the cavity index). We see that \( \gamma \) varies from 1 to 3 for the DBR’s currently employed in vertical cavity devices. The fractional bandwidth is the ratio of mirror band stop width to the Bragg wavelength, \( \Delta \lambda/\lambda_c \). We plot the bandwidth contours so that we can estimate the validity of the approximation that the magnitude of reflectivity is constant. Of particular interest are the \( Al,Ga_{1-x}As-Al,Ga_{1-x}As \) \((x \neq 0) \) mirror systems which are prevalent in SEL’s. These mirrors have \( 1 < \gamma < 1.2 \) and \( 0 < \Delta \lambda/\lambda_c < 10\% \).

III. SPONTANEOUS EMISSION AND THE HARD MIRROR

Several researchers use the hard mirror construction for the study of spontaneous emission. Rogers et al. have used \( L_{pen} \) as an experimental fitting parameter in their lumped mirror model for spontaneous emission in a microcavity [12]. Similarly, Björk, et al. have used \( L_{pen} \) as a fitting parameter for their numerical calculations of spontaneous emission beam divergence [13]. We investigate the validity of such hard mirror constructions for treating spontaneous emission.
The proportionality constant, $\gamma$, between the two penetration depths, $L_1 = \gamma L$, versus the low and high indices of the DBR (normalized to the cavity index). Also shown is the fractional bandwidth of the DBR.

### B. The Hard Mirror Resonator

Once the effective hard mirror resonator is constructed, we find analytic expressions for the resonator transmission, reflection and the beam divergence of spontaneous emission (the angular width of the central lobe). The field transmission through a hard mirror Fabry–Perot resonator is:

$$ t = \frac{t_1 t_2 e^{jk_e L'_c}}{1 - r_1 r_2 e^{2jk_e L'_c}} $$

(10a)

$$ = \frac{t_1 t_2 r_1 r_2 e^{2jk_e L'_c}}{1 - r_1 r_2 e^{2jk_e L'_c} - j_1 - j_2} $$

(10b)

where $r_{1,2}$ are the complex hard mirror reflections with phase shifts $\phi_{1,2}(\omega)$. The effective cavity length, $L'_c$, is the physical cavity length plus the diffraction penetration into the two mirrors:

$$ L'_c = L_c + L_{D1} + L_{D2} $$

(11)

Since $L'_c$ includes the penetration into the DBR’s, it is no longer a multiple of half-wavelengths. The phase shift upon reflection must be included in the transmission in order to regain $t^* = 1$ for a resonant plane wave. The power transmission is then,

$$ T = \left| t \right|^2 = \frac{|t_1|^2 |t_2|^2}{1 - 2R \cos(2k_e L'_c - \phi_1 - \phi_2) + R^2} $$

(12a)

$$ = \frac{1 - 2R \cos(2k_e L'_c - \phi_1 - \phi_2) + R^2}{2R} $$

(12b)

where $R$ is the power mirror reflectivity, $R = r_1 r_2$. We can appreciate the utility of this approximation by comparing the hard mirror resonator with the DBR resonator. Fig. 4 shows a comparison of the analytic expression for the power reflectivity ($R = 1 - T$) for the exact one-wavelength GaAs–AlAs 15-period DBR resonator and the equivalent hard mirror resonator. We see that for angles in the stop band of the DBR resonator the agreement is excellent. Equation (12) allows us to analytically explore the optical properties of the DBR resonator.

The full width at half maximum of the transmission peak is especially relevant to spontaneous emission. The spontaneous emission intensity is greatest on resonance and falls off with increasing $\theta$, as we move off resonance. Therefore at a single frequency, the spontaneous emission divergence angle will be approximately equal to the full width at half maximum of the resonance transmission peak [13].

The full width at half maximum of the transmission peak, $\Delta \theta$, can be expressed as follows:

$$ \frac{T(k_\pm = k \cos(\Delta \theta/2))}{T(k_\pm = k)} = \frac{1}{2} $$

$$ = \frac{1 - 2R \cos(2k_e L'_c - \phi_1 - \phi_2) + R^2}{1 - 2R \cos(2k_e L'_c - \phi_1 - \phi_2) + R^2} $$

(13)

For high-Q resonators (where the transmission peak is narrow), we obtain

$$ \left( \frac{\Delta \theta}{2} \right)^2 = \frac{1}{k_e L'_c} \left( 2k_e L'_c - \phi_1 - \phi_2 - 2k_e L'_c + \arccos \left( \frac{-1 - R^2 + 4R \cos(2k_e L'_c - \phi_1 - \phi_2)}{2R} \right) \right) $$

(14a)

$$ \left( \frac{\Delta \theta}{2} \right)^2 = \frac{1}{k_e L'_c} \left( 2(k_e - k_e) L'_c + \arccos \left( \frac{-1 - R^2 + 4R \cos(2(k_e - k_e) L'_c)}{2R} \right) \right) $$

(14b)

where

$$ L'_c = L_c + L_{r1} + L_{r2} $$

(15)
Fig. 5. The full width half maximum of the angular distribution for spontaneous emission from an atom in the center of a Fabry-Perot resonator utilizing GaAs–AlAs and Si/SiO$_2$ DBR’s. The angular spread at the Bragg frequency is plotted as a function of the number of DBR mirror periods. The comparison is between the numerically calculated beam divergence and the analytic result using $L_D$.

At the center frequency, (14) reduces to the simple expression:

$$\frac{\Delta \theta}{2} = \frac{1}{k_x L'_c} \left( \text{arccos} \left( \frac{1 - R^2 + 4R}{2R} \right) \right)$$

where we have used an approximation for small angles

$$y \approx \text{arccos} \left( 1 - \frac{y^2}{2} \right).$$

Our hard mirror construction coupled with (14) analytically specifies the divergence of the spontaneous emission from a planar microcavity. We see from Fig. 5 that the exact numerical solution agrees with the analytic expression for both AlAs–GaAs ($\gamma = 1.2$) and SiO2/Si ($\gamma = 3.1$). However, since $L_D$ is exact only for paraxial beams there is noticeable error for lower finesse cavities where there is more spontaneous emission power at large incidence angles.

Our analytic expression also agrees with the numerical calculations of G. Björk, et al. [13]. For a resonator using two Al$_{1.25}$Ga$_{0.75}$As–AlAs DBR’s having 93% reflectivities and an Al$_{0.3}$Ga$_{0.7}$As cavity, they have shown that the FWHM of the spontaneous emission beam divergence at a wavelength of 786 nm is 21 degrees. Using our analytic expressions for $L'_c$ and $r_c$ in (16), we obtain a beam divergence of 20.5 degrees. The good agreement confirms that the lumped mirror model of [13] can be constructed analytically.

The hard mirror, $r_h$, is unique in that there is no dependence on the incident angle of the light in either the magnitude or the phase of the reflectivity. A single dielectric interface has a magnitude of reflectivity that varies strongly with incident angle and a metal reflector has a phase that varies strongly with incident angle [15]. So while the DBR boundary condition appears complex, its unique resemblance to the hard mirror allows us to greatly simplify the calculation of field related quantities.

IV. SPONTANEOUS EMISSION AND IMAGE DIPOLES

The numerical calculation of spontaneous emission in microcavities has relied on a steady state analysis. This analysis is valid when the dipole decorrelation time (inversely proportional to the homogeneous linewidth) and spontaneous emission lifetime are much longer than the photon lifetime in the cavity [16], [17]. The cavity field must lose all memory of previous emission processes before another photon is emitted. Without this requirement, the steady state analysis, which assumes an infinite coherence time, would not agree well with atomic experiments of researchers such as Drexhage, et al. [18]. We shall restrict our analysis to this short photon lifetime case.

In addition, semiconductor materials have emission linewidths resulting primarily from inhomogeneous broadening. Assuming that emission rates are always much smaller than the inhomogeneously broadened linewidth, we split the problem into two parts. We first consider a single frequency component with an angular spread, then weight this intensity by the free-space lineshape function for spontaneous emission. We repeat this for all frequency components and then reconstruct the reflected spontaneous emission.

A. The Simple Hard Mirror

Once we construct a hard mirror boundary that has no dependence on incident angle, we replace a single hard mirror with a single image element (Fig. 6). In the case of a Fabry–Perot resonator, we construct an infinite array of image dipoles. For resonators with reflectivities less than unity each image element will have a smaller oscillator strength after reflection through the boundary. The $n$th dipole will have a moment of

$$p_n = p_0 (r_{h1} r_{h2})^{n|n|/2}$$

for $n$ even

$$= p_0 \frac{2 (r_{h1} r_{h2})^{(n+1)/2}}{(1 + \frac{n}{|n|}) r_{h2} + (1 - \frac{n}{|n|}) r_{h1}}$$

for $n$ odd.

For a symmetric resonator with $r_{h1} = r_{h2}$ the above expressions simplify to

$$p_n = p_0 r_n^{n|n|}.$$  

For an array of image atoms, the interpretation of the altered density of modes at different cavity spacings is replaced by the concept of interference between the dipole and its images. For example, inhibited emission is viewed as destructive
interference of the image emissions with those of the physical dipole. As the cavity dimensions increase, the density of modes approaches the free-space value or equivalently the image elements move away from the physical dipole washing out the interference. The image array, however, presents a more natural treatment of radiation in lossy resonators.

With the construction of the equivalent array and the steady state approximation, we can immediately determine the classical field profile within the resonator by using a superposition of the full fields radiated by each dipole [20]:

\[ E_r = \frac{p_n \sin \theta}{2\pi r^2} \left[ 1 + \frac{1}{ikr} \right] e^{-jkr} \]

\[ E_\theta = \frac{p_n \sin \theta}{4\pi r} \left[ 1 + \frac{1}{ikr} - \frac{1}{(kr)^2} \right] e^{-jkr} \]

\[ E_\phi = 0. \]  

(21)

Here \( \theta \) is measured relative to the direction of the dipole moment, whereas \( \phi \) is relative to the reflector normal. Fig. 7 shows the total electric field in the plane perpendicular to the hard mirrors (\( \theta_i = \pi/2 \)) for a horizontal dipole.

\[ E(x, y) = \sum_n p_n e^{-ikr} \left[ \frac{1}{(kr)^2} + i \frac{kr}{(kr)^2} - i \frac{(kr)^2}{(kr)^2} \right] \]  

(22)

such that

\[ k r = \frac{2\pi}{\lambda} \sqrt{n L_\perp + (-1)^n |z_0| - y^2 + x^2}. \]  

(23)

The summation is over the dipole radiators where \( L_\perp \) is the effective cavity spacing \( |z_0| \) is the distance from the real dipole to the midpoint of the effective cavity and \( p_n \) is the dipole moment of the \( n \)th dipole. Naturally, there is a singularity in the field at the location of the physical dipole.

The resulting field pattern for this linear array of Hertzian dipoles provides an excellent visualization tool. Fig. 7(b) shows the field patterns for a one-wavelength symmetric resonator utilizing the hard mirror equivalent for DBR-4. The most striking feature is the transverse structure of the intracavity field. The complex transverse structure is a result of the interference between the self-field (or free-space field) of the physical dipole and the image fields. We can subtract the self-field from the field-pattern leaving only a slow roll-off in the field pattern along the transverse direction (Fig. 7(c)). Within the complex field pattern of Fig. 7(b), we can discern the cavity eigenmode—representative of the spontaneous emission coupling efficiency into the cavity eigenmode. Lastly, in Fig. 7(d) we examine a resonator with the same one-wavelength distance between DBR's but with mirrors having a greater penetration depth (DBR-6). Since the dipole separa-
tions are greater for this resonator, the cavity does not greatly alter the free-space field pattern. As a result, the spontaneous emission lifetimes adjacent to DBR’s with large penetration depths are not expected to be altered much from their free-space values.

In order to calculate the spontaneous emission rate for the atomic dipole, we superpose only the far-field contribution from each dipole and integrate over all angles to determine the spontaneous emission power in the cavity. For a dipole in front of a single mirror we obtain the following relations for the emission time normalized to the free-space decay time [18]:

\[ \frac{\tau_0}{\tau_\perp} = 1 - \frac{3}{2} \int_0^1 r_e(1 - u^2) \cos(2kd - \phi) du \] (24)

and

\[ \frac{\tau_0}{\tau_\parallel} = 1 + \frac{3}{4} \int_0^1 r_e(1 + u^2) \cos(2kd - \phi) du \] (25)

where \( u = \cos \theta \), \( \tau_\perp \), and \( \tau_\parallel \) are the spontaneous emission lifetimes for a dipole perpendicular to the plane of the reflector and for randomly oriented dipoles parallel to the plane of the reflector, respectively. As mentioned before, the magnitude and phase of the reflector are generally a function of the incident angle thereby preventing an analytic solution to the above integrals. However, using the hard mirror reflectivity we obtain the following closed form solutions for the spontaneous emission time of the atom:

\[ \frac{\tau_0}{\tau_\perp} = 1 - \frac{3}{2} \frac{r_e}{(2kd)^3} \left[ 2(2k^2d^2 + 1) \sin(\phi) - 4kd \cos(2kd - \phi) + 2 \sin(2kd - \phi) \right] \] (26)

and

\[ \frac{\tau_0}{\tau_\parallel} = 1 + \frac{3}{4} \frac{r_e}{(2kd)^3} \left[ 2(2kd^2 - 1) \sin(\phi) + 4kd \cos(2kd - \phi) + 2(4k^2d^2 - 1) \sin(2kd - \phi) \right] \] (27)

where, as in (8), \( \phi = 2(k_c - k)L_r + 2kL_D \) also \( d = z + L_D \) where \( z \) is the distance from the atom to the DBR.

The radiation adjacent to the hard mirror differs significantly from that adjacent to an ideal conducting mirror. The constant phase shift of the mirror introduces additional terms in (26) and (27) suggesting that increasing \( L_D \) is more significant than displacing the DBR (increasing \( z \)). Fig. 8 shows \( \tau_0/\tau_\perp \) and \( \tau_0/\tau_\parallel \) as a function of \( z \) for an idealized reflector and DBR’s4-6, \( L_D = 0.0, 0.297, 0.632, \) and 0.948 \( \mu m \). We see that modulation of the emission rates by the reflector is diminished with increasing \( L_D \). In addition, we observe that the curves in Fig. 8(a) and (b) receive a global shift as a result of the fixed phase shift (\( \phi \)). The result for vertical dipoles is that for some hard mirror boundaries the dipole emission is never inhibited as \( z \) varies (e.g., DBR-6) and for others it is never enhanced (DBR-4).

This same approach can be applied to the microcavity with two hard mirrors. In this case, the dipole emission by each of the images contributes to a total field that can be represented by a geometric series. The resulting emission rates are

\[ \frac{\tau_0}{\tau_\perp} = \frac{3}{2} \int_0^1 I_\perp(u)(1 - u^2) du \] (28)

and

\[ \frac{\tau_0}{\tau_\parallel} = \frac{3}{4} \int_0^1 I_\parallel(u)(1 + u^2) du \] (29)

where (30) and (31) are found at the bottom of the page, and where \( d_1 \) and \( d_2 \) are the distances from the dipole to the reflectors. \( I_\perp \) and \( I_\parallel \) express how the cavity modifies the power

\[ I_\perp(u) = \frac{1 - R^2 - (1 - r_{c1}^2)r_{e1}\cos(2kd_1u - \phi_1) - (1 - r_{c2}^2)r_{e2}\cos(2kd_2u - \phi_2)}{1 - 2R\cos(2kd_1u - \phi_1 + 2kd_2u - \phi_2) + R^2} \] (30)

and

\[ I_\parallel(u) = \frac{1 - R^2 + (1 - r_{c1}^2)r_{e1}\cos(2kd_1u - \phi_1) + (1 - r_{c2}^2)r_{e2}\cos(2kd_2u - \phi_2)}{1 - 2R\cos(2kd_1u - \phi_1 + 2kd_2u - \phi_2) + R^2} \] (31)
carried in a plane wave traveling in the direction \( \theta_i \). Naturally \( I_\perp = I_\parallel = 1 \) for \( r_3 = r_2 = 0 \). Appendix II provides further discussion of how incident fields are modified by the cavity: This provides some useful intuition into the origin of \( I_\perp \) and \( I_\parallel \).

The above expressions coupled with the field profiles emphasize the influence of diffraction in the resonator (and DBR) on "controlling" spontaneous emission.

**B. Introducing a Hole in the Hard Mirror**

Our success in analytically predicting the beam divergence in Section III might lead us to the conclusion that (26)-(31) apply directly to DBR resonators. We saw in Fig. 4 that the hard mirror resonator approximates the DBR resonator for angles less than \( \theta_s \), where the reflectivity is approximately constant, whereas the above derivation assumed a constant magnitude of reflectivity as \( \theta_i \) varied from 0 to \( \pi/2 \).

A simple solution is suggested by the numerical results of [4] and [13], they show that for \( \theta_s < \theta_i < \theta_c \), the spontaneous emission intensity is approximately equal to the free-space intensity. This is reasonable since the pass band usually represents a region of lowered reflectivity, implying that the boundary condition does not significantly alter the field in this direction. Under this approximation we can repeat the above integrations with the added condition that \( r_\perp = 0 \) in the pass band:

\[
|r_\parallel| = r_\parallel: \quad \begin{cases} 
0 < \theta_i < \theta_s \text{ or } u_1 < u < 1 & (32a) \\
0: \quad \theta_s < \theta_i < \theta_c \text{ or } u_2 < u < u_1 & (32b)
\end{cases}
\]

where \( u_1 = \cos \theta_c, u_2 = \cos \theta_s \),

\[
\sin \theta_c = \frac{2n_H n_L}{n_{ex}(n_H + n_L)}
\]

and [4]

\[
\sin \theta_s = \frac{n_{ex}}{n_\parallel} \sqrt{1 - \frac{4n_\parallel^2}{n_H + n_L}}. \quad (34)
\]

In addition, we observe that for angles greater than the resonator critical angle there is no spontaneous emission—we assume that spontaneous emission into guided modes is negligible. We shall refer to this approximate boundary condition as the segmented hard mirror (SHM), Fig. 9. The emission rates for dipoles in an SHM resonator are

\[
\frac{\tau_0}{\tau_\perp} = \frac{3}{2} \int_{u_1}^{u_\parallel} I_\parallel(u)(1 - u^2)du + \frac{3}{2} \int_{u_2}^{u_\parallel} 1 - u^2 du \quad (35)
\]

and

\[
\frac{\tau_0}{\tau_\parallel} = \frac{3}{4} \int_{u_1}^{u_\parallel} I_\parallel(u)(1 + u^2)du + \frac{3}{4} \int_{u_2}^{u_\parallel} 1 + u^2 du \quad (36)
\]

In order to test the applicability of the SHM, we compare how the SHM and DBR modify the power carried in various plane wave components as a function of the angle. The results for a symmetric AlAs resonator embedded in GaAs utilizing two 17-period GaAs–AlAs DBR’s and the segmented hard mirror (SHM) equivalents are shown in Fig. 10. This is done for isotropic radiators (neglecting the \( (1 \pm u^2) \) dependence) in both one-wavelength and half-wavelength resonators. We see that the analytic spontaneous emission intensity approximates the exact intensity for all angles in a piecewise manner. The integral of the exact and approximate intensity agree to better than 0.3% for both curves.
C. Spontaneous Emission Coupling into the Cavity Eigenmode

So far we have been concerned with the calculation of total spontaneous emission lifetimes. For device applications, the relevant parameter is the amount of spontaneous emission that couples into the cavity eigenmode. This number is representative of the spontaneous coupling efficiency of a microcavity LED into a fiber, for example. The ratio of the spontaneous power into the cavity eigenmode (with divergence angle \( \Delta \theta \)) versus the spontaneous emission power in all directions is \( \beta = P_{\Delta \theta}/P_\theta = \tau_i/\tau_{\Delta \theta} \). For illustrative purposes we confine ourselves to a single frequency and consider a single component of the spontaneous emission coupling constant, \( \beta_n \).

It is important to realize that \( \beta_n \) is not the spontaneous emission coupling efficiency into the lasing mode. The mode that we are considering is the 'cold cavity eigenmode.' A calculation of the spontaneous emission coupling efficiency into the lasing mode requires a rigorous description of the self-consistent eigenmode of the laser resonator including gain and dissipative boundary conditions \([21],[22]\). For our simple case, we arrive at the expression for the spontaneous emission coupling into the passive cavity mode along one direction

\[
\frac{\tau_\theta}{\tau_{\Delta \theta}} = \frac{3}{8} \int_{u'}^1 I_\theta(u)(1 + u^2)du \quad (37a)
\]

and the spontaneous emission factor

\[
\beta_n = \frac{\tau_\theta}{\tau_{\Delta \theta}} = \frac{1}{2} \int_{u_1}^1 I_\theta(u)(1 + u^2)du + \int_{u_2}^{1} 1 + u^2du \quad (37b)
\]

where \( u' = \cos(\Delta \theta/2) \) and \( \Delta \theta \) is given by (14). \( \beta_n \) is the the ratio of (37a) to (36). Since \( \beta_n \) can vary by several orders of magnitude whereas the total emission rate rarely deviates greatly from the free-space value (37) is a simple approximate expression for coupling efficiency in a DBR resonator.

The SHM has allowed us to obtain simple expressions for the spontaneous beam divergence, the spontaneous emission lifetime and the spontaneous emission coupling efficiency. These expressions offer simple interpretations for emission modified by the DBR boundary. Our theoretical analysis has also shown that the closed form expressions obtained above approximate the exact DBR results closely.

V. CLASSICAL RADIATION EXPERIMENTS AND SPONTANEOUS EMISSION

A. Theory

We also verify our prescription for an equivalent array construction by using macroscopic dipole oscillators. Our experiments use a dipole antenna as the radiation source. As with spontaneous emission, the cavity alters the coupling strength into individual modes as well as the density of modes. The altered decay rate and therefore radiation resistance can be observed by monitoring the reflected power returning through the waveguide that is used to feed the dipole.

For a given array size, the driving-point impedance can be expressed as a summation of the dipole self-impedance and contributions from mutual coupling between the physical dipole and images [20]. The location of each dipole along the z-axis is given by

\[
z_n = nL_\ell + (-1)^n z_0 \quad (38)
\]

where \( L_\ell \) is the cavity length and \( z_0 \) is the location of the physical dipole relative to the cavity center. The driving point impedance is then

\[
z_{in} = R_{in} + jX_{in} = \sum_n \left( \frac{I_n}{I_0} \right) Z_{no} = \sum_n \tau_\theta^{\|} Z_{no} \quad (39)
\]

where \( Z_{no} \) is the self-impedance of the (physical) dipole, and \( Z_{n0} \) is the mutual impedance between the dipole and its th image separated by a distance (\( z_n \) to \( z_0 \)). Dipole-cavity resonance occurs when \( X_{in} = 0 \). For thin, center-fed dipoles of length \( l \) and radius \( a \), a single expression for both the self and mutual impedance can be found using the induced EMF method [20], [23],

\[
Z_{no} = j\frac{\eta}{4\pi} \int_{-1/2}^{1/2} \sin k(l/2 - |x|) \sin^2(\kappa l/2) \left[ \frac{e^{-jkR_1}}{R_1} + \frac{e^{-jkR_1}}{R_1} - 2 \cos(kl/2) e^{-jkR_0} \right] d\chi \quad (40a)
\]

where

\[
R_m = \sqrt{(|z_n - z_0| + a)^2 + (\chi + ml/2)^2}. \quad (40b)
\]

Equations (39) and (40) provide a simple means for investigating the influence of mutual coupling on the dipole impedance as a function of array spacing and frequency.

The real part of the self-impedance is the free-space radiation resistance from the dipole and each of the mutual coupling terms represents perturbations on the free-space radiation rate. As in first-order perturbation theory for weak atom-field coupling, the induced EMF method does not include a self-consistent solution for the dipole currents (or dipole moments). Equation (39) is a simple expression representing the interference interpretation for the altered dipole emission rates.

Since the induced EMF method does not have explicit dependence on the incidence angle, it is difficult to introduce the effects of a finite mirror band width into this treatment. The method of complex images [24] allows one to apply the induced EMF method to the SHM boundary, but a simple image construction is sufficient for testing how well a single image element approximates the DBR boundary. Since we neglect the fact that the mirror has a finite band stop, our theory is expected to overestimate the achievable inhibition or enhancement by the DBR.

B. Experiments

The alteration of the dipole emission rate by a single DBR was observed. We use a microwave vector network analyzer to directly measure radiation rates from a dipole antenna. These measurements were made at 3.71 GHz with an HP8720 vector.
network analyzer, using an unbalanced 2.7 cm dipole, centered from a semi-rigid coaxial cable. The millimeter wave DBR consisted of 5.5 periods of air and Rexolite 1422 (a nearly peak reflectivity was .9848 assuming no losses). This structure fed from a semi-rigid coaxial cable. The millimeter wave DBR had lossless dielectric with an index of refraction of 1.56). The theoretical curve. Using only the high and low indices, the antenna, the theory curve was specified exactly. Upon comparison with location. It is important to note that there are no fitting parameters in the construction approximates the DBR's influence on the dipole emission rate. This is a strong correlation in the behavior of impedance with location. It is important to note that there are no fitting parameters in the theoretical curve. Using only the high and low indices, the dimensions of the DBR and the dimensions of the dipole antenna, the theory curve was specified exactly. Upon comparison, we see that the agreement greater than a quarter wavelength away is good suggesting that our technique for the construction of the equivalent dipole array is reasonable. The experiment confirms the results of the Section IV showing that an image element constructed according to the hard mirror construction approximates the DBR's influence on the dipole emission rate.

VI. CONCLUSION

This paper has sought to explicitly incorporate the unique features of microcavities that utilize distributed mirrors. We have demonstrated that the essential features of the DBR mirror—the band stop and the penetration depths—can be used to construct an equivalent resonator (SHM) that facilitates the analysis of spontaneous emission. Using the SHM boundary, we have been able to obtain closed form expressions for the spontaneous emission beam divergence and the spontaneous emission lifetime as well as approximate expressions for the spontaneous emission coupling factor. The above expressions are directly applicable for comparison of different resonator geometries. One must, however, keep in mind the approximations that we have made:

1) the scalar beam approximation, which allows us to neglect the difference in TE and TM reflectivities under the assumption that the DBR's are high-finesse and that the Brewster angle is within the DBR pass band, and

2) the approximation that the spontaneous emission enhancement/inhibition is negligible in the passband of the DBR resonator. While valid for most materials, the approximation will not hold for DBR's with very high index steps between layers. These DBR's can have high reflectivities over small ranges even within the passband.

Our analysis highlights some of the limitations in using the DBR to "control" spontaneous emission. The fields radiated by the physical dipole and its images interfere strongly in small resonators. For this reason, microcavity devices typically consist of \( \lambda \) or \( \lambda/2 \) cavities. However, penetration of light into the DBR's increases the effective resonator length. For GaAs–AlAs DBR's, a physical cavity length of \( \lambda \) gives an effective cavity length of more than 3\( \lambda \), dramatically reducing the interference effects. The resonator boundary also does not significantly alter the spontaneous emission radiated into the pass band of the resonator \( (\theta_s < \theta_f < \theta_c) \). Both of these properties severely limit the achievable inhibition or enhancement of spontaneous emission.

APPENDIX I

DESIGN OF DBR's

The design of distributed mirrors has typically been restricted to obtaining high reflectivity mirrors in desirable material systems. As SEL's mature, DBR designs that have specific diffraction or temporal (pulse delay) properties may be required. Once analytic expressions have been obtained, algorithms for the design of distributed mirrors can be easily implemented. This appendix will focus on the simple algorithm that was used to design the DBR's in Table I and II.

It has been shown that the diffraction of a DBR is specified by the diffraction range, \( \chi_D \), where [10]

\[
\chi_D = \frac{\lambda_D L_D}{n_i}
\]

(A1)

and the pulse delay, \( \tau \), has the functional dependence [11]

\[
\tau = \frac{2n_i}{c} L_T
\]

(A2)

Equations A1 and A2 provide a quick summary of the relevant parameters. This paper specifically sought to examine the effects of various \( \gamma \), where \( L_D = \gamma L_r \). Therefore, the design of DBR's 1–6 kept \( n_i, \lambda_r, \tau \) and the relevant \( L_{pen} \) constant while recalculating \( n_L, n_H \) and the number of periods, \( m \). The system of simultaneous equations that needed to be solved were (A3)–(A5) found at the bottom of the next page [8], [10], [11].

Newton's method was used to iteratively solve the above system of equations. Since we only investigated structures where \( n_{ex} < n_H \) and \( n_L < n_i \), we had an added restriction that \( m \) must be an even integer. To satisfy this, we first used (A3)–(A5) to obtain a solution where \( m \) was approximately an even integer, then we solved the equations leaving \( m \) as fixed and allowing \( \gamma \) to vary slightly. In such a way we were able
to obtain a set of "user defined" structures that had reasonable values for the design parameters.

One may also be interested in specifying the fractional bandwidth of the DBR in addition to the diffraction and temporal properties. From Table I and II we see that the fractional bandwidth varied as we changed \( \gamma \). In fact, one can see from (A3-5) and (A6), \[ \frac{\Delta \lambda}{\lambda_c} = 4 \text{arcsin} \left( \frac{1 - \frac{n_L}{n_H}}{1 + \frac{n_L}{n_H}} \right) \] (A6)

that keeping \( n_i, \lambda_c, r_c \) and \( L_{pen} \) constant does not allow enough degrees of freedom to vary the bandwidth. For practical application, the most reasonable way to increase the degrees of freedom is to make \( n_i \) a free variable.

Although we have developed this simple algorithm for the verification of theory, additional constraints for material availability and desirability can be implemented in the non-linear optimization problem that DBR design poses. One may, for example, wish to optimize the modal reflectivity while minimizing the fractional bandwidth (to increase the mode suppression ratio). The above analytic expressions may in this way be useful for the design of VCSEL structures.

**APPENDIX II**

**DERIVATION OF INTRACAVITY FIELD INTENSITY**

As discussed in Section IV, \( I_1 \) and \( I_2 \) represent the influence of the cavity boundary on the power radiated into different plane wave components. The previous derivation started with the field radiated by the dipole in free-space and by image construction determined how the cavity modified the emission. The reciprocity principle suggests that propagating plane waves into the resonator and observing the modification of the plane wave intensity by the boundaries should result in the same expressions for \( I_11 \) and \( 111 \). To this end, we propagate plane waves of unit amplitude into the hard mirror resonator.

As in (10a) the resonator transmission is

\[ t = \frac{b_{h1}e^{ikL_c}L_c'}{1 - r_{h1}r_{h2}e^{2ikL_c}'L_c'} \] (A7)

For a *unit amplitude* plane wave incident from the left, we can use the above expression to determine the field amplitude for the forward and backward propagating waves in the resonator. For simplicity, we shall look only at the fields in the center of the resonator. The output wave is just the fraction of the forward traveling wave in the cavity that is transmitted:

\[ t = a(0)b_{h1}e^{ikL_c}L_c'/2 \]

We can combine these two equations to obtain an expression for the plane wave amplitude in terms of hard mirror parameters

\[ a(0) = \frac{t_{h1}e^{ikL_c}L_c'/2}{1 - r_{h1}r_{h2}e^{2ikL_c}'L_c'} \]

Similarly the backward propagating wave is the reflected portion of the forward traveling wave:

\[ b(0) = a(0)r_{h1}e^{ikL_c}L_c' \]

The field at the center of the resonator is therefore

\[ E_i(0) = a(0) + b(0) = a(0)(1 + r_{h1}e^{ikL_c}L_c') \]

\[ = \frac{t_{h1}e^{ikL_c}L_c'/2}{1 - r_{h1}r_{h2}e^{2ikL_c}'L_c'}(1 + r_{h1}e^{ikL_c}L_c') \]

The intracavity intensity for the plane wave incident from the left is then

\[ E_i(0)E_i^*(0) = \frac{|t_{h1}|^2}{|1 - r_{h1}r_{h2}e^{2ikL_c}'L_c'|^2}(1 + r_{h1}e^{ikL_c}L_c')^2 \]

\[ = \frac{(1 - r_{c1}^2)(1 + 2r_{c2} \cos(k_cL_c' - \phi_1 + \phi_2) + r_{c2}^2)}{1 - 2r_{c1}r_{c2} \cos(2k_cL_c' - \phi_1 - \phi_2) + r_{c1}^2r_{c2}^2} \]

Similarly the intensity for a plane wave of unit amplitude incident from the right is:

\[ E_r(0)E_r^*(0) = \frac{(1 - r_{c2}^2)(1 + 2r_{c1} \cos(k_cL_c' - \phi_1 + \phi_2) + r_{c1}^2)}{1 - 2r_{c1}r_{c2} \cos(2k_cL_c' - \phi_1 - \phi_2) + r_{c1}^2r_{c2}^2} \]

From (31) we see that

\[ I_1(d_1 = d_2 = L_c'/2) = E_r(0)E_r^*(0) + E_i(0)E_i^*(0) \]

We notice that \( I_1(r_{c1}, r_{c2}) = I_1(-r_{c2}, -r_{c1}) \). This is because the dipole perpendicular to the plane of a reflector with \( r_c = |r_c| \) is out of phase with its image whereas a dipole parallel to the plane of the reflector is in phase with its image.
have in this way demonstrated the reciprocity principle for radiation in the hard mirror resonator.

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