Example plane wave problem

A wave traveling through a non-magnetic material ($\mu = \mu_0$) is described by an electric field:

$$\bar{E}(x,t) = 4\sin(2\pi \times 10^7 t - 0.8x)\,\hat{z} \text{ [V/m]}$$

Is this a plane wave? If so, find the phase velocity, dielectric constant ε_r and wave impedance η of the material, and the accompanying magnetic field intensity $\bar{H}(x,t)$

Solution

This appears to be a plane wave for the following reasons:

- It is a traveling wave, since it is a function of $\omega t \pm kx$
- The direction of propagation ($\hat{x}$) is perpendicular to the field polarization ($\hat{z}$)
- It has a uniform amplitude everywhere (4 V/m)

By inspection we can find the frequency and propagation constant as

$$\omega = 2\pi \times 10^7 \text{ [rad/s]}$$

$$k = 0.8 \text{ [rad/m]}$$

From the relationship $k = \omega / v$, we can find the phase velocity as

$$v = \frac{\omega}{k} = 7.85 \times 10^7 \text{ [m/s]}$$

(Sanity check: should be less than 3×10^8 [m/s] !)

The velocity is related to the material parameters, so we can get the dielectric constant as follows:

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_r \varepsilon_0}} = \frac{c}{\sqrt{\varepsilon_r}} \quad \Rightarrow \quad \varepsilon_r = \left(\frac{c}{v}\right)^2 = \boxed{14.6}$$

The wave impedance is then

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{14.6}} = 98.7\,\Omega$$

To find the magnetic field, the first step is to convert the electric field to phasor form:

$$\bar{E}(x,t) \Rightarrow \tilde{\bar{E}}(x) = -4j\,e^{-j0.8x}\,\hat{z}$$

Now we get the magnetic field from

$$\tilde{\bar{H}} = \frac{1}{\eta}\hat{x} \times \tilde{\bar{E}} = \frac{-4j}{98.7} e^{-0.8x}(-\hat{y}) = 0.041 j e^{-0.8x}\,\hat{y}$$

Now convert back to the time domain

$$\bar{H}(x,t) = \mathrm{Re}\{\tilde{\bar{H}}(x) e^{j\omega t}\} = -H_0 \sin(\omega t - kx)\,\hat{y}$$

where $H_0 = 0.041$ [A/m]