

Computer simulation of electric field lines

L Kirkup

Electric fields due to stationary charges are often represented by lines that possess certain properties. The construction of so called electric field lines attempts to make the otherwise difficult concept of the electric field more tangible to the novice. Textbooks frequently contain field line diagrams due to common charge arrangements such as a single isolated charge or a dipole but avoid fields due to more complex charge configurations. The ever increasing power and availability of the microcomputer make it an ideal tool for aiding the introduction of the electric field while at the same time permitting fields due to non-standard charge configurations to be studied.

Properties of field lines

Field lines in general possess one property of immediate importance: a tangent drawn to a field line at any point is parallel to the field at that point. A further property is that the number of field lines per unit area (measured perpendicular to the direction of the field) is proportional to the magnitude of the field at that point. It is possible to use the first property of the field to calculate the path of a field line (Merrill 1976). The first step is to determine the electric field at a point in space by summing the contribution to the field at that point due to each charge in turn. Namely,

$$\mathbf{E} = \sum_{i=1}^n \frac{q_i}{4\pi\epsilon_0 r_i^2} \hat{\mathbf{r}}_i \quad (1)$$

Les Kirkup is a lecturer at Paisley College of Technology. He graduated from Sheffield University, gained an MSc from Bedford College London and was a lecturer at Luton College of HE for two years. His main interests are the development of software for physics teaching and the mechanisms involved in electroplating alloys. He was a co-author of *Essential Physics* (1983 London: Pitman).

where \mathbf{E} is the electric field at the point in question, n is the number of charges to be considered, q_j refers to the value of the j th charge, r_j is the distance from the j th charge to the point in question and $\hat{\mathbf{r}}_j$ is the unit vector along r_j . The following discussion is limited to fields in two dimensions. Note that although the treatment provided here is restricted to working in rectangular coordinates, it would be quite possible to develop this exposition in terms of polar coordinates.

Figure 1a shows the x and y components of the electric field at a point on a field line. Now move a small displacement D along the field line from the point (x, y) to $(x + \Delta x, y + \Delta y)$. If D is small enough the triangle formed by the vectors in figure 1a and that formed by the vectors in figure 1b are similar. The position of the point on the end of the small line segment, length D , can now easily be calculated, since

$$\Delta x = D \cos \theta = \frac{DE_x}{(E_x^2 + E_y^2)^{1/2}} \quad (2a)$$

and

$$\Delta y = D \sin \theta = \frac{DE_y}{(E_x^2 + E_y^2)^{1/2}} \quad (2b)$$

where E_x and E_y are the x and y components of the electric field respectively and θ is the angle between the direction of the electric field vector and the x axis.

These equations can be used as the basis of a field line plotting algorithm. Equations (2a) and (2b) can be combined to give the standard differential equation

$$dy/dx = E_y/E_x \quad (3)$$

It is worth noting that there are a number of numerical methods that could have been adopted to solve equation (3). These include the Runge-Kutta and Adam-Moulton methods of solving differential equations (Gerald 1978). The Euler method for the solution of differential equations used in the program presented here, while not as accurate as those, was found to be adequate in this instance.

The microcomputer

The most important feature of a microcomputer for the present purpose is good resolution graphics. The program developed was written for a BBC model B which has a number of graphics modes. The mode selected for the program presented here gives a resolution of 320×256 pixels (Coll 1982). It should be possible to modify the code for other suitable microcomputers such as the Apple IIe, Sinclair Spectrum or the Commodore 64. As the program adopts a 'step by step' approach to

determining the path of the electric field lines a vast amount of memory space is not necessary. Indeed the program developed for the BBC 'B' is contained in just over 1100 bytes of memory.

As the number of charges that have to be dealt with increases, the time taken to complete a field line increases. This is because the number of calculations required to determine the field at a point increases linearly with the number of charges present. One method of decreasing computation time would be to write an appropriate machine code routine, though this would inevitably increase the complexity of the program. The program in this article was written in BASIC and the typical time for a complete field line plot varies from about a minute for one charge to above ten minutes for ten charges. It is worth noting that the BBC microcomputer's speed of computation is greater than that found on many other small microcomputers and therefore when working in BASIC the plotting time may be longer on other machines.

The program

Although equations (2a) and (2b) are sufficient to enable a field line plotting algorithm to be written there are a number of potential (no pun intended!) problems to examine before writing a full program. A field line originating from a positive charge will go off to infinity if all other charges nearby possess a positive sign. However should one or more negative charges exist close to a positive charge there is a good chance that a field line will terminate on such a charge. A routine must therefore be written to deal with field lines approaching the edge of the screen to evaluate the possibility of the field line returning on to the screen.

If a field line arrives at a neutral point (where the electric field strength is zero) the line will stop abruptly. There are two possible courses of action. The first is to recognise that a neutral point has been reached and simply terminate the plot, continuing with another field line if appropriate. The second is to trace over the field line with the background colour, thereby erasing the line. The author favours the latter approach as it is less likely to cause confusion in the mind of the user—it can look as if a field line originating on one positive charge terminates on another positive charge! In the interests of brevity, however, the former approach has been adopted in the following program.

The direction of the electric field is usually indicated by arrows drawn on the field lines. The program does not incorporate an arrow drawing routine but this and other suggestions for improvement are given in the conclusion. To

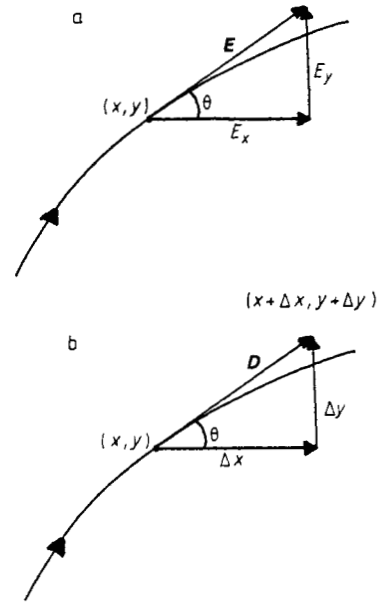


Figure 1 a, Components of the electric field at the point (x, y) on a field line. b, $(x + \Delta x, y + \Delta y)$ is the new point on the field line calculated from the field at the point (x, y)

account for the relative magnitude of the charges an adjustment can be made to the number of field lines originating (or terminating) at each charge. The program (figure 2) contains three procedures, the functions of which are outlined below.

PROCinitialise. This prints the sign of the charges at appropriate positions on the screen (as specified by line 100). Flags F and G are used to determine whether the screen contains:

- all positive charges,
- all negative charges, or
- a combination of positive and negative charges.

The arrays $X(I\%)$, $Y(I\%)$, $Q(I\%)$ contain the values for the x and y coordinates and the magnitude of the charges respectively. These can be modified by changing the values in line 100. The variable $N\%$ contains the number of charges to be considered.

PROCfield. This plots the electric field lines. The starting point (in the case of positive charges) is taken to be a fixed distance from the '+' sign. Lines 320–380 deal with the conditions for terminating a particular field line. When a field line nears a neutral point it is found that the x and y coordinates oscillate between two or three fixed values, effectively bringing the plot to a halt. Lines 320 and 330 detect this 'oscillation' and terminate the plotting of the line. Lines 340–360 deal with field lines that go off the screen. Lines 370–390

```

LIST
10 REM Field line plot program
20 REM by L.Kirkup July '84
30 DIMQ(12),X(12),Y(12)
40 MODE4
50 PROCinitialise:PROCFIELD
60 END
70
80 DEFFPROCinitialise:
90 F=0:G=0
100 NX=2:Q(1)=1:X(1)=500:Y(1)=540:Q(2)=1:X(2)=850:Y(2)=540
110 FORI%=1TON%
120 IFO(I%)<0THENF=1ELSEG=1
130 NEXT
140 FORI%=1TON%:MOVEX(I%)-12,Y(I%)+10
150 IFO(I%)>0THENVDU5,43ELSEVDU5,45
160 NEXT:VDUA
170 IFF=1ANDG=0THENFORI%=1TON%:Q(I%)=-Q(I%):NEXT
180 ENDFPROC
190
200 DEFFPROCFIELD
210 x1=0:y1=0:x2=0:y2=0:x3=0:y3=0
220 GCOL0,1
230 FORJ%=1TON%
240 IFO(J%)<0 THEN420
250 FORL%=1TO12
260 XO=X(J%)+30*COS(RAD(30*L%)):YO=Y(J%)+30*SIN(RAD(30*L%)):MOVEXO,YO
270 X1=X0:Y1=Y0:dx=0:dy=0:dD=15
280 REPEAT:EX=0:EY=0:PROCSUM
290 dx=dD*EX/Em:dy=dD*EY/Em:X1=X1+dx:Y1=Y1+dy
300 DRAWX1,Y1
310 FL=FALSE
320 x3=x2:y3=y2:x2=x1:y2=y1:x1=X1:y1=Y1
330 IF INT(x1)=INT(x3)ANDINT(y1)=INT(y3)THENFL=TRUE
340 IF ((G=1ANDF=0)OR(G=0ANDF=1))AND(X1>1200ORY1>1020ORX1<0ORY1<0)THENFL=TRUE
350 IF X1>1350ORY1>1200ORX1<-1000ORY1<-1000THEND=200ELSEdD=15
360 IF X1>1000ORY1>1000ORX1<-1000ORY1<-1000THENFL=TRUE
370 FORF%=1TON%
380 IFSQR((ABS(X1-X(F%))2+(ABS(Y1-Y(F%))2))<30THENFL=TRUE
390 NEXT
400 UNTILFL=TRUE
410 NEXT
420 NEXT
430 ENDFPROC
440
450 DEFFPROCSUM
460 FORI%=1TON%
470 x=X1+dx/2-X(I%):y=Y1+dy/2-Y(I%)
480 r=(SDR(x*x+y*y))3
490 EX=EX+Q(I%)*x/r:EY=EY+Q(I%)*y/r
500 NEXT
510 Em=SDR(EX*EX+EY*EY)
520 IFE=0THENE=1
530 ENDFPROC

```

Figure 2 Program listing

calculate whether a field line is near to a negative charge.

PROCSUM. This calculates the x and y components of the electric field at the point at the end of the last segment of field line drawn on the screen. Lines 460–500 perform a summation to determine the electric field at that point due to all the charges on the screen. Line 510 calculates the magnitude of the electric field.

Field line diagrams

Figures 3–7 show the variety of field line diagrams that can be obtained with slight modifications to the program. (The main improvement is the addition of an arrow drawing routine.) Upon completion of the field line plot the screen was 'dumped' to a dot matrix printer. Figures 6 and 7 exhibit a common problem encountered with the field line program. When charges of the same sign are close together it is possible for the field lines between the charges to be too abundant. This gives the impression that the electric field in these regions is stronger than it should be. A modification to the program is required if this is to be avoided. Figures 5 and 6

have been drawn with a greater number of field lines originating from each charge. Modifying lines 250 and 260 in the program will achieve the same end.

Conclusion

This article describes the implementation of an electric field line plotting routine on a microcomputer. The program can be much improved. Among other things it should allow for the easy entry of charge data, deal more effectively with field lines travelling toward neutral points, take into account the magnitude of each charge and draw arrows on the field lines. Additionally it would be of interest to know if working in polar rather than rectangular coordinates would improve the overall efficiency of the program. A natural extension to this program would be to include an equipotential line plotting routine (see Merrill 1976).

It is hoped that the program and the ideas contained within will serve as a starting point for other more sophisticated programs. In addition to electric fields it is possible to treat other vector fields such as the gravitational and magnetic fields

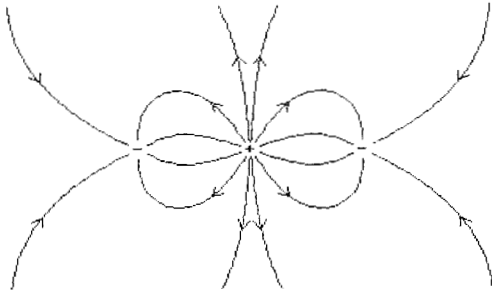


Figure 3 Electric field line pattern due to a quadrupole arrangement of charges

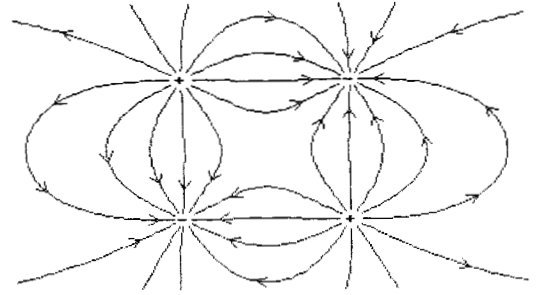


Figure 4 Field due to four charges of equal magnitude, two positive and two negative

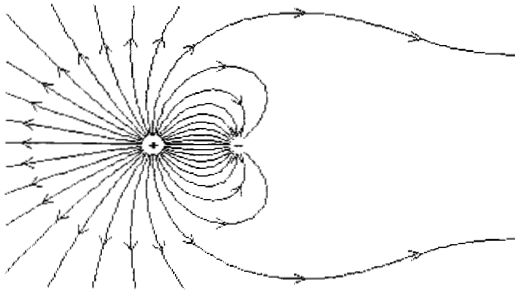


Figure 5 Field due to two charges. The positive charge is twice the magnitude of the negative charge

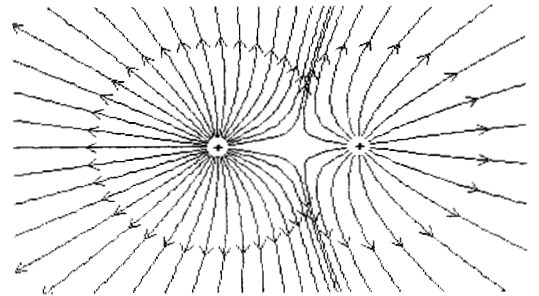


Figure 6 Field due to two positive charges. The charge on the left is twice the magnitude of the other

in a similar manner. The author would be pleased to make available a field line plotting program to anyone contacting him at the Department of Physics, Paisley College of Technology, High Street, Paisley, Renfrewshire PA1 2BE.

References

- Coll J 1982 *BBC Microcomputer User Guide* (London: British Broadcasting Corporation) p 301
 Gerald C F 1978 *Applied Numerical Analysis* (New York: Addison-Wesley) ch. 5
 Merrill J R 1976 *Using Computers in Physics* (Boston: Houghton Mifflin) ch. 4

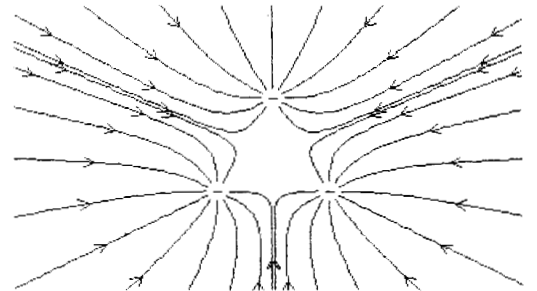


Figure 7 Field due to three charges of equal magnitude

Women in engineering booklet

The Industry/Education Unit of the Department of Trade and Industry has published a 24 page booklet entitled *What IS Engineering?* Written by

Rosemary West (a former president of the Womens Engineering Society) the booklet contains profiles of women working in many branches of engineering. It also provides further contacts and sources of information for those wishing to pursue an engineering career. Copies are available free of charge from the Womens Engineering Society, 25 Foubert's Place, London W1V 2AL.