

A Brief Note on Field Theory

The following is an excerpt from the lecture notes of Professor Rodney Cole, University of California at Davis, entitled “Introduction to Classical Fields: Divergence, Gradient, Curl, and Other Things That Go Bump in the Night”. I think it is a well written introduction for students studying field theory, as it addresses the one of the most fundamental issues of field concepts, but one which is paradoxically left out of a typical curriculum. That is, what is a field?

You are already familiar with the physics of forces, and you understand that an object can only be accelerated if a force acts upon it. We have however avoided the fundamental questions of how objects place forces on one another. We have found forces for which the bodies do not need to be in contact, for example, gravity and the electromagnetic force. They can act at a distance. This “action at a distance” has stymied many of the great minds, and the full understanding of the problem has come only within the last fifty years. Unlike other theories, it is not the work of an isolated genius, but it is the culmination of centuries of work by many people.

The basic question is how can one object place a force on another object that is some distance away without any apparent contact whatsoever? Something must go between them! We will invent a quantity to carry the force between the objects. This something is called the field. We will direct our attention from the forces to the fields themselves. This course is a study of classical field theories. What properties do the fields have? And how do we describe them?

Classical field theories suffer one major drawback: they do not actually answer the question posed. It turns out, as we shall see, that one cannot touch or feel a classical field. More importantly, classical fields are not conserved at all! They can be created and destroyed. Well, if they cannot be seen or felt and they are not conserved, do they exist? There is no way to tell. We are thus led to the horrible realization that there is no way to know whether the fields we have invented actually exist. Might they be nothing more than a mathematical trick?

This dilemma has been solved by combining field theory, relativity, and quantum mechanics into a powerful theory with the unwieldy name of Quantum Electrodynamics (QED for short). In QED the electric and magnetic fields are combined to form discrete packets or quanta called photons. These photons sometimes behave as particles and they can be seen and counted by machines. In fact, under special circumstances it is possible for the human eye to “see” one photon.

In relativity, mass and energy are equivalent. It is possible, therefore, to create mass from photons and to annihilate matter and antimatter to produce photons. This is done quite easily in modern accelerators. Relativity, then, gives us a general conservation law called the Conservation of Mass–Energy, and although the number of photons is not conserved and the mass is not conserved, the total amount of mass plus photons is conserved. Thus, in modern physics it is possible to show that packets of fields in the form of photons do really exist and useful conservation laws can be formulated.

While QED may be too complicated to study here, we will introduce you to the basics behind classical field theory. Uses of classical fields range from thermodynamics and hydrodynamics to electromagnetics and computer science. They are found in virtually every field of science that uses sophisticated mathematics.

In ECE134, we will deal with electromagnetic fields exclusively. Classical electromagnetic theory (as opposed to QED) does a wonderful job of explaining the macroscopic phenomena that we will be interested in. It is only when events occurring at atomic dimensions are studied that one might really need to address the issue of what a field is composed of, and even there classical models often prove useful.