

Reading Assignment:

Cheng: Review Chapters 1-2, study sections 3.1–3.6

Schey: review material as needed

Cartoon Guide: Chapters 12-13

Homework #1**Due: Friday 2 October 2009**

- 1) We have already seen in our limited study that the equations of electromagnetics often involve a vector \bar{R} that describes the separation between the *source location* and the *observation point* at which we want to determine the fields. If we describe the observation point by the position vector \bar{r} and the source point by \bar{r}' , then $\bar{R} = \bar{r} - \bar{r}'$. In rectangular coordinates it is easy to see that

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \quad (1.1)$$

Find R when the source and observation points are described in a) cylindrical coordinates, or b) spherical coordinates.

- 2) In class we used the following identity in our derivation of the electrostatic potential

$$\nabla \left(\frac{1}{R} \right) = -\frac{\hat{R}}{R^2} \quad (1.2)$$

where R is given by (1.1), and \hat{R} is the unit vector in the direction of \bar{R} . Prove the result (1.2) by carrying out the gradient operation on $1/R$. Note that (1.2) is nothing more than the vector equivalent of the more familiar expression

$$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

- 3) In each problem below, find the divergence, $\nabla \cdot \bar{A}$. Note that the operation takes different forms in each coordinate system (summarized on the inside back cover of Cheng), and I have used the same notation for angles as in Cheng:
- $\bar{A} = x^2 \hat{x} + yz \hat{y} + xy \hat{z}$ (rectangular coordinates)
 - $\bar{A} = r \sin \phi \hat{r} + 2r \cos \phi \hat{\phi} + 2z^2 \hat{z}$ (cylindrical coordinates)
 - $\bar{A} = \sin \theta \hat{\theta} + \sin \phi \hat{\phi}$ (spherical coordinates)
- 4) Similarly, in each problem below, find the gradient, $\nabla \Phi$
- $\Phi(x, y, z) = 5e^{-z} \sin(2x) \ln y$ (rectangular coordinates)
 - $\Phi(r, \phi, z) = r^2 z \cos \phi$ (cylindrical coordinates)
 - $\Phi(r, \theta, \phi) = r \sin^2 \theta \cos \phi$ (spherical coordinates)

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- 5) Find the total charge on a circular disk of radius a , centered at the origin of the $z=0$ plane, assuming the charge density is given by

$$\rho_s(r) = \cos\left(\frac{\pi r}{2a}\right) \quad (1.3)$$

where r is the radius in cylindrical coordinates. This will give you some practice with integration in cylindrical coordinates.

- 6) A certain charge distribution gives rise to the so-called *Yukawa* potential, which in spherical coordinates is

$$\Phi(\bar{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} e^{-r/a} \quad (1.4)$$

where a is some constant. The potential exists everywhere in the range $0 \leq r < \infty$:

- Find the electric field
- Find the charge density
- Find the total charge

Note that:
$$\int x e^{px} dx = \frac{e^{px}}{p} \left(x - \frac{1}{p} \right)$$

- 7) An array of positively-charged ions fills the region between two large conducting plates as shown below. The plates have a cross-sectional area A . The charges are distributed uniformly across the device with a density of N_d ions per cubic meter. The conductor at $x=0$ is held at ground potential (0 Volts) and the conductor at $x=d$ is held at a potential of V_0 Volts. Find an expression for the potential and electric field between the plates as a function of x , V_0 , and N_d . (Hint: solve Poisson's equation in one dimension. In problems like this we assume that the plate dimensions that are so large compared with d that the potential will be uniform in y and z).

