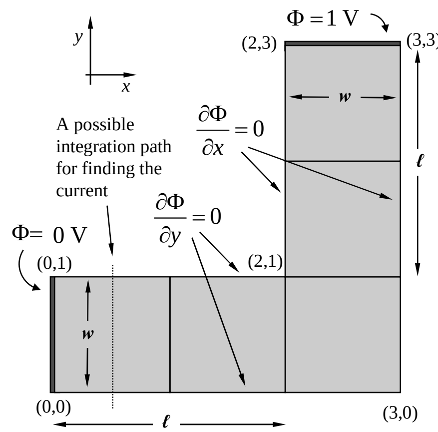
Reading Assignment:

Cheng: Chapter 6
Cartoon Guide: Chapters 17-18

Homework #5 Due: Friday 30 October 2009

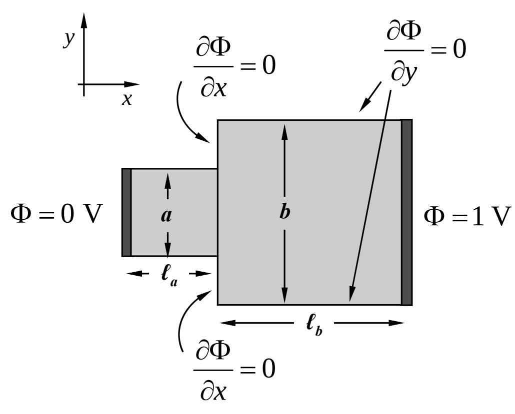
1) In this problem we will use a numerical PDE solver to explore the reduction in resistance due to a corner in a thin-film resistor as shown here. This is a very thin sheet of conducting material with contacts covering each end. Five squares of material are shown ($\ell = 2w$). Find the true resistance, which is less than $5r_{sh}$ for reasons we discussed in class. From this, deduce the contribution of a corner. Include a plot of the equipotentials with your solution. Note that you will need to compute the total current by integrating the current density across the device width; one possible integration path is shown in the figure. This problem can also be solved analytically using a technique called conformal mapping,



$$R = \frac{2r_{sh}}{\pi} \ln\left[\cosh\frac{\pi}{2}\left(\frac{2\ell}{w}+1\right)\right] \tag{1.1}$$

Compare your numerical result against the exact formula (problems like this with known answers are helpful in debugging a program and establishing confidence in your numerical methods). Provide evidence of individual work.

2) Now let's use the PDE solver to explore spreading resistance. In this problem, a very thin sheet of conducting material with contacts on both ends has a step change in width. In this figure, two square of material are shown ($\ell_a = a$ and $\ell_b = b$), but the resistance is slightly large than $2r_{sh}$ because of the spreading resistance. Find the total resistance of the structure, and deduce the spreading resistance contribution from your answer. This is another problem that can be solved analytically by conformal mapping, albeit with a nontrivial effort, yielding



$$\Delta R_{sp} = \frac{r_{sh}}{2\pi}\left[\frac{a^2+b^2}{ab}\ln\left(\frac{b+a}{b-a}\right)+2\ln\left(\frac{b^2-a^2}{4ab}\right)\right] \tag{1.2}$$

Compare this result against your numerical calculation.

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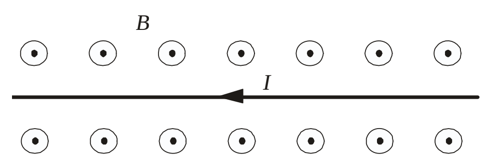
3) This problem considers the maximum allowable current *pulse* of duration Δt that can be sustained by a conducting wire, neglecting any possible changes in resistance due to the TCR. First show that for pulse widths that are small compared with the thermal time constant ($\Delta t \ll \tau_{th}$) the fusing current for the wire is given by

$$I_{fuse} \approx \frac{\pi d^2}{4}\sqrt{\frac{c_p \rho_m}{\rho \Delta t}(T_{melt} - T_0)}$$

where d is the diameter of the wire and ρ is the electrical resistivity. The idea here is that the pulse is so short we can neglecting convection and radiation terms in comparison to the internal energy term.

4) A long horizontal Copper wire carries a current *I* and is oriented such that it flows perpendicular to the Earth's magnetic field *B* as shown below. The wire has a 1 mm diameter and a linear mass density of m_g = 0.1 kg/m. If the Earth's magnetic field is $B \approx 20\ \mu\mathrm{T}$:



a. What current would be required to levitate the wire?
b. The maximum DC current density that can be safely supported by copper wires is 10^6 A/cm^2. Will it be possible for the wire in this problem to support the necessary current to observe magnetic levitation?

5) The circuit shown below uses two identical springs to support a *W*=10cm long horizontal wire with a mass of 5 g. A current is driven through the wire by a 12-Volt battery and a resistor $R = 4\Omega$. In the absence of a magnetic field, the weight of the wire causes the springs to stretch a distance of 0.2 cm each. When a uniform magnetic field *B* is applied (normal to the page), the strings are observed to stretch by an additional 0.5 cm. What is the intensity of the magnetic flux density *B*?