Yukawa Potential

$$\Phi = \frac{q}{4\pi\epsilon_0 r} e^{-r/a}$$

From $\nabla^2 \Phi = -\rho/\epsilon_0$ we can find the charge density

<u>but</u>: $\nabla^2 \Phi$ undefined at $r=0$ because of singularity

write

$$\Phi = \underbrace{\frac{q}{4\pi\epsilon_0 r}}_{\text{point charge at origin}} - \underbrace{\overbrace{\frac{q}{4\pi\epsilon_0 r}\left[1 - e^{-r/a}\right]}^{g(r)}}_{\text{non-singular at } r=0} \qquad (1)$$

we know that $\rho = q\,\delta(r)$ for the point charge.

what is charge density associated w/ $g(r)$ term?

$$\nabla^2 g(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial g}{\partial r}\right) = -\frac{e^{-r/a}}{r a^2}\frac{q}{4\pi\epsilon_0}$$

so $$\rho(r) = \boxed{q\,\delta(r) + \frac{q}{4\pi a^2 r} e^{-r/a}}$$

this is the charge density associated with (1)