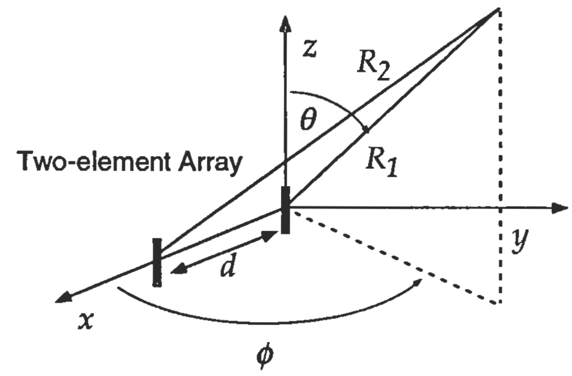
Antenna Array Theory

Two-Element Array

The simplest array consists of two antennas, as shown below. Since Maxwell's equations are linear, the principal of superposition applies just as in circuit theory, and so the total field due to both antennas is just the vector sum of the fields due to each individually.



The electric field produced by any antenna in the far field can usually be written as

$$\overline{E} = \overline{E}_0 F(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

where $F(\theta,\phi)$ describes the radiation pattern of the single antenna, and is called the element pattern. the total field from two identical antennas spaced by d is then

$$\overline{E}_{total} = \overline{E}_1 + \overline{E}_2 = F(\theta,\phi)\left[\overline{E}_1\frac{e^{-j\beta R_1}}{R_1} + \overline{E}_2\frac{e^{-j\beta R_2}}{R_2}\right]$$

Assuming that each antenna is excited with identical amplitudes but allowing for a phase-shift ξ between the antennas, that is, $E_2 = E_1 e^{j\xi}$, gives

$$\overline{E}_{total} = E_1 F(\theta,\phi)\left[\frac{e^{-j\beta R_1}}{R_1} + \frac{e^{j\xi}e^{-j\beta R_2}}{R_2}\right]$$

In the far-field where $r \gg d$, the two distances R_1 and R_2 are approximately the same. However, even a small difference could produce a significant phase-delay between the two signals, hence we must use a better approximation in the phase terms. To first order, we get

$$R_2 \simeq R_1 - d\sin\theta\cos\phi$$

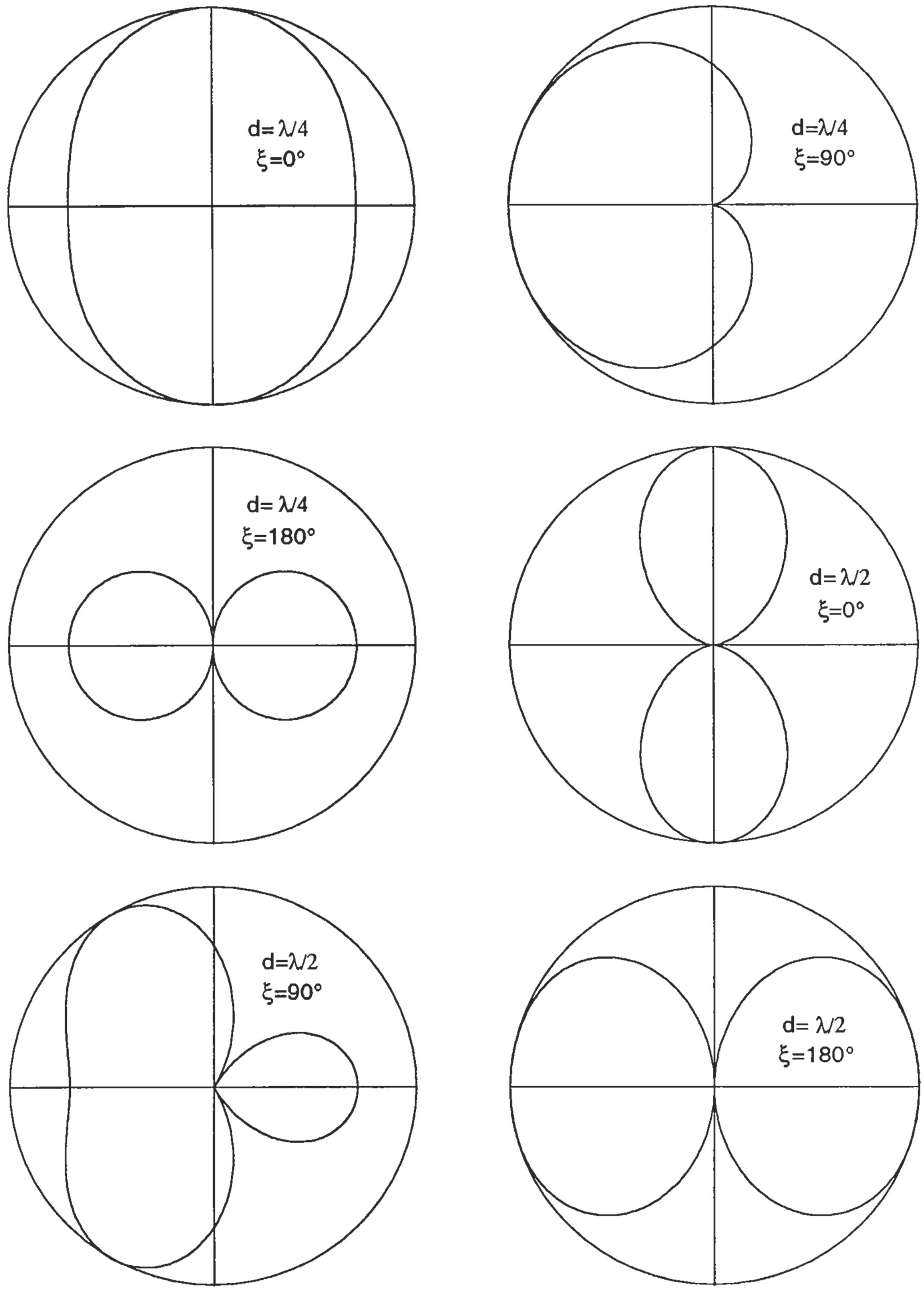
We can still use $R_1 \approx R_2$ in the amplitude terms, however. Using these far-field approximations gives the result

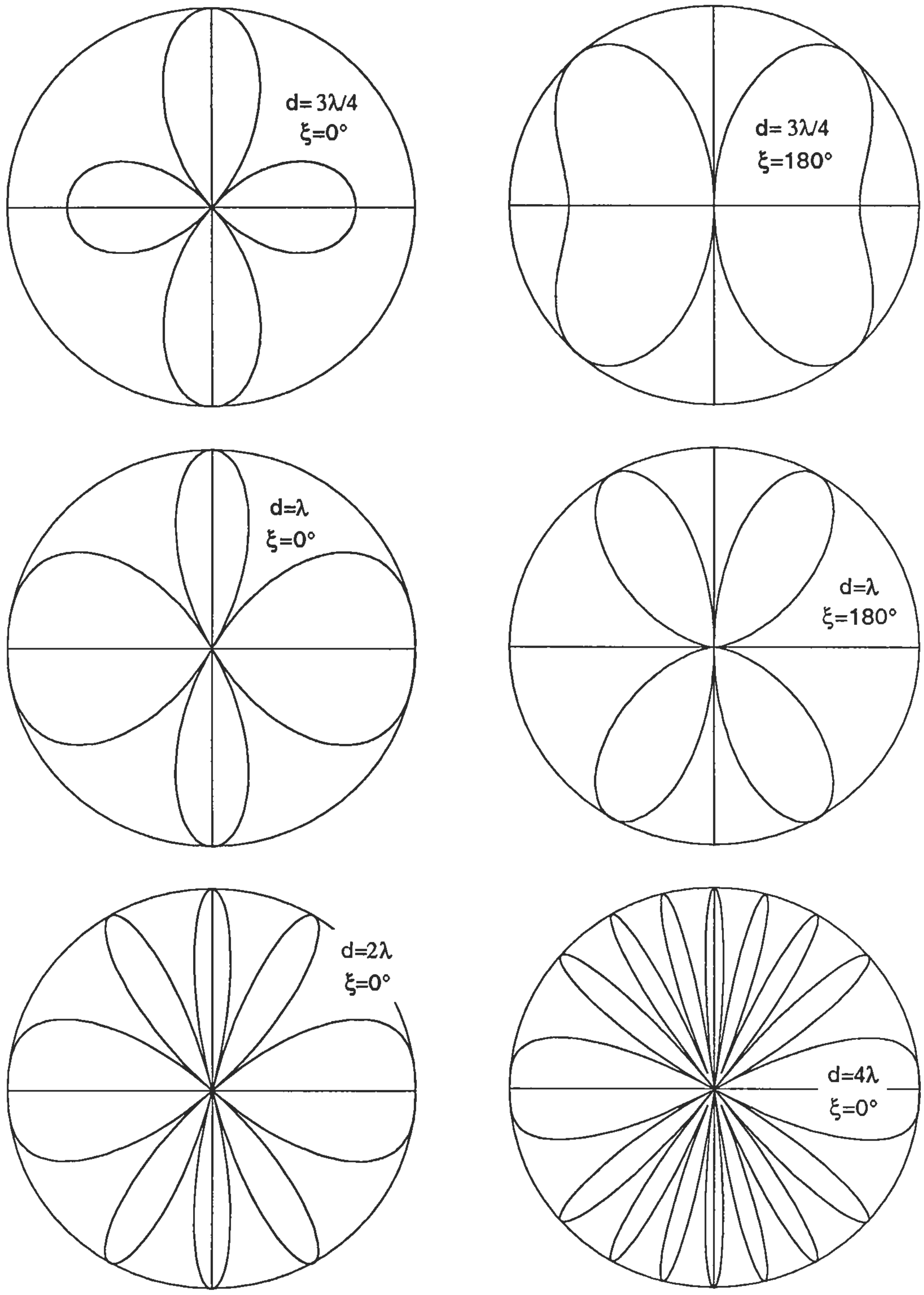
$$|\overline{E}| = \frac{2E_1}{r}|F(\theta,\phi)|\left|\cos\frac{\psi}{2}\right| \qquad \text{where} \quad \psi = \beta d\sin\theta\cos\phi + \xi$$

The $|\cos(\psi/2)|$ term is called the array factor, since it describes the interference pattern for two antennas, *regardless of what kind of antenna is used.* This equation illustrates the principal of pattern multiplication, which states that the total radiation pattern of an array is just the element factor multiplied by the array factor.

Two-Element Array Patterns

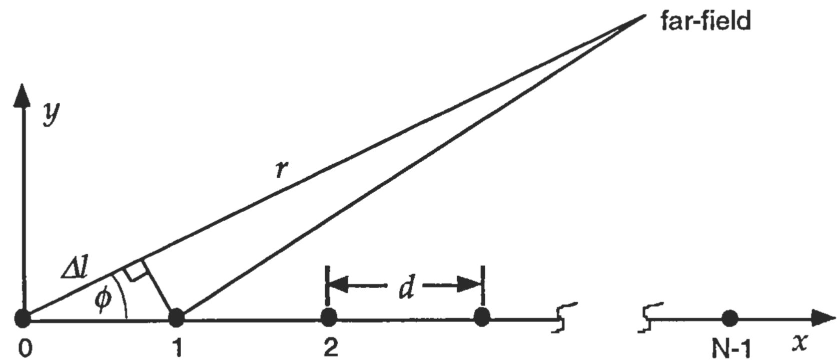
The array factor for a two-element array has been plotted in polar form for several different values of phase-shift ξ and spacing d below. Notice as the spacing increases, the patterns become more complicated. For spacings greater than a wavelength, grating lobes start to appear.




d= 3λ/4
ξ=0°
d= 3λ/4
ξ=180°
d=λ
ξ=0°
d=λ
ξ=180°
d=2λ
ξ=0°
d=4λ
ξ=0°

General Uniform Linear Arrays

We can generalize the two-element result to N elements, using the diagram below.



If we assume that each element is excited by the same amplitude signal (uniform excitation), and that there is a constant phase progression along the array (that is, there is a phase shift of ξ between neighboring antennas), then we can write the normalized array factor

$$|A(\psi)| = \frac{1}{N}\left|1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(N-1)\psi}\right|$$

This geometric series can be summed analytically to gives

$$|A(\psi)| = \frac{1}{N}\left|\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right|$$

where again $\psi = \beta d \sin\theta\cos\phi + \xi$. If this array factor is plotted graphically, it is found to be a multi-lobed pattern, with one large lobe (the main beam) located at $\psi = 0$. Denoting this main beam direction by ϕ_0, we find

$$\phi_0 = \cos^{-1}\left(\frac{-\xi}{\beta d}\right)$$

Note that the beam points in a direction determined by the element spacing and phase shift. This possibility of scanning a beam of radiation by controlling the phase shift between antennas is the principal of operation behind phased-array radars.

The null locations can be found from the array factor to be at the locations

$$\text{null points} \qquad N\psi/2 = \pm k\pi \quad k = 1, 2, 3, \ldots$$

The width of the beam is normally found as the angular distance between the first nulls on either side of the main lobe. This the beamwidth decreases as the number of elements increases, so a large array has a very small beamwidth and therefore a high directivity. the sidelobes can be similarly determined at the points

$$\text{sidelobes} \qquad N\psi/2 = \pm(2m+1)\pi/2, \quad m = 1, 2, 3, \ldots$$

It is found that the first sidelobe level is down about 13.5 dB for large arrays.

Although we have assumed uniform excitations, this is not absolutely necessary, In fact, it has been found that the sidelobe levels of an array pattern can be reduced or even eliminated by properly adjusting the output amplitude of each antenna. A special case is the binomial array, where the amplitudes are selected according to binmial coefficients, and which has no sidelobes at all, at the expense of a larger beamwidth.

Phased-Array Example

The array factor for an 8-element phased array, with spacing $d = \lambda/2$, has been plotted below for several values of phase shift ξ. For $\xi = 0°$, the beam is pointing in the *broadside* direction, whiloe at $\xi = 180°$ the beam points in the *end-fire* direction. As the beam is scanned from bradside towards end-fire, the beamwidth increases. In this example, the array radiates into all of space (both in front and behind the array), but in practice a ground plane is usually placed behind the array to direct the radiation in one direction.