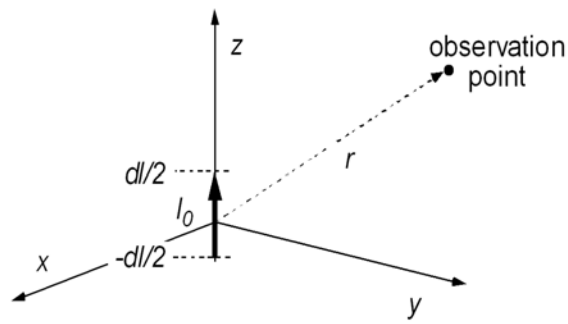
Fundamentals of Electromagnetic Radiation

Hertzian Dipole

The most elementary source of radiation is the oscillating electric dipole, or Hertzian dipole. We can model this source as an infinitisimally thin filament of current of length dl in the $\hat{z}$ direction, as shown below.



The current is assumed to be spatially constant over the length dl and time-harmonic. Mathematically the current density is

$$\overline{J}(x',y',z') = \begin{cases} I\,\delta(x')\delta(y')\hat{z} & -dl/2 < z' < dl/2 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

where $\delta(x)$ is the Dirac delta function. This current produces the vector potential

$$\begin{aligned} \overline{A}(x,y,z) &= \frac{\mu}{4\pi}\int_{V'} \frac{\overline{J}(x',y',z')e^{-\jmath kR}}{R}\,dV' \simeq \frac{\mu}{4\pi}\int_{-dl/2}^{dl/2} \frac{I\hat{z}\,e^{-\jmath kr}}{r}\,dz' \\ &= \frac{\mu}{4\pi}\frac{e^{-\jmath kr}}{r} I dl\,\hat{z} \end{aligned} \tag{2}$$

Converting this to spherical coordinates and calculating the fields from

$$\overline{H} = \frac{1}{\mu}\nabla\times\overline{A} \qquad \overline{E} = \frac{1}{\jmath\omega\epsilon}\nabla\times\overline{H}$$

gives

$$\begin{aligned} \overline{H} &= -\hat{\phi}\frac{I\,dl}{4\pi}k^2\sin\theta\left[\frac{1}{\jmath kr} + \frac{1}{(\jmath kr)^2}\right]e^{-\jmath kr} \\ \overline{E} &= -\frac{Idl}{4\pi}\eta k^2\left\{2\hat{r}\cos\theta\left[\frac{1}{(\jmath kr)^2} + \frac{1}{(\jmath kr)^3}\right] + \hat{\theta}\sin\theta\left[\frac{1}{\jmath kr} + \frac{1}{(\jmath kr)^2} + \frac{1}{(\jmath kr)^3}\right]\right\}e^{-\jmath kr} \end{aligned} \tag{3}$$

In the Far-Field of the antenna where $kr \gg 1$, the terms in (3) that are proportional to $1/kr$ dominate, giving

$$\text{Far-Field:} \quad \begin{aligned} \overline{H} &= H_\phi\,\hat{\phi} = \jmath\frac{I\,dl}{4\pi}\left(\frac{e^{-\jmath kr}}{r}\right)k\sin\theta\,\hat{\phi} \\ \overline{E} &= E_\theta\,\hat{\theta} = \jmath\frac{I\,dl}{4\pi}\left(\frac{e^{-\jmath kr}}{r}\right)k\eta\sin\theta\,\hat{\theta} \end{aligned} \tag{4}$$

The radiation in the far-field behaves much like a plane wave, with the field intensity dropping off as $1/r$. If we calculate the time-average Poynting vector $\overline{P}_{ave} = \frac{1}{2}\text{Re}\left\{\overline{E}\times\overline{H}^*\right\}$ using the complete field expressions (3), the same result is obtained as when the fields (4) are used. Thus the terms in (3) proportional to $1/r$ are considered as responsible for the average power transfer away from the antenna, while the other terms in (3) are interpreted as contributing to stored (reactive) energy in the immediate vicinity of the antenna.

Hertzian Dipole Parameters

Using the far-field expressions (4) the time-averaged poynting vector is

$$\overline{P}_{ave} = \frac{1}{2}\mathrm{Re}\left\{\overline{E} \times \overline{H}^*\right\} = \hat{r}\,\frac{(I\,dl)^2}{32\pi^2}\eta k^2\,\frac{\sin^2\theta}{r^2} \tag{5}$$

giving a total radiated power of

$$P_r = \int_0^{2\pi}\int_0^{\pi}(\overline{P}_{ave}\cdot\hat{r})\,r^2\sin\theta\,d\theta\,d\phi = \frac{1}{2}R_r I^2 \tag{6}$$

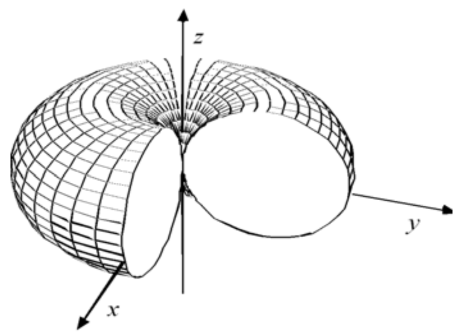
where R_r is defined as the radiation resistance,

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \tag{7}$$

The gain is found to be

$$G(\theta,\phi) = \frac{4\pi r^2\overline{P}_{ave}}{P_r} = \frac{3}{2}\sin^2\theta \tag{8}$$

and so the directivity is $D = G_{max} = 1.5$. The half-power beamwidth is found from (8) to be 90°. Equation (8) also indicates that the radiation pattern is constant with ϕ, and describes a torus as shown below.

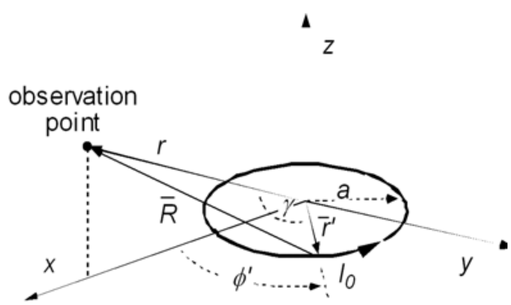


If the dipole is made from a metal wire of radius a, the radiation efficiency is

$$\eta_r = \left[1 + \frac{R_s\lambda^2}{160\pi^3 a\,dl}\right]^{-1} \qquad \text{where} \quad R_s = \sqrt{\pi f\mu/\sigma} \tag{9}$$

Magnetic Dipole

A similar analysis of the short current loop pictured below reveals radiated fields nearly identical to (4), if the roles of electric and magnetic fields are interchanged. This duality is a result of the symmetry of Maxwell's equations. As a result, we can think of the current loop as a magnetic dipole.



The far-field of the magnetic dipole is described by

$$\overline{E} = \hat{\phi}\,\frac{120\pi^2 IA}{\lambda^2}\sin\theta\frac{e^{-jkr}}{r} \qquad \overline{H} = -\hat{\theta}\,\frac{E_\phi}{\eta} \tag{10}$$

where $A = \pi a^2$ is the area of the loop. The radiation resistance can then be found as

$$R_r = \frac{320\pi^4 A^2}{\lambda^4} \tag{11}$$