

Reflection and Refraction of Plane Waves

Polarization

The polarization of a plane wave describes how the *direction* of the electric field vector changes with time. Until now we have just assumed that the E -field points linearly in one direction, usually along a coordinate axis, such as $\overline{E} = \hat{x}E_0e^{-jk_0z}$. However, there could also be a \hat{y} component to this field, since \hat{y} is also perpendicular to the direction of propagation. Furthermore, there could also be a *phase-shift* in time between these two components. Therefore, a more general expression for a plane wave travelling in the \hat{z} direction would be

$$\overline{E} = [\hat{x}E_x + \hat{y}E_y e^{j\phi}] e^{-jk_0z}$$

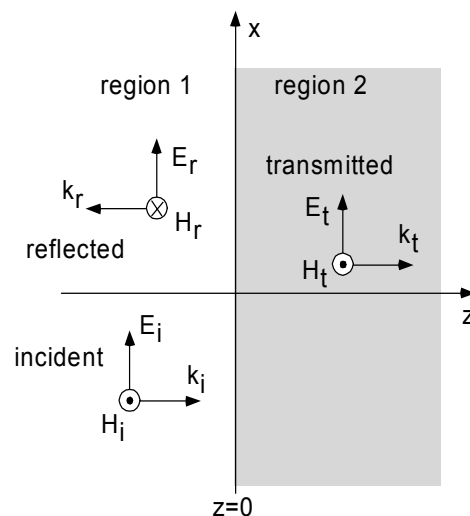
where E_x and E_y are the magnitudes of the \hat{x} and \hat{y} components, and ϕ is the phase shift between the two components. The different types of polarization represent different conditions on these variables:

- **Linear Polarization:** $\phi = 0$ When there is no phase shift in time between the elements, then the direction of the E -field does not vary with time, and we get linear polarization.
- **Circular Polarization:** $\phi = \pm 90^\circ$, $E_x = E_y$ When the components are 90° out of phase and equal in magnitude, then locus of the E -field vector traces out a circle in time, and the wave is circularly polarized.
- **Elliptical Polarization:** For any other possible situation other than the two above, the E -field traces an ellipse in time, and therefore is called elliptical polarization. This is the most general case.

In the cases of circular and elliptical polarization, the field vector can rotate in one of two possible directions. If the thumb points in the direction of propagation as the fingers of the right hand are curled in the direction of the rotation, then the wave is called right circular, otherwise it is left-circular polarization.

Normal Incidence on Plane Boundaries

Plane waves normally incident on plane boundaries can be treated in ways that are strikingly similar to reflections on transmission lines. The situation is shown in the figure below.



The fields can be written as

$$\begin{aligned}\overline{E}_i &= \hat{x}E_{i0}e^{-jk_1z} & \overline{E}_r &= \hat{x}E_{r0}e^{jk_1z} & \overline{E}_t &= \hat{x}E_{t0}e^{-jk_2z} \\ \overline{H}_i &= \hat{y}\frac{E_{i0}}{\eta_1}e^{-jk_1z} & \overline{H}_r &= -\hat{y}\frac{E_{r0}}{\eta_1}e^{jk_1z} & \overline{H}_t &= \hat{y}\frac{E_{t0}}{\eta_2}e^{-jk_2z}\end{aligned}$$

Enforcing the boundary conditions for continuity of tangential field components gives the reflection coefficient and transmission coefficient

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{and} \quad \tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

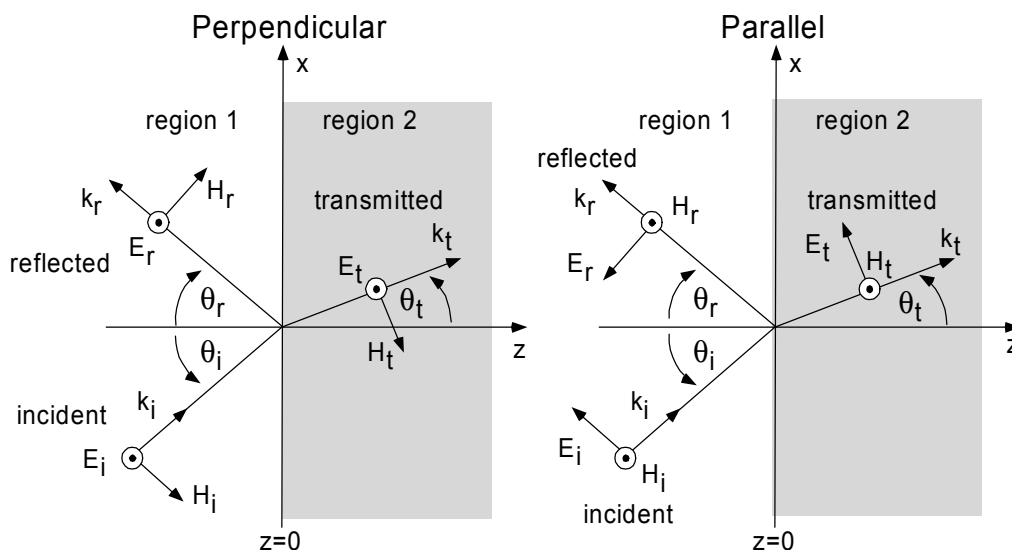
If region-2 is a perfect conductor, then $\eta_2 = 0$ and we get $\Gamma = -1$ and $\tau = 0$, so there is no signal transmitted into the conductor. Just as with transmission lines, we can define a wave impedance which describes the ratio of *total* electric field to *total* magnetic field at any point in space. This gives the familiar impedance transformation formula

$$Z_{in}(-l) = \eta_1 \frac{\eta_2 + j\eta_1 \tan k_1 l}{\eta_1 + j\eta_2 \tan k_1 l}$$

We can then treat situations like quarter-wave matching problems and multiple dielectric boundaries just like we did with transmission lines.

Oblique Incidence on Plane Boundaries

Oblique incidence on plane boundaries is more complicated for two reasons: the field components and direction of propagation no longer point along coordinate axes, and polarization of the waves becomes important. The picture below and on the following page illustrates the different polarizations.



The two polarizations are defined with respect to a plane of incidence, which is an imaginary plane containing both the normal vector to the boundary and the \vec{k}_i vector of the incident signal. Parallel polarization has the E -field pointing parallel to the plane of incidence.

The analysis proceeds by writing down expression for the various field components and enforcing the boundary conditions. Doing this for both cases of polarization gives us Snell's Laws and the Fresnel Equations. Snell's laws are

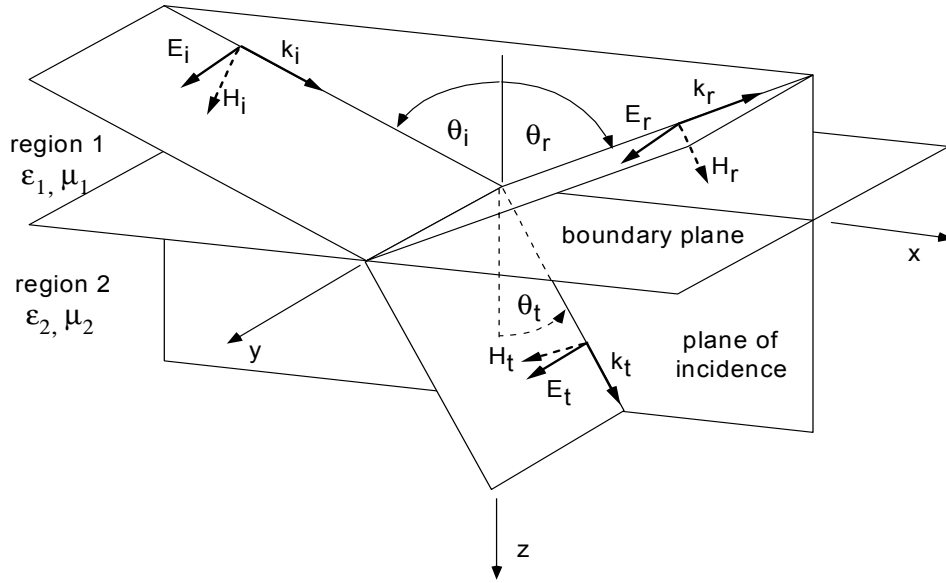
$$\text{Snell's law of reflection: } \theta_i = \theta_r$$

$$\text{Snell's law of refraction: } \sqrt{\epsilon_1} \sin \theta_i = \sqrt{\epsilon_2} \sin \theta_t$$

Snell's law of refraction is usually written in terms of the index of refraction, which is defined as $n_1 = \sqrt{\epsilon_1}$. (This is a somewhat unfortunate choice of notation, because n is often confused with η .) The Fresnel

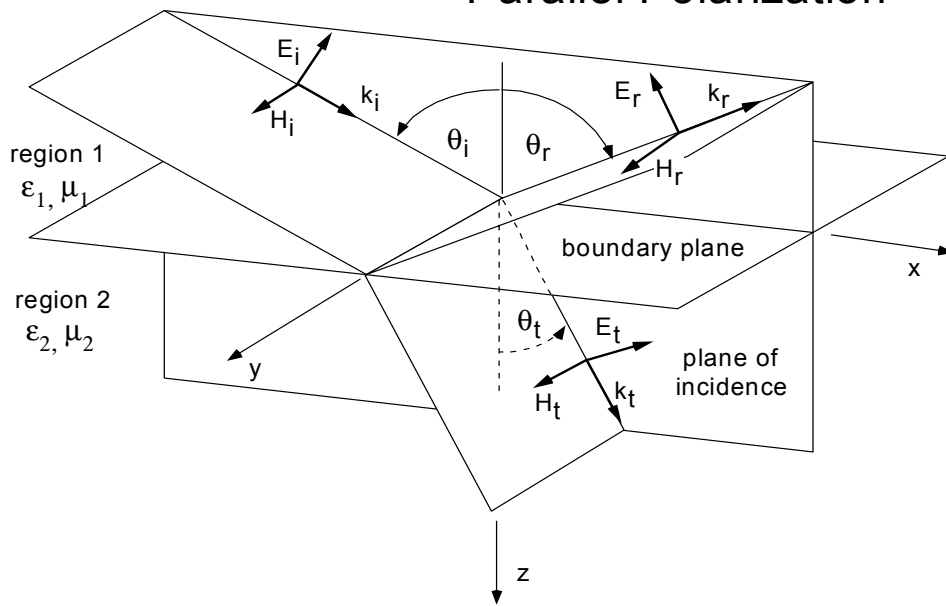
Parallel Polarization

Perpendicular Polarization



Perpendicular Polarization

Parallel Polarization



equations are

$$\begin{aligned} \text{Perpendicular: } \Gamma_{\perp} &= \frac{\eta_2 \sec \theta_t - \eta_1 \sec \theta_i}{\eta_2 \sec \theta_t + \eta_1 \sec \theta_i} & \text{Parallel: } \Gamma_{\parallel} &= \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \\ \tau_{\perp} &= \frac{2\eta_2 \sec \theta_t}{\eta_2 \sec \theta_t + \eta_1 \sec \theta_i} & \tau_{\parallel} &= \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \end{aligned}$$

Comparing these expression with the case of normal incidence, we see that they are similar in form except that the wave impedances have been modified to incorporate some angular dependence. We can define a generalized wave impedance and phase constant for incidence at any angle on dielectric boundaries, which for the i th region are written as

$$\begin{aligned} \text{phase constant: } \beta_i &= k_i \cos \theta_i \\ \text{perpendicular wave impedance: } Z_i &= \eta_i \sec \theta_i \\ \text{parallel wave impedance: } Z_i &= \eta_i \cos \theta_i \end{aligned}$$

In the case of parallel polarization, we can always find an angle of incidence such that $\Gamma_{\parallel} = 0$, provided that $\mu_1 = \mu_2$ and $\epsilon_1 \neq \epsilon_2$. we call this angle the Brewster Angle, and denote it by θ_B . Setting the numerator of $\Gamma_{\parallel} = 0$ and using Snell's law of refraction gives us the simple expression for Brewster angle

$$\tan \theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

At the Brewster angle, waves with parallel polarization will be completely transmitted into medium 2. We could also find a similar angle for perpendicular polarization such that $\Gamma_{\perp} = 0$, but this would require materials of different permeabilities and identical permittivities, something which does not often occur in nature.

Another important angle occurs if the wave travels from a medium of high permittivity (ϵ_1) into a medium of lower permittivity (ϵ_2). In this case we encounter the critical angle for total internal reflection. At angles of incidence greater than or equal to this critical angle, there is no transmitted signal into region-2, and $\theta_t = 90^\circ$. From Snell's law we get

$$\theta_c = \sin^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

Physically there is still some field penetration into region-2, but the fields strength decays exponentially away from the boundary. This is called an evanescent field. This evanescent field actually propagates along the boundary.