Microstrip

Despite it's structural simplicity, the microstrip line is difficult to analyze rigorously. However extensive research has provided numerous approximate modelling techniques which are adequate for many design purposes. The following are a collection of some of the more useful formulae for microstrip circuit design.

1 EQUATIONS FOR MICROSTRIP LINES

The following variables are used in the formulae below:

ε_r	substrate dielectric constant
h	substrate thickness
w	microstrip physical line width
Z_0	characteristic impedance
ε_{eff}	microstrip effective permittivity
l_{eo}	effective open-end length extension
w_{eff}	microstrip effective width

In the following, f is in GHz and h is in millimeters.

1.1 Static Microstrip Synthesis

Find the required w/h ratio given desired characteristic impedance and substrate parameters (see [1]-[2]). Valid for low frequencies.

$$\frac{w}{h} = \begin{cases} \left[\dfrac{\exp H'}{8} - \dfrac{1}{4\exp H'}\right]^{-1} & Z_0 > (44 - 2\varepsilon_r) \\ \dfrac{2}{\pi}\left[d_\varepsilon - \ln(2d_\varepsilon + 1)\right] + & Z_0 < (44 - 2\varepsilon_r) \\ \qquad \dfrac{\varepsilon_r - 1}{\pi\varepsilon_r}\left(\ln d_\varepsilon + 0.293 - 0.517/\varepsilon_r\right) & \end{cases} \tag{1}$$

$$\text{where} \quad \begin{aligned} H' &= \frac{Z_0\sqrt{2(\varepsilon_r + 1)}}{119.9} + \frac{1}{2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right) \\ d_\varepsilon &= \frac{59.95\pi^2}{Z_0\sqrt{\varepsilon_r}} \end{aligned}$$

1.2 Static Microstrip Analysis

Find the low-frequency characteristic impedance given the physical dimensions of the line (see [1]-[2]):

$$Z_0(0) = \begin{cases} \dfrac{119.9}{\sqrt{2(\varepsilon_r+1)}}\left[\ln\left(4h/w + \sqrt{16(h/w)^2+2}\right)\right. & w/h < 3.3 \\ \qquad \left. -\dfrac{1}{2}\left(\dfrac{\varepsilon_r-1}{\varepsilon_r+1}\right)\left(\ln\dfrac{\pi}{2} + \dfrac{1}{\varepsilon_r}\ln\dfrac{4}{\pi}\right)\right] & \\ \dfrac{119.9\pi}{2\sqrt{\varepsilon}}\left\{\dfrac{w}{2h} + \dfrac{\ln 4}{\pi} + \dfrac{\ln(e\pi^2/16)}{2\pi}\left(\dfrac{\varepsilon_r-1}{\varepsilon_r^2}\right)\right. & w/h > 3.3 \\ \qquad \left. +\dfrac{\varepsilon_r+1}{2\pi\varepsilon_r}\left[\ln\dfrac{\pi e}{2} + \ln\left(\dfrac{w}{2h}+0.94\right)\right]\right\}^{-1} & \end{cases} \tag{2}$$

1.3 Static Effective Permittivity

Find the low-frequency effective permittivity given either electrical or physical characteristic of the microstrip (see [1]-[2]):

$$\varepsilon_{eff}(0) = \begin{cases} \dfrac{\varepsilon_r+1}{2}\left[1 - \dfrac{1}{2H'}\left(\dfrac{\varepsilon_r-1}{\varepsilon_r+1}\right)\left(\ln\dfrac{\pi}{2} + \dfrac{1}{\varepsilon_r}\ln\dfrac{4}{\pi}\right)\right]^{-2} & w/h < 1.3 \\ \dfrac{\varepsilon_r+1}{2} + \dfrac{\varepsilon_r-1}{2}\left(1 + 10\dfrac{h}{w}\right)^{-0.555} & w/h > 1.3 \end{cases} \tag{3}$$

where H' is defined as in (1), or equivalently as a function of w/h given by

$$H' = \ln\left(4h/w + \sqrt{16(h/w)^2+2}\right)$$

1.4 Effective Permittivity Dispersion

Gives frequency-dependent effective permittivity, obtained by curve-fitting to rigorous analytical results (see [7]):

$$\epsilon_{\text{eff}}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{\text{eff}}(0)}{1 + P(f)} \tag{4}$$

$$\begin{aligned} \text{where} \quad P(f) &= P_1 P_2 \left[(0.1844 + P_3 P_4)\, 10fh\right]^{1.5763} \\ P_1 &= 0.27488 + [0.6315 + 0.525/(1+0.157fh)^{20}]w/h \\ &\qquad -0.065683\exp(-8.7513w/h) \\ P_2 &= 0.33622[1 - \exp(-0.03442\epsilon_r)] \\ P_3 &= 0.0363\exp(-4.6w/h)\left\{1 - \exp[-(fh/3.87)^{4.97}]\right\} \\ P_4 &= 1 + 2.751\left\{1 - \exp[-(\epsilon_r/15.916)^8]\right\} \end{aligned}$$

with a 0.6% accuracy over the ranges $0.1 \leq w/h \leq 100$, $1 \leq \epsilon_r \leq 20$, $0 \leq h/\lambda_0 \leq 0.13$, and $\epsilon_{\text{eff}}(0)$ is calculated from (3).

1.5 Impedance Dispersion

Gives frequency-dependent characteristic impedance, obtained by curve-fitting to rigorous analytical results, using the power-current definition for impedance (see [2]):

$$Z_0(f) = Z_0(0)\,(R_8/R_9)^{R_{12}} \tag{5}$$

where

$$\begin{aligned}
R_1 &= 4.766\,\exp[-3.228\,(w/h)^{0.641}] \\
R_2 &= 0.016 + (0.0514\,\epsilon_r)^{4.524} \\
R_3 &= 1.206 - 0.3144\,\exp(-0.0389\,\epsilon_r^{1.4})\,\left[1 - \exp(-0.267\,(w/h)^7)\right] \\
R_4 &= 1 + 1.275\,\left[1 - \exp(-0.00463\,R_1 \epsilon_r^{1.674}\,(hf/18.37)^{2.745})\right] \\
R_5 &= \frac{5.086\,R_2\,(hf/28.84)^{12}}{0.384 + 0.386\,R_2}\,\frac{\exp[-22.2\,(w/h)^{1.92}]}{1 + 1.3\,(hf/28.84)^{12}}\,\frac{(\epsilon_r - 1)^6}{1 + 10(\epsilon_r - 1)^6} \\
R_6 &= \left[0.0962 + (19.47/hf)^6\right]^{-1} \\
R_7 &= \left[1 + 0.00245\,(w/h)^2\right]^{-1} \\
R_8 &= 0.9408\,\epsilon_{\text{eff}}^{R_4}(f) - 0.9603 \\
R_9 &= (0.9408 - R_5)\,\epsilon_{\text{eff}}^{R_4}(0) - 0.9603 \\
R_{10} &= 0.707\,(0.00044\,\epsilon_r^{2.136} + 0.0184)\,(hf/12.3)^{1.097} \\
R_{11} &= 1 + 0.0503\,\epsilon_r^2\,R_6\,\left\{1 - \exp[-(w/15h)^6]\right\} \\
R_{12} &= R_3\,\left\{1 - 1.1241\,(R_7/R_{11})\exp[-0.026(hf)^{1.1566} - R_{10}]\right\}
\end{aligned}$$

The accuracy is specified to less than 1% for the ranges $0.1 \leq w/h \leq 10$, $1 \leq \epsilon_r \leq 18$, and $0 \leq h \cdot f \leq 30$ GHz·mm.

1.6 Microstrip Effective Width

The planar-waveguide model (see [3]) replaces a microstrip of width w by an ideal parallel-plane transmission line of width w_{eff}, where

$$w_{eff}(f) = \frac{h\eta_0}{Z(f)\sqrt{\epsilon_{\text{eff}}(f)}} \tag{6}$$

and where $Z(f)$ and $\varepsilon_{eff}(f)$ are calculated using the models described in (4) and (5)

1.7 Open-End Length Correction

Gives the effective length extension due to fringing at microstrip open circuits (see [5]-[6]):

$$\frac{l_{eo}}{h} = \frac{1}{2\pi}\,\frac{w/h + 0.366}{w/h + 0.556}\left\{0.28 + \frac{\varepsilon_r + 1}{\varepsilon_r}\,[0.274 + \ln(w/h + 2.518)]\right\} \tag{7}$$

This is a static result from Hammerstad [5], and as noted by Leir [8] is valid for the extremely wide lines sometimes used for patch antennas, unlike a previous expression from Hammerstad [4].

If wide lines are not being used, the more accurate and frequency-dependent expression due to Jansen *et al.* [6] should be used, which is given by

$$\frac{l_{eo}}{h} = R_1 R_3 R_5 / R_4 \tag{8}$$

where
$$\begin{aligned}
R_1 &= 0.434907 \frac{\epsilon_{\text{eff}}^{0.81} + 0.26}{\epsilon_{\text{eff}}^{0.81} - 0.189} \frac{(w/h)^{0.8544} + 0.236}{(w/h)^{0.8544} + 0.87} \\
R_2 &= 1 + \frac{(w/h)^{0.371}}{2.358\epsilon_r + 1} \\
R_3 &= 1 + \frac{0.5274 \tan^{-1}\left[0.084(w/h)^{1.9413/R_2}\right]}{\epsilon_{\text{eff}}^{0.9236}} \\
R_4 &= 1 + 0.0377 \tan^{-1}\left[0.067(w/h)^{1.456}\right] \\
&\quad \times \{6 - 5\exp\left[0.036(1 - \epsilon_r)\right]\} \\
R_5 &= 1 - 0.218\exp(-7.5w/h)
\end{aligned}$$

with a quoted accuracy of 2.5% over the range $0.01 \leq w/h \leq 100$ and $\varepsilon_r < 50$. The effective dielectric constant $\varepsilon_{eff}(f)$ should be calculated using the formula in (4).

1.8 Fringing Capacitance

The fringing capacitance of an open-circuited microstrip line is calculated from the open-end length extension as (see [1]-[2])

$$C_{fringe} \approx \frac{l_{eo}\sqrt{\varepsilon_{eff}}}{cZ_0} \tag{9}$$

References

1. T. C. Edwards, *Foundations for Microstrip Circuit Design*, Wiley, Great Britian, 1981.
2. R. K. Hoffmann, *Handbook of Microwave Integrated Circuits*, Translation from the German of *Integrierte Mikrowellenschaltungen*, Artech House, 1987.
3. G. Kompa and R. Mehran, “Planar Waveguide Model for Calculating Microstrip Components”, *Electron. Lett.*, vol. 11, pp. 459-460, 1975.
4. E. Hammerstad and O. Jensen, “Accurate Models for Microstrip Computer Aided Design”, *1980 IEEE MTT-S International Microwave Symposium Digest*, (Washington), pp. 407-409, 1980.
5. E. Hammerstad, “Computer-Aided Design of Microstrip Couplers with Accurate Discontinuity Models”, *1981 IEEE MTT-S international microwave symposium digest*, *Los Angeles*, pp. 54-56.
6. M. Kirschning, R. H. Jansen, N. H. L. Koster, “Accurate Model for Open-End Effect of Microstrip Lines”, *Electron. Lett.*, vol. 17, pp. 123-125, 1981.
7. M. Kirschning and R, H. Jansen, “Accurate Model for Effective Dielectric Constant with Validity up to Millimeter-Wave Frequencies”, *Electron. Lett.*, vol. 18, pp. 272-273, Jan 1982.
8. E. Lier, “Improved Formulas for Input Impedance of Coax-fed Microstrip Patch Antennas”, *Proc. IEE*, vol. 129, Pt. H, pp. 161-164, Aug 1982.