Final Exam

This exam is to be completed INDIVIDUALLY. You are free to use any external reference material, but if you use any equations or concepts that were not discussed in class or the course texts, please indicate the source (especially important if you are wrong). You are also free to use computer tools like Mathematica and MatLab, but submit a well-commented, neatly printed, full listing of your work. Please return the exam to me by 6:00 PM Friday, 23 March 2001. If you are leaving town, feel free to FAX me your exam at (805) 893-5947 (make it legible!!). If you need to reach me by phone, my numbers are (805) 893-7113 (Office) and (805) 964-4235 (Home).

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1) In 1934, Čerenkov discovered experimentally that all liquids and solids emit visible radiation when bombarded by fast-moving electrons moving at constant velocity through the material. Find a quantitative expression in the far-field for the electric field radiated by an electron moving with velocity v along the $\hat{z}$-axis by the following procedure:

   a) Express the time-dependent current density of the moving electron in terms of delta functions in the cylindrical coordinate system (note the problem has azimuthal symmetry). Fourier transform this current density to the frequency domain.

   b) In the Lorentz gauge the vector potential must then satisfy an equation of the form

   $$(\nabla^2 + k^2)\overline{A}(r,z) = \overline{F}(z)\frac{\delta(r)}{2\pi r}$$

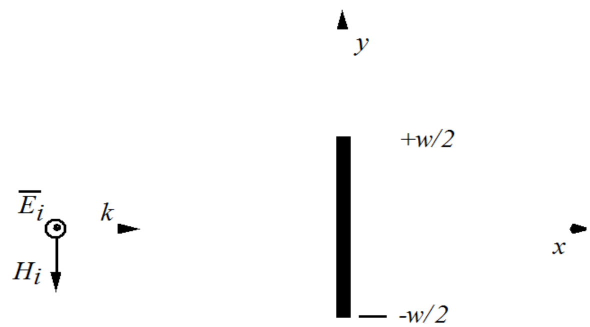
   where $\overline{F}(z)$ is a function that you should have an expression for from (a). Write $\overline{A}(r,z) = \overline{F}(z)g(r)$, and derive the equation governing $g(r)$. This should look familiar! Solve for $g(r)$.

   c) Use your results to determine the far-field electric field, using the asymptotic (far-field) form of $g(r)$. Your result should have the form of a plane wave, for which the constant phase-front forms a cone around the $\hat{z}$ direction. Find the angle that the $\overline{k}$ vector makes with the $\hat{z}$-axis, and find the critical velocity for this *Čerenkov radiation* to occur.

   d) You have just computed the spectrum function for the electric field, ie. the amplitude and phase of the field as a function of frequency. How can we compute the actual time-domain waveform observed in the far-field? There are some subtleties associated with this problem. Do <u>NOT</u> try to solve the problem numerically, but describe how one would do so, considering things like limits of integration, material properties, etc.. Could we use the method of stationary phase to get an analytic approximation?

Note that this analysis ignores the fact that as the charges radiate, they slow down due to the radiation reaction, and finally cease to radiate when the speed falls below the critical velocity. The problem is further complicated by the fact that when the particle is slowing down it is decelerating, which is itself another mechanism for emitting radiation in addition to the Čerenkov effect. The problem is impossible to solve analytically without approximation.

2) In this problem we consider an accurate numerical calculation of the scattered fields from a long, thin PEC strip. This is essentially a 2D problem, as shown below, where the strip is assumed to be infinitely long in the z direction (no variation of fields or currents in z). To keep things simple, we further assume that the incident electric field is linearly polarized in the z direction and travels along the x axis, so that the induced currents on the strip will be z-directed. For this special case only a scalar Green's function is required; you do not need dyadics! Prove this to yourself before continuing.



The Green's function for a line source at location $\overline{\rho}'$ is

$$G(\overline{\rho}, \overline{\rho}') = \frac{-\jmath}{4} H_0^{(2)}(k|\overline{\rho} - \overline{\rho}'|)$$

I. Find the scattered fields rigorously by the following procedure:

a) Set up an integral equation for the unknown currents on the strip by enforcing the boundary condition on the electric field. Assume the following: the strip is infinitesimally thin; that the incident frequency is 10 GHz; that the strip width is one wavelength, $w = \lambda$.

b) Solve by the method of moments using point matching and pulse basis. The only tricky part here is in evaluating the matrix elements where the source and observation points coincide. Fortunately, it is an integrable singularity, and you can use the small-argument approximation for the Hankel function to evaluate the integral analytically. The remaining integrals must be done numerically.

c) Once you have found the current distribution, find the scattered fields using techniques developed in class. Make a plot of the radar cross section (RCS) as a function of angle around the object.

II. Compare your result of (I.c) with that obtained by a simple physical optics (PO) approximation to the current on the strip.

III. Compare your result of (I.c) with that obtained by a more intelligent approximation to the current on the strip which includes the edge effect.

Lots of partial credit will be awarded on this question. The emphasis here is not on getting the exact answer to the tenth decimal place, but rather in showing that you understand the underlying concepts. So please document your efforts! Think about the problem a little before tackling it: what sort of current distribution do you expect? What scattered field pattern would you expect? How can you test that your answer(s) are correct on physical grounds? Don't make your life miserableby using zillions of expansion modes, or by trying to solve nasty integrals analytically, or by trying to make your program handle arbitrary 2D geometries. Write a simple, clean code to solve the problem as stated.