Homework #1 **Due: Friday, 19 January 2001**

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1) Read the handout on linear vector spaces, the Sturm-Liouville equation, and eigenfunction expansions.

2) Read the handout on vector potentials and TE/TM field decomposition. Using the TE/TM theory, write explicit expressions for all six components of $\overline{E}$ and $\overline{H}$ in both rectangular and cylindrical coordinates in terms of the scalar solutions to the Helmoholtz equation, φ^{TE} and φ^{TM}, for the pilot vector $\overline{c} = \hat{z}$ (*ie.* find the TE/TM to $\hat{z}$ fields).

3) Show from the one-dimensional Sturm-Liouville equation that, if the normalized eigenfunction ϕ_n is known, the associated eigenvalue is

$$\lambda_n = \int_a^b \phi_n \mathcal{L}\, \phi_n \, dx$$

4) Derive the conditions under which the scalar wave equation operator $\nabla^2 + k^2$ is self-adjoint, assumign a real scalar product of the form

$$\langle \psi | \phi \rangle = \iiint \psi \phi \, dV$$

5) A certain one-dimensional field problem reduces to that of solving the following equation

$$\frac{d^2 E(x)}{dx^2} + k^2 E(x) = -\mu J(x)$$

in the range $0 < x < a$, subject to the boundary conditions $E(0) = E(a) = 0$.

a) Solve this equation for $J(x) = 1$ using well-known ODE techniques (ie. variation of parameters). A good math handbook or Schaum's Outlines can help you here.
b) Solve the same problem using a Fourier series solution for the field.
c) According to the uniqueness theorem, the solutions in (a) and (b) shoudl be the same. Prove this by analytical or numerical means. An acceptable numerical proof woudl be to evaluate both solutions for $E(x)$ numerically and superimpose them on the same graph for comparison. Use the math program of your choice (Mathematica, MATLAB, MathCAD).
d) Extra credit: suggest a physical situation corresponding to this mathematical problem.
e) More extra credit: derive a green's function for this problem.