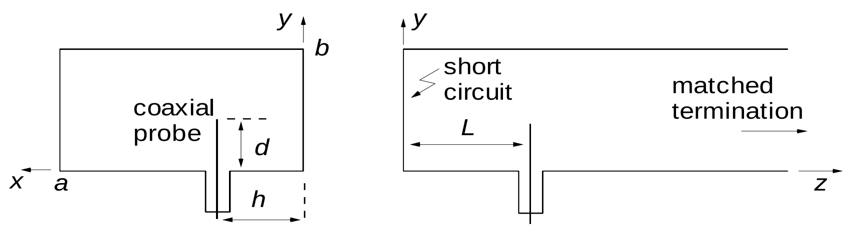
Homework #3 **Due: Friday, 9 February 2001**

1) A rectangular waveguide has the center conductor of a coaxline line extending down a distance d into its interior, located a distance h from a sidewall. One end of the guide is terminated in a short circuit a distance L behind the probe. The current along the probe is time-harmonic at frequency ω, which is low enough so that only the dominant mode can propagate. The probe current must vanish at the end, a so can be written as $\overline{J} = I_0 \sin k(d-y)\delta(x-h)\delta(z-L)$.



a) Using the theory developed in class, find the amplitudes of the excited modes in the waveguide.
b) Since only the dominant mode propagates, only this mode can carry power away from the probe. Find an expression for the average power P radiated by the probe. What values of L maximizes the power for a given probe current I_0?
c) Find the radiation resistance (the equivalent load resistance seen by the generator on the coaxial line) defined by $P = I_0^2 R_{rad}/2$. By varying the parameters h and d, design for a match to a $50\,\Omega$ coaxial line for efficient transfer of energy. Make a plot of R_{rad} versus frequency for your design.

2) Thin metal diaphragms are used in waveguides to make filters, resonators, and impedance matching circuits. Consider the symmetrical metal diaphragm in a rectangular PEC waveguide, shown below, which is modelled by the equivalent transmission-line circuit shown on the right (Z_1 is the wave impedance of the dominant mode). Using the variational expressions developed in the class notes, find an expression for the equivalent shunt admittance using your best guess as to what the fields in the aperture are, assuming the wave incident on the obstacle is the dominant mode, and assuming only the dominant mode can propagate. Evaluate your expression over a frequency range appropriate for a waveguide with $a = 2b = 2.286\,\text{cm}$, assuming a diaphragm opening of $c = a/2$. Is this a capacitive or inductive discontinuity?