ECE 201B ELECTROMAGNETIC THEORY WINTER 2001

Reading: Handouts on diffraction, stationary phase

Homework #5 **Due: Friday, 9 March 2001**

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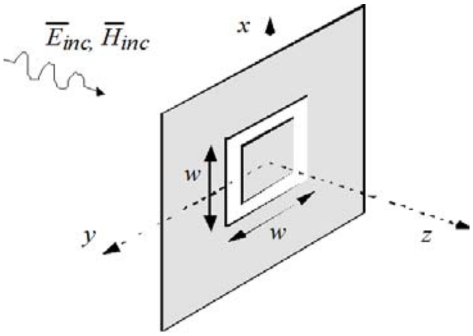
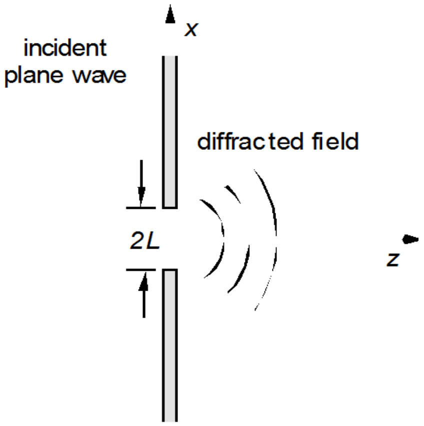
1) Using the vector diffraction theory developed in class, compute the far-field (*Fraunhoffer*) diffraction pattern from the aperture in a PEC sheet as shown below. The aperture is an electrically thin slot forming a square loop of side length w. Assume that the excitation is a linearly polarized plane wave with $\overline{E}_{inc} = E_0\hat{x}$.



2) We have treated vector diffraction formally in terms of the Green's functions in class. This is by no means the only approach possible. Another method that has intuitive appeal is the so-called *plane-wave spectrum representation* for the fields. Consider the 2D slit diffraction problem below:



The "eigenmodes" of the wave equation for $z > 0$, satisfying the radiation boundary condition at infinity, are just plane waves of the form

$$e^{-\jmath(k_x x + k_z z)} \qquad \text{where} \qquad k_x^2 + k_z^2 = k^2$$

Therefore an arbitrary field for $z > 0$ can be represented as a linear superposition of the eigenmodes. The eigenmodes form a continuous spectrum so we must write the superposition as an integral

$$\overline{E}(x, z) = \int_{-\infty}^{\infty} \overline{E}(k_x) e^{-\jmath(k_x x + k_z z)} dk_x$$

where $\overline{E}(k_x)$ is the *spectrum function*, found by enforcing the boundary condition at $z = 0$ and using the properties of the Fourier transform. Since this representation of the field is related to a spatial

Fourier transform it is sometimes called a *spectral domain* method. Note also that the summation in dk_x covers all possible values of the k vector, and involves both propagating and evanescent waves since $k_z = \sqrt{k^2 - k_x^2}$.

Find the far-field (Fraunhofer region) diffraction pattern assuming an $\hat{x}$-polarized plane wave of magnitude E_0 is incident from $z < 0$ (a "uniformly illuminated" aperture) using the following method: (a) First, find the spectrum function as described above. (b) By observing the behavior of the integrand, make an appropriate Taylor expansion of k_z. (c) The far fields can then be evaluated by the method of stationary phase. Show that the resulting field expression is given by

$$\overline{E}(x,z) \approx \hat{x} E_0 L \sqrt{2\jmath k/\pi z} \frac{\sin(kLx/z)}{kLx/z} e^{-\jmath(kx^2/2z)} e^{-\jmath kz}$$

which is exactly the same field as obtained in class by other means.

3) Consider radar scattering from a large planar conductor of width W in the x direction and length L in the y direction. Assume the plate is far enough from the radar that the incident wave is a plane wave. The incident wave arrives at an angle of θ with respect to the normal. Using the induction theorem, reduce this scattering problem to a current in free space (assume the plate is large enough so that image theory holds). Find the backscattered field using our far-field diffraction theory formalism, and compute the *backscattering cross section*, or *radar cross section* (RCS) according to

$$\sigma_{\text{rcs}} = \lim_{R \to \infty} \left(4\pi R^2 \frac{P_{\text{scatt}}}{P_{\text{inc}}} \right)$$

where P_{scatt} is the scattered power density in the direction of the radar set. Your result should reduce to the classic $\sigma = 4\pi(\text{Area})^2/\lambda^2$ for the case of normal incidence.