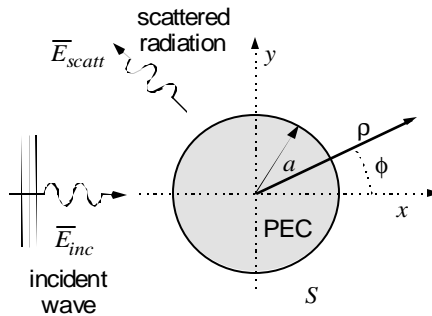


Reading: Handouts

Homework #6

Due: Friday, 16 March 2001

Consider the scattering of a plane wave by a long conducting cylinder of radius  $a$ , as shown in the figure below



For simplicity, we will consider only TE excitation of unit amplitude, such that

$$\bar{E}_{inc}(\rho, \phi) = \hat{z}e^{-jkx} = \hat{z}e^{-jk\rho\cos\phi}$$

This field will excite a  $\hat{z}$ -directed current on the surface of the cylinder, which can be represented as

$$J(\rho, \phi) = \hat{z}I(\phi)\delta(\rho - a)$$

- a) Using expressions for the fields in terms of the 2D Green's function (developed in the handout on auxiliary potentials), first demonstrate that the scattered field can be represented as

$$\bar{E}_{scatt}(\rho, \phi) = -\hat{z}\frac{a\omega\mu}{4} \int_0^{2\pi} I(\phi')H_0^{(2)}(kR) d\phi'$$

where

$$R = \sqrt{a^2 + \rho^2 - 2a\rho\cos(\phi - \phi')}$$

where  $H_0^{(2)}(x)$  is the Hankel function of the second kind.

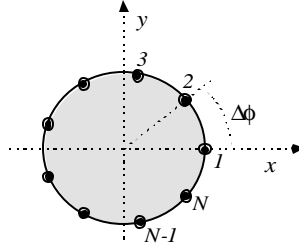
- b) Next, apply the boundary conditions to generate an integral equation for the unknown current on the cylinder.
- c) Apply the Method-of-Moments procedure to generate a matrix equation, using the following procedure. Expand the unknown current as

$$I(\phi) = \sum_{n=1}^N i_n f_n(\phi)$$

where  $f_n(\phi)$  are the basis functions. We will use a so-called "pulse" basis, and enforce the boundary condition at  $N$  discrete points on the cylinder (point-matching) given by

$$\phi_m = (m - 1)\Delta\phi = (m - 1)\frac{2\pi}{N}$$

These points are depicted below:



The pulse basis functions are defined by

$$f_n(\phi) = \begin{cases} 1 & |\phi - \phi_n| \leq \Delta\phi/2 \\ 0 & \text{otherwise} \end{cases}$$

Derive a matrix equation for the unknown expansion coefficients  $i_n$  and show that the matrix elements are related to integrals of the form

$$\int_{\phi_n - \Delta\phi/2}^{\phi_n + \Delta\phi/2} H_0^{(2)} \left( ka \sqrt{2[1 - \cos(\phi_m - \phi)]} \right) d\phi'$$

where  $m = 1 \dots N$  and  $n = 1 \dots N$ .

- d) Solve this equation numerically for the case of  $a = \lambda/2$ , and make a plot of the current distribution. Note that the matrix elements along the diagonal ( $m = n$ ) must be handled carefully due to a singular integrand. Using the small argument expansion for the Hankel function, show that this is an integrable singularity, and evaluate analytically for those cases. The small argument expansion is

$$H_0^{(2)}(x) \approx 1 - j\frac{2}{\pi} [\ln(x/2) + \gamma]$$

where  $\gamma = 0.5772156$  is the Euler's constant (called **EulerGamma** in *Mathematica*).

- e) Using the asymptotic expansion for  $H_0^{(2)}(x)$  when  $x \gg 1$ , find a general expression for the far-field scattering pattern in terms of the approximate representation of the current, and identify the differential scattering cross section. Make a plot of the scattering pattern for  $a = \lambda/2$ .
- f) Find the monostatic radar cross section for this object, defined in 2D as

$$\sigma_{RCS} = \lim_{\rho \rightarrow \infty} 2\pi\rho \frac{|\overline{E}_{scatt}|^2}{|\overline{E}_{inc}|^2}$$

Note that we *could* have improved the computational efficiency by exploiting the symmetry of the problem to reduce the number of unknowns: the current on the top half of the cylinder should have the same form as the bottom half, so that  $i_n = i_{N-n}$ . In very large and complex problems, it is often necessary to make use of such symmetries to keep the size of the matrix manageable.