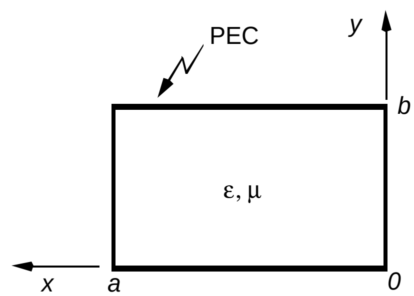
NORMALIZED WAVEGUIDE MODES

1 RECTANGULAR WAVEGUIDE



Assuming $e^{-\gamma z}$ dependence we have

$$\begin{aligned}
&\text{TE modes } (e_z = 0): && \text{TM modes } (h_z = 0): \\
&h_z = A_{mn}^{\mathrm{TE}} \cos(k_x x)\cos(k_y y) && e_z = A_{mn}^{\mathrm{TM}} \sin(k_x x)\sin(k_y y) \\
&e_x = \jmath\omega\mu A_{mn}^{\mathrm{TE}} \frac{k_y}{k_c^2}\cos(k_x x)\sin(k_y y) && e_x = \gamma_{mn} A_{mn}^{\mathrm{TM}} \frac{k_x}{k_c^2}\cos(k_x x)\sin(k_y y) \\
&e_y = -\jmath\omega\mu A_{mn}^{\mathrm{TE}} \frac{k_x}{k_c^2}\sin(k_x x)\cos(k_y y) && e_y = \gamma_{mn} A_{mn}^{\mathrm{TM}} \frac{k_y}{k_c^2}\sin(k_x x)\cos(k_y y)
\end{aligned} \tag{1}$$

where

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b} \qquad k_c^2 = k_x^2 + k_y^2 = \gamma_{mn}^2 + k^2$$

Note that for TE modes, either m or n can be zero, but both cannot be zero simultaneously. For TM modes, neither m or n can ever be zero; the lowest order TM mode is TM_{11}.

Enforcing the desired normalization condition

$$\int_0^a \int_0^b \hat{e}_{t,mn} \cdot \hat{e}_{t,m'n'}\, dy\, dx = \delta_{mm'}\delta_{nn'} \tag{2}$$

we find, for the TE modes,

$$A_{mn}^{\mathrm{TE}} = \frac{k_c}{\jmath\omega\mu}\sqrt{\frac{\epsilon_{m0}\epsilon_{n0}}{ab}} \qquad \text{where} \quad \epsilon_{ij} = \begin{cases} 1 & i = j \\ 2 & i \neq j \end{cases} \tag{3}$$

and for the TM modes,

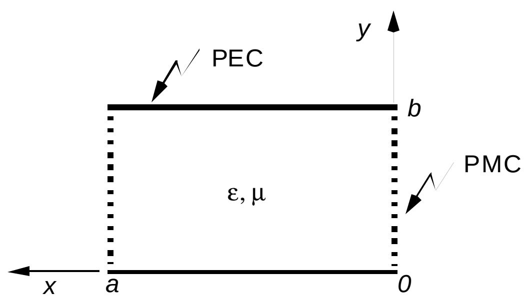
$$A_{mn}^{\mathrm{TM}} = \frac{k_c}{\gamma_{mn}} \frac{2}{\sqrt{ab}} \tag{4}$$

So we can define normalized TE and TM field vectors for the transverse fields as

$$\hat{e}_{t,mn}^{\mathrm{TE}} = \frac{1}{k_c} \sqrt{\frac{\epsilon_{m0}\epsilon_{n0}}{ab}} \left[\hat{x} k_y \cos(k_x x)\sin(k_y y) - \hat{y} k_x \sin(k_x x)\cos(k_y y)\right] \tag{5a}$$

$$\hat{e}_{t,mn}^{\mathrm{TM}} = \frac{1}{k_c} \frac{2}{\sqrt{ab}} \left[\hat{x} k_x \cos(k_x x)\sin(k_y y) + \hat{y} k_y \sin(k_x x)\cos(k_y y)\right] \tag{5b}$$

2 IDEAL PARALLEL-PLATE WAVEGUIDE



Assuming $e^{-\gamma z}$ dependence we have

$$\begin{array}{ll} \text{TE modes } (e_z = 0): & \text{TM modes } (h_z = 0): \\ h_z = A_{mn}^{\mathrm{TE}} \sin(k_x x)\cos(k_y y) & e_z = A_{mn}^{\mathrm{TM}} \cos(k_x x)\sin(k_y y) \\ e_x = \jmath\omega\mu A_{mn}^{\mathrm{TE}} \frac{k_y}{k_c^2} \sin(k_x x)\sin(k_y y) & e_x = \gamma_{mn} A_{mn}^{\mathrm{TM}} \frac{k_x}{k_c^2} \sin(k_x x)\sin(k_y y) \\ e_y = \jmath\omega\mu A_{mn}^{\mathrm{TE}} \frac{k_x}{k_c^2} \cos(k_x x)\cos(k_y y) & e_y = -\gamma_{mn} A_{mn}^{\mathrm{TM}} \frac{k_y}{k_c^2} \cos(k_x x)\cos(k_y y) \end{array} \tag{6}$$

where

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b} \qquad k_c^2 = k_x^2 + k_y^2 = \gamma_{mn}^2 + k^2$$

Note that for TE modes, $m \geq 1$ and $n \geq 0$, and for TM modes, $m \geq 0$ and $n \geq 1$.

Enforcing the desired normalization condition

$$\int_0^a \int_0^b \hat{e}_{t,mn} \cdot \hat{e}_{t,m'n'}\, dy\, dx = \delta_{mm'}\delta_{nn'} \tag{7}$$

we find, for the TE modes,

$$A_{mn}^{\mathrm{TE}} = \frac{k_c}{\jmath\omega\mu} \sqrt{\frac{2\epsilon_{n0}}{ab}} \qquad \text{where} \quad \epsilon_{ij} = \begin{cases} 1 & i = j \\ 2 & i \neq j \end{cases} \tag{8}$$

and for the TM modes,

$$A_{mn}^{\mathrm{TM}} = \frac{k_c}{\gamma_{mn}}\sqrt{\frac{2\epsilon_{m0}}{ab}} \tag{9}$$

So we can define normalized TE and TM field vectors for the transverse fields as

$$\hat{e}_{t,mn}^{\mathrm{TE}} = \frac{1}{k_c}\sqrt{\frac{2\epsilon_{n0}}{ab}}\left[\hat{x}k_y \sin(k_x x)\sin(k_y y) + \hat{y}k_x \cos(k_x x)\cos(k_y y)\right] \tag{10a}$$

$$\hat{e}_{t,mn}^{\mathrm{TM}} = \frac{1}{k_c}\sqrt{\frac{2\epsilon_{m0}}{ab}}\left[\hat{x}k_x \sin(k_x x)\sin(k_y y) - \hat{y}k_y \cos(k_x x)\cos(k_y y)\right] \tag{10b}$$

This structure also supports a TEM mode with $e_z = h_z = 0$. The normalized TEM field vector is simply

$$\hat{e}_{t,mn}^{\mathrm{TEM}} = \frac{1}{\sqrt{ab}}\hat{y} \tag{11}$$