Reading: Balanis Chapters 2-3

Homework #1 **Due: Wednesday, 16 April 2003**

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1) An array of identical, phase-locked point sources (isotropic radiators) placed along the $\hat{z}$-axis and spaced a distance d apart, has a normalized field pattern described by

$$f(\theta, \phi) = \frac{1}{N} \frac{\sin\left(\frac{N}{2} kd \cos\theta\right)}{\sin\left(\frac{1}{2} kd \cos\theta\right)}$$

where N is the number of elements and k is the propagation constant (there is no ϕ dependence). Assume $N = 5$ and $d = \lambda_0/2$. Use *Mathematica*, *MATLAB*, or any other math program to:

a) Plot the power pattern (in dB) for any ϕ =constant plane. Do both a rectangular and polar plot.

b) Make a 3D plot (spherical or cylindrical) using a linear scale for simplicity.

c) Calculate the maximum directivity D_0 and beam solid angle Ω_A

d) Find the 3 dB beamwidth (HPBW), first-null beamwidth (FNBW), and first sidelobe level.

e) Many planar arrays are made on a substrate with a ground plane, which restricts the radiation to a half-space. Suppose the array elements can be modelled as "semi-point sources"; that is, sources that radiate isotropically into 2π steradians. Using the same parameters as above, what would be the directivity for this case?

2) In class we used only the far-field terms in the field expressions for a Hertzian dipole to show that the total average power passing through a spherical surface is

$$P = \eta \frac{\pi}{3} I^2 \left(\frac{dl}{\lambda}\right)^2 \quad \text{Watts}$$

a) Using the complete equations for the electric and magnetic fields of a Hertzian dipole, show that this same result is obtained in the near field as well as the far field. This must be true, of course; otherwise the power would "pile up" somewhere.

b) Since these near field terms do not contribute to average power flow away from the antenna, we attribute their presence to stored (reactive) energy in the vicinity of the antenna. Using the complex Poynting vector, the total complex time-averaged power can be written as

$$P = \frac{1}{2} \oiint (\overline{E} \times \overline{H}^*) \cdot d\overline{S} = P_{rad} + \jmath 2\omega(\tilde{W}_m - \tilde{W}_e)$$

where $\tilde{W}_m$ and $\tilde{W}_e$ are the time-averaged magnetic and electric energy densities, respectively. Is the reactive part of the input impedance of the Hertzian dipole capacitive or inductive?

3) The surface resistivity of a conductor is given by $R_s = \sqrt{\pi f \mu / \sigma}\ \Omega/\square$, where σ is the conductivity of the metal. For a wire of radius a and length l carrying a uniform current, the total loss resistance will be

$$R = \frac{l}{2\pi a} R_s$$

Calculate the radiation efficiency for a $\lambda/80$ Hertzian dipole operating at 3 MHz in free-space. Assume that the dipole is made from copper wire ($\sigma = 5.8 \times 10^7$ S/m) with $a = 1.024$ mm.

4) Do problem 2.41 in Balanis.

5) Do problem 2.53 in Balanis.

6) Do problem 2.61 in Balanis.