Reading: Balanis Chapter 6-7, plus Handouts

Homework #4 **Due: Wednesday, 7 May 2003**

1) In our discussion of the line-source synthesis problem, we noted that the "Sinc" pattern $\mathrm{SF}(u) = \sin u/u$ resulting from a uniform line source is typically unsatisfactory in terms of its rather large sidelobes near the main beam. A simple way to address this deficiency is to multiply the number of zeros as follows:

$$\mathrm{SF}(u) = \left(\frac{\sin u}{u}\right)^N$$

Using the Fourier synthesis approach, find a numerical solution for the current distribution that will produce this pattern over a visible space of $|u| \leq 5\pi$, assuming N=3. Make a plot of the magnitude and phase of the current distribution and estimate the electrical length L/λ that is required.

2) Consider a uniformly-fed N-element array of $\hat{y}$-directed Hertzian dipoles spaced equally along the z-axis:

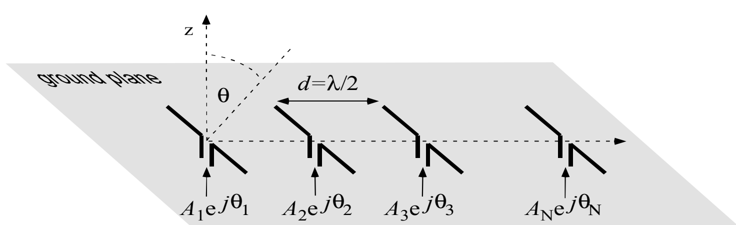
   a) Derive an exact analytical expression for the maximum directivity of this array. Note that

$$\left[\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right]^2 = N + 2\sum_{k=1}^{N-1}(N-k)\cos(2k\psi/2)$$

   b) Assume the array is fed for broadside radiation. Using the result of part (a), find the directivity in the limit of $kd \to 0$. Does this answer make sense?

   c) Evaluate your expression for directivity for a uniform broadside array with $N = 10$ and $d = \lambda/2$. Compare with the approximate result of Balanis.

3) The figure below shows a linear array of N dipoles, uniformly separated by $d = \lambda/2$ along the H-plane. The dipoles are electrically short, and are located $\lambda/4$ above a ground plane. Generate a set of **Polar** plots showing the H-plane pattern for this array as we attempt to scan the beam away from the broadside direction. Assume the N=4 elements are driven uniformly with a constant phase progression, and vary the phase shift from 0° to 180° in 30° increments. Ignore mutual coupling in this computation.



For each phase shift that you use, compute the direction of the main beam, the first sidelobe level, and the maximum directivity. Although not as accurate as the numerical integration routines built into the math programs, it is usually simpler to use a crude numerical approximation to the directivity integral as follows

$$\oiint P(\theta,\phi)\sin\theta\, d\theta\, d\phi \approx \sum_i \sum_j P(\theta_i,\phi_j)\sin\theta_i\,\Delta\theta\,\Delta\phi$$

How do the patterns change if the dipoles were rotated so that scanning takes place in the E-plane of the dipoles? In both cases, comment on the behavior of the beam as a function of scan angle, and the implications for a practical system.