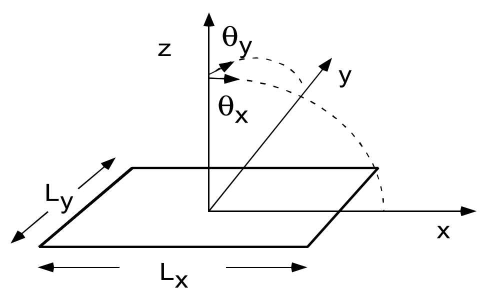
Reading: Balanis Chapter 10-12,15, plus Handouts

Homework #6 **Due: Wednesday, 4 June 2003**

1) Show by direct integration of the power pattern that the effective area of a large, uniformly illuminated rectangular aperture (or array) is equal to the physical area, assuming that the array radiates in only one direction (upwards), not two. Hint: the calculation is fairly simple if L_x and L_y are large enough so that $\sin\theta_x \approx \theta_x$ and $\sin\theta_y \approx \theta_y$ for angles where the radiated power is appreciable. Using θ_x and θ_y is somewhat easier than the usual θ, ϕ. Also, can you approximate the limits of integration for the integrals of the form

$$\int \frac{\sin^2 x}{x^2}\, dx$$

in some helpful way?).



2) Prove the more general result that the aperture efficiency is given by:

$$\eta_a \equiv \frac{A_{eff}}{A_{physical}} = \frac{\langle E_a(x,y)\rangle^2}{\langle {E_a}^2(x,y)\rangle}$$

where E_a is the tangential field at any point in the aperture, the fields are in phase and in the same direction over the aperture, and $\langle\rangle$ means a spatial average over the aperture. From this result it is obvious that a uniformly illuminated aperture has an aperture efficiency of unity and all tapered aperture distributions have smaller efficiencies. Using this expression sometimes provides a very simple way to calculate A_{eff} and hence the maximum gain of aperture antennas. As discussed in Balanis (§12.9), we can use a plane wave expansion

$$f(k_x, k_y) = \iint E_a(x, y, 0) e^{\jmath(k_x x + k_y y)}\, dx\, dy$$

and via stationary phase methods it is shown that

$$|E(r, \theta, \phi)| = \frac{|f(k'_x, k'_y)|}{r\lambda}$$

where k'_x and k'_y are the stationary phase points:

$$k'_x = k_0 \sin\theta\cos\phi \qquad k'_y = k_0 \sin\theta \sin\phi$$

Note that since the aperture fields are all in phase (by assumption), $|E|$ and the radiated power density will be a maximum on the z-axis. To find the total radiated power, you could integrate over the whole antenna pattern, but there is an easier way: calculate the power flux just above the aperture, and from this deduce D_0 and hence A_{eff}.

3) Consider the problem of radiation from a rectangular aperture on a dielectric substrate (printed slot antenna). To make the analysis simpler, it is common to assume a semi-infinite substrate; this assumption is justifiable when the substrate is electrically thick. The geometry of the problem is shown below. The aperture is driven by a uniform current I_0 across the slot (in the $\hat{x}$-direction) and located at the point $y = y_0$. This current will excite an electric field in the aperture. For thin slots, the field will be in the $\hat{x}$-direction, given by $E_a(x, y)\hat{x}$. Derive, <u>but do not try to solve</u>, the integral equation for the unknown aperture field (or equivalent magnetic current) by applying field equivalence principles on both sides of the slot, and enforcing the boundary conditions at the aperture plane (see figure 12.5 in Balanis). The resulting equation is analogous to Pocklington's equation for the dipole antenna, and can be solved by Method of Moments.