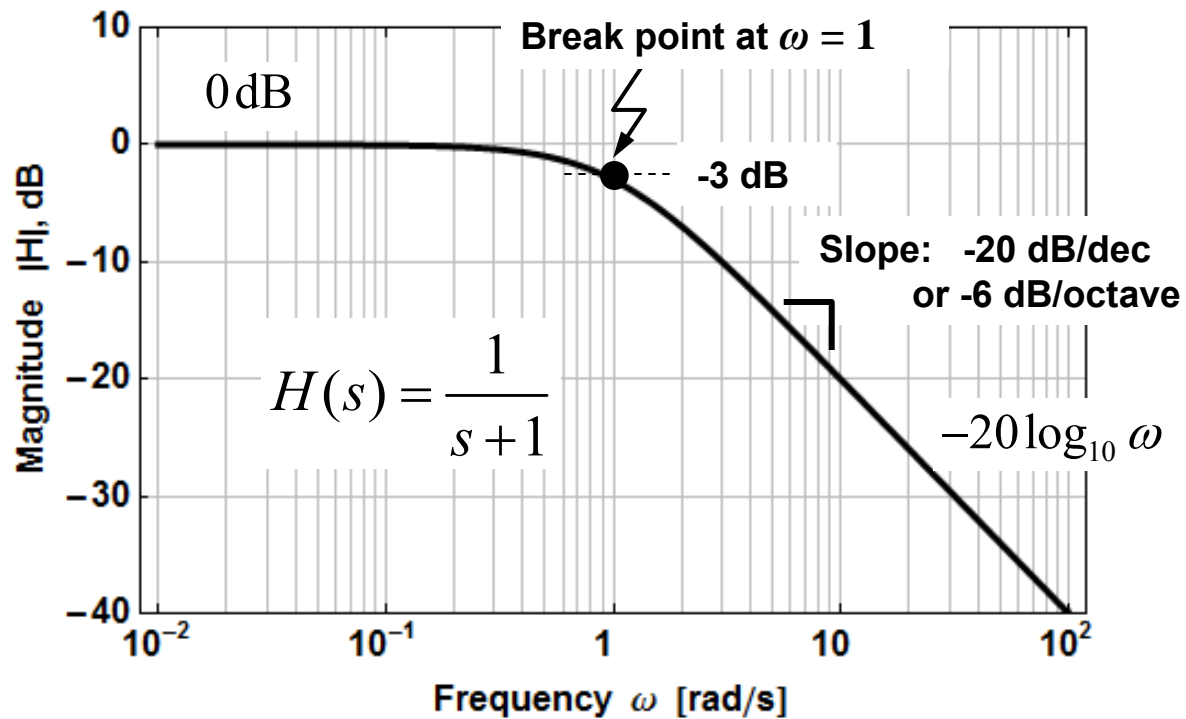
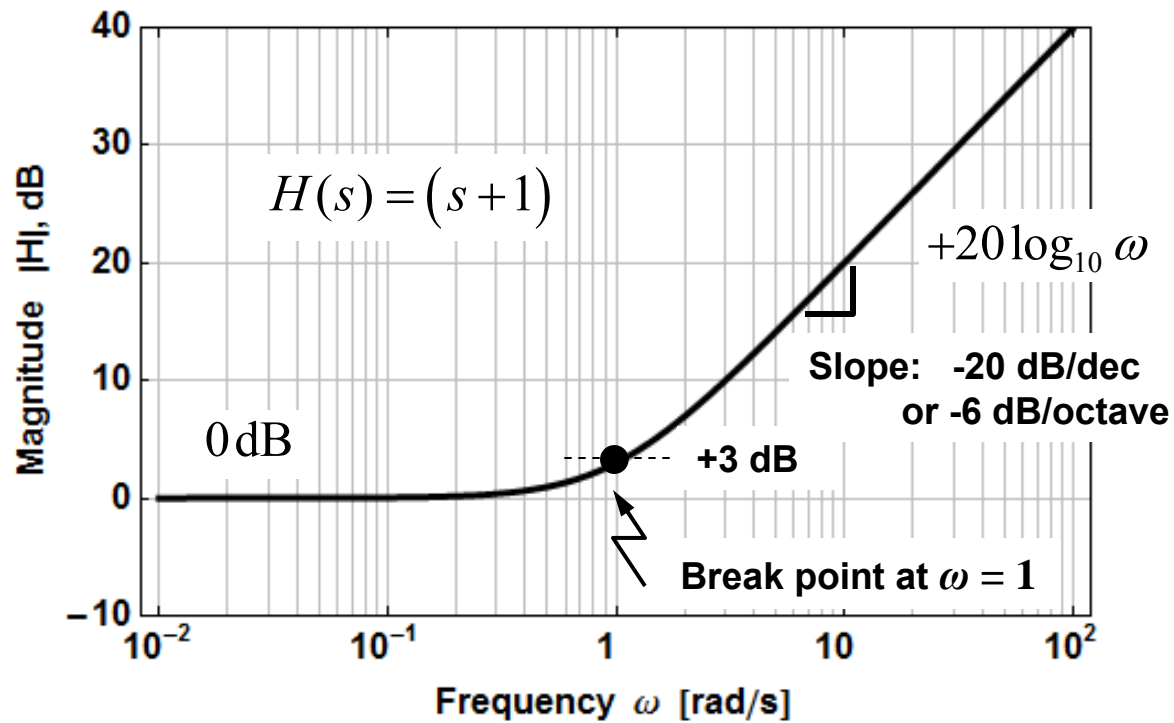


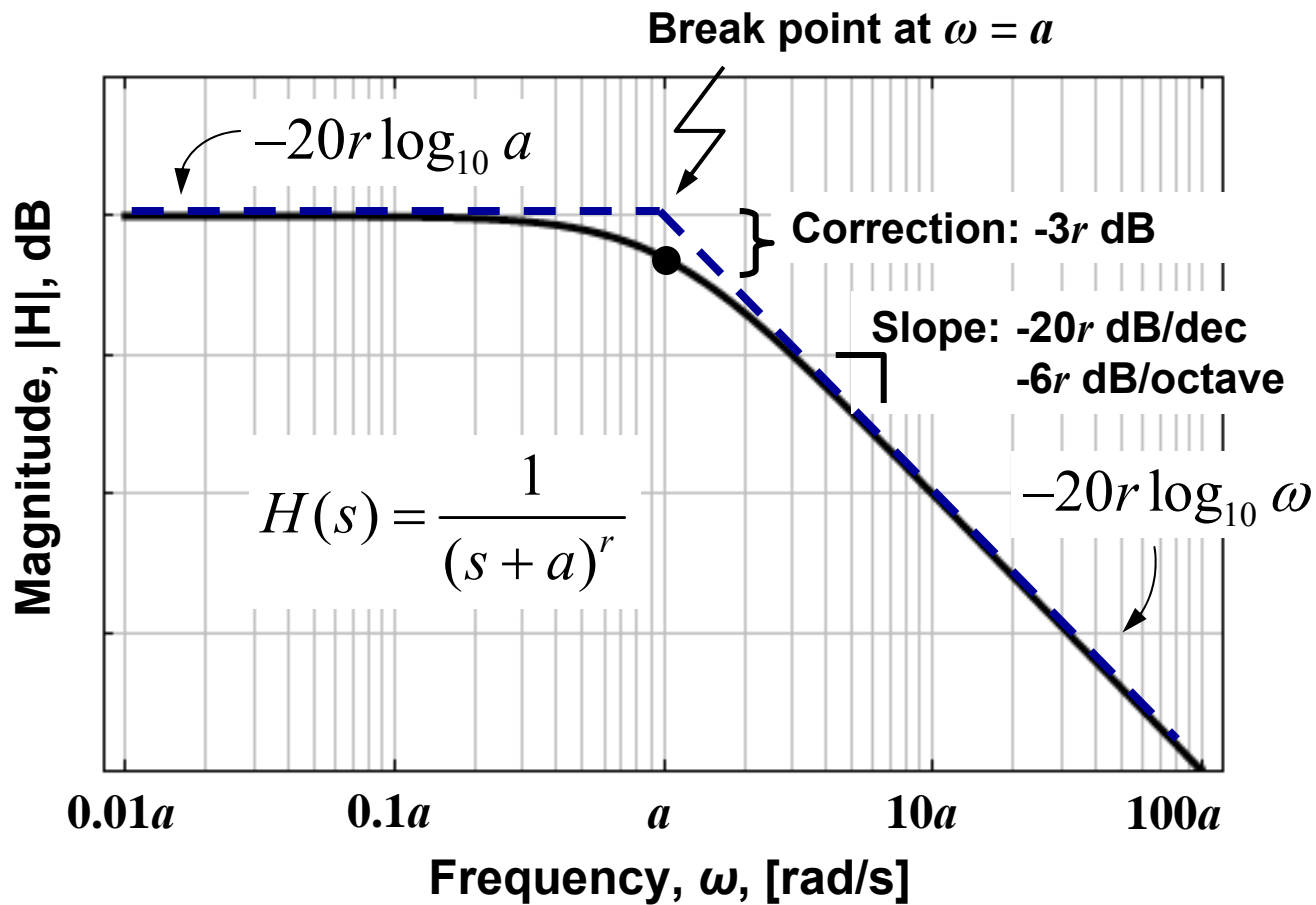
Simple Pole – Amplitude



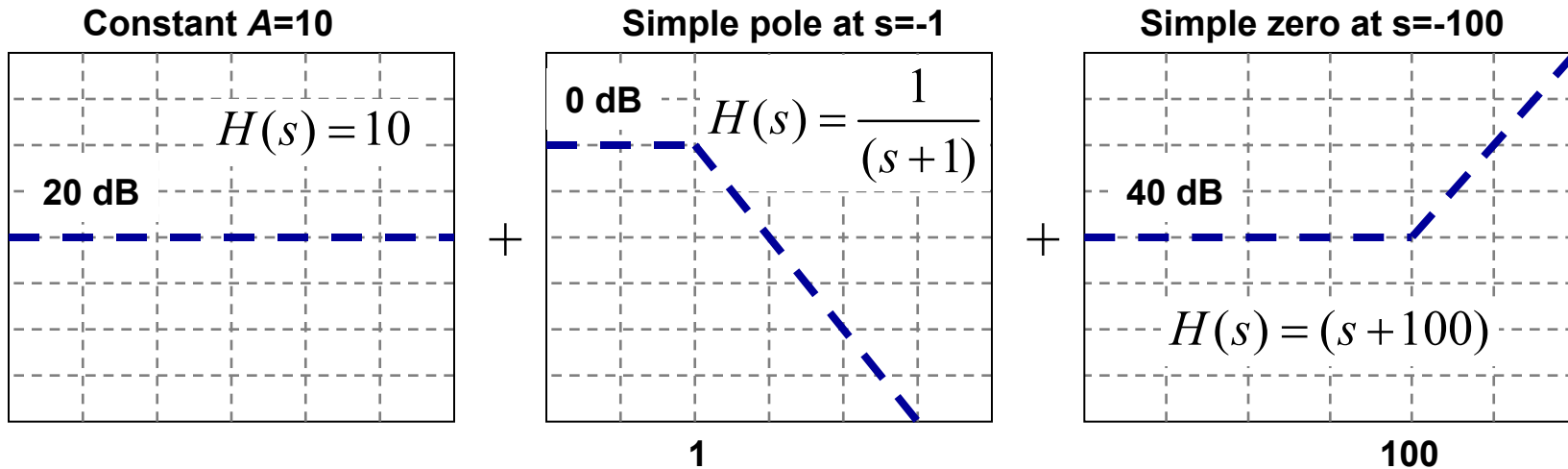
Simple Zero



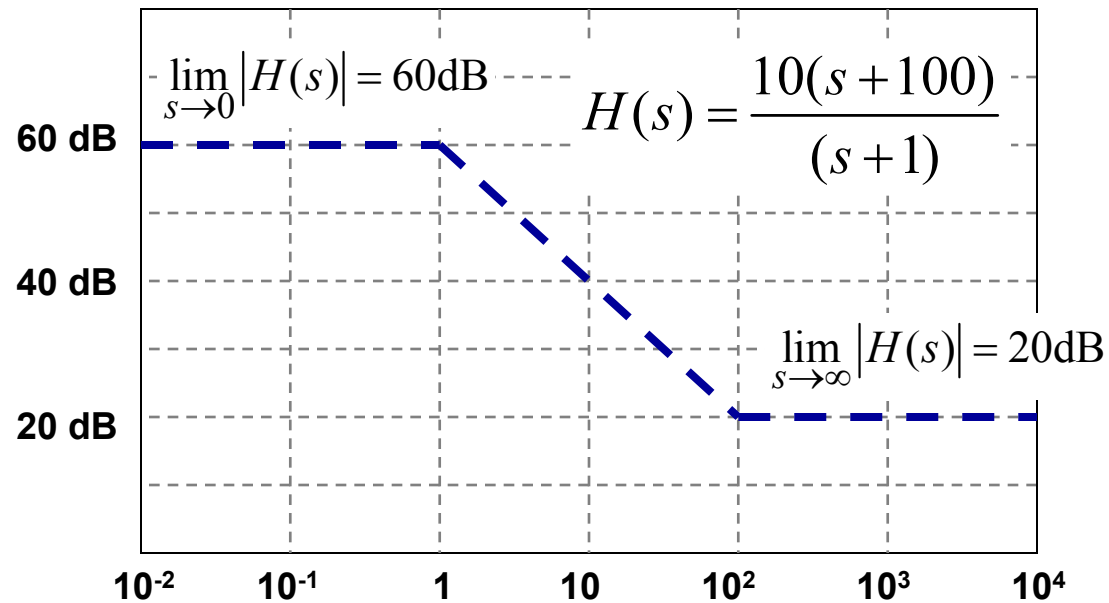
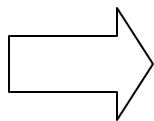
Bode Straight-Line Plot



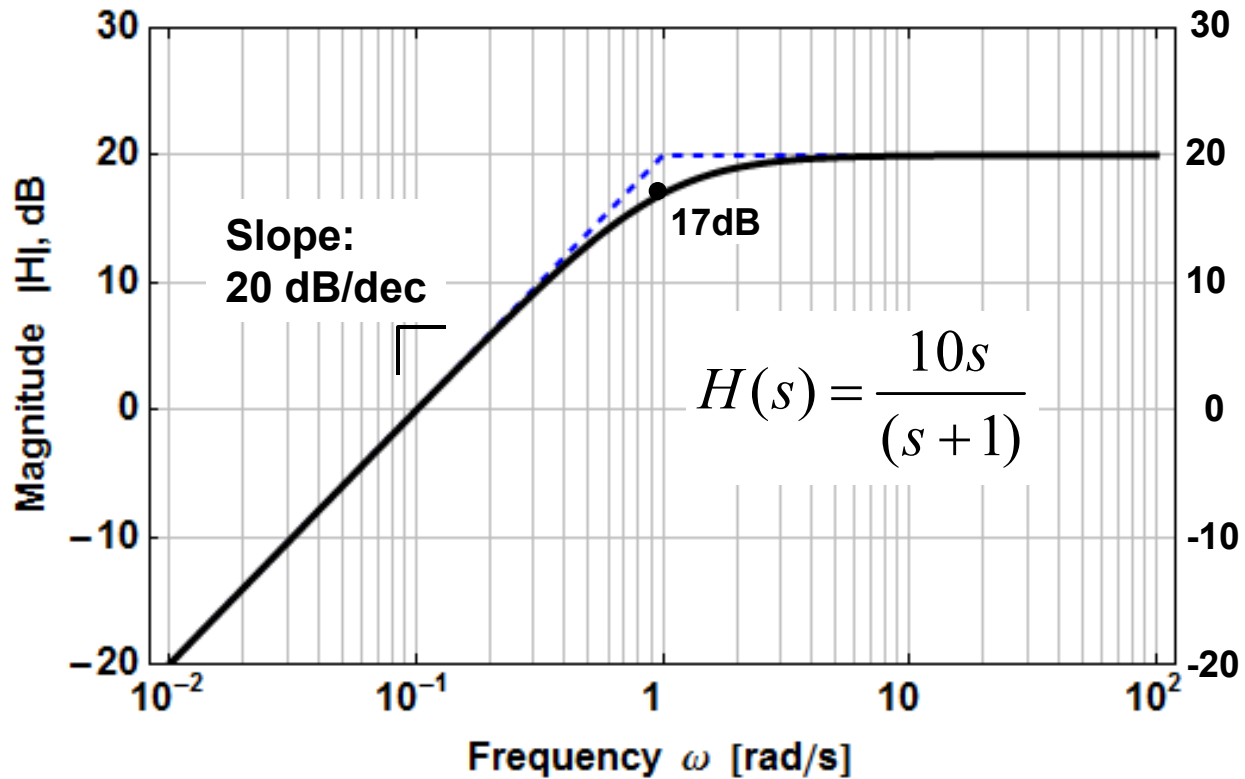
Combination of Pole and Zero



Add together



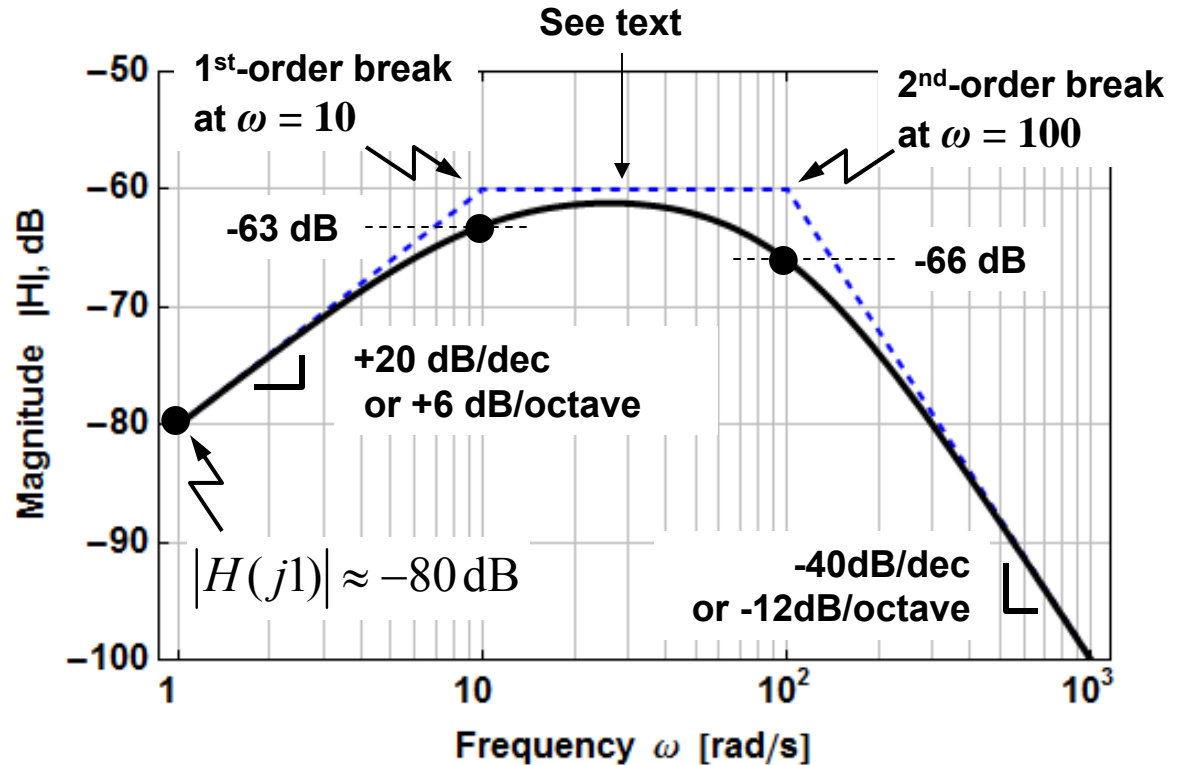
1st-order High-Pass



Finding a Reference

$$H(s) = \frac{10s}{(s+10)(s+100)^2}$$

$$\begin{aligned} |H(j1)| &= \frac{10|j1|}{|j1+10||j1+100|^2} \\ &= \frac{10}{\sqrt{1+10^2} \left(\sqrt{1+100^2}\right)^2} \\ &\approx 10^{-4} \Rightarrow -80 \text{ dB} \end{aligned}$$

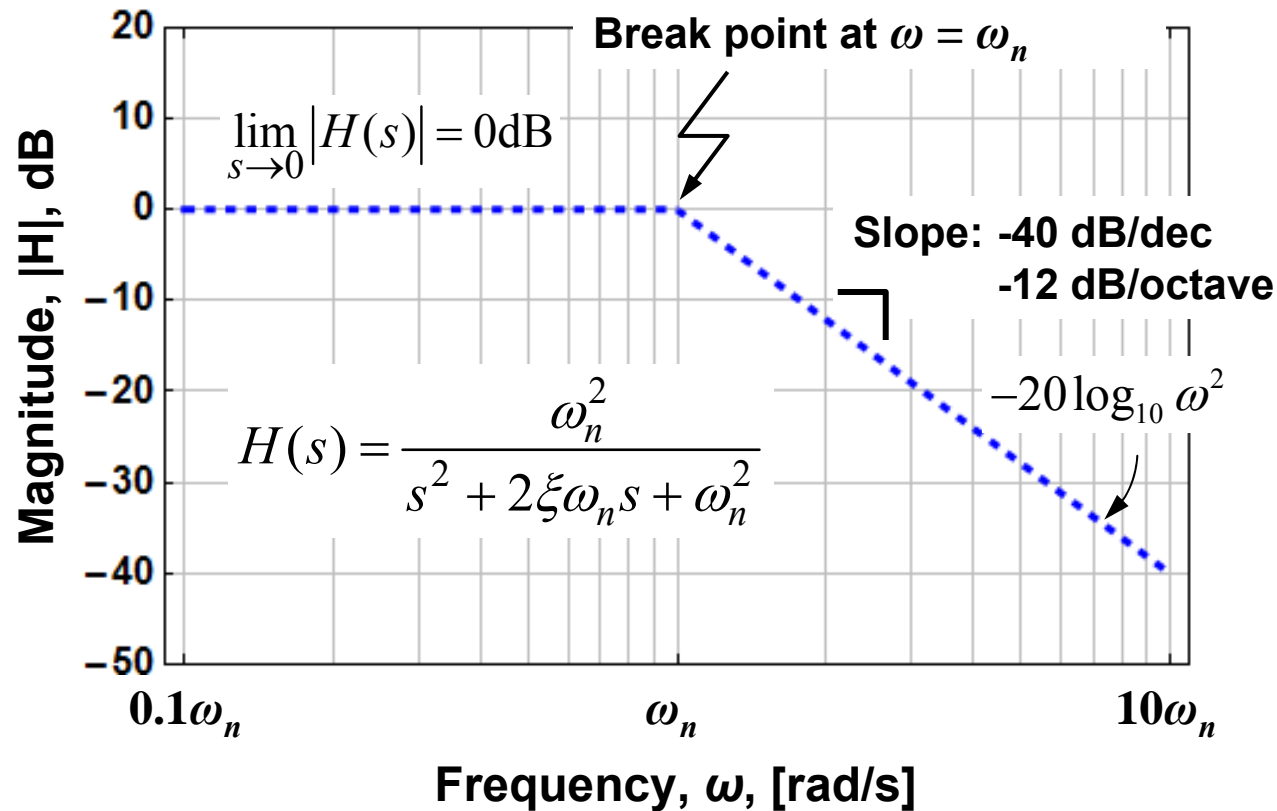


$$|H(j10)| \approx \underbrace{\left[\frac{10s}{s+10} \right]}_{\approx 10 \text{ for } \omega \gg 10} \times \underbrace{\left[\frac{1}{(s+100)^2} \right]}_{\approx 10^{-4} \text{ for } \omega \ll 100} \approx 10^{-3} \Rightarrow -60 \text{ dB}$$

terms with poles or zeros at or below $\omega=10$ terms with poles or zeros at or above $\omega=10$

General 2nd-order Asymptotes

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad |H(j\omega)| = \frac{\omega_n^2}{|\omega_n^2 - \omega^2 + j2\xi\omega_n\omega|}$$

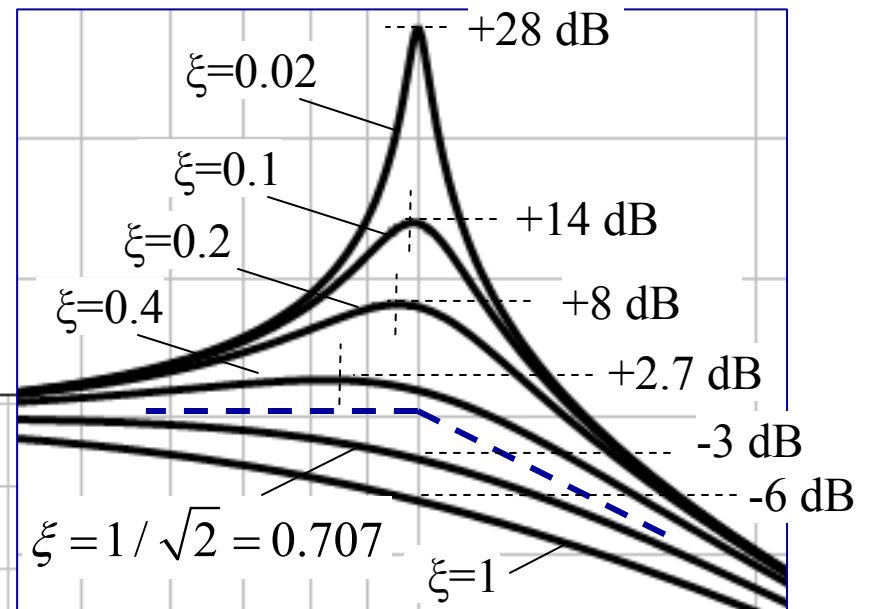
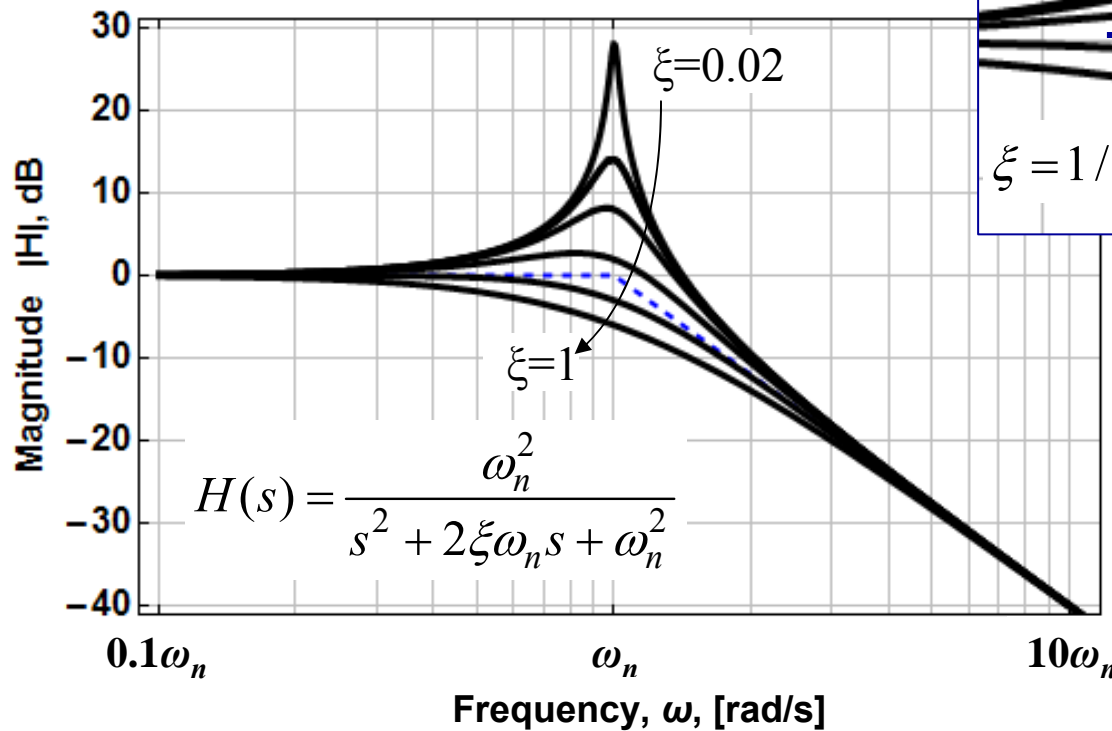


General 2nd-order Low-Pass



$$|H(j\omega)| = \frac{\omega_n^2}{|\omega_n^2 - \omega^2 + j2\xi\omega_n\omega|}$$

At break point: $|H(j\omega_n)| = \frac{1}{2\xi}$



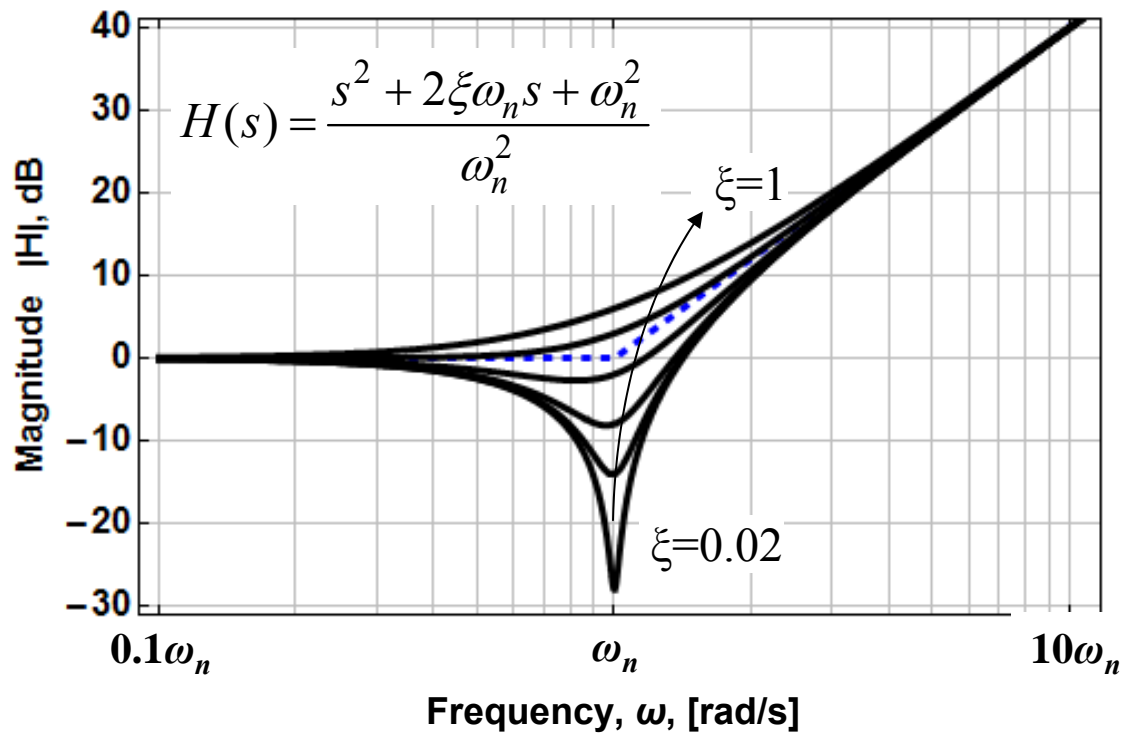
Peak location and value:

$$\frac{\partial}{\partial \omega} |H(j\omega)| = 0$$

$$\omega_p = \omega_n \sqrt{1 - 2\xi^2}$$

$$|H(j\omega_p)| = \frac{1}{2\xi\sqrt{1 - \xi^2}}$$

Complex Zero Pair

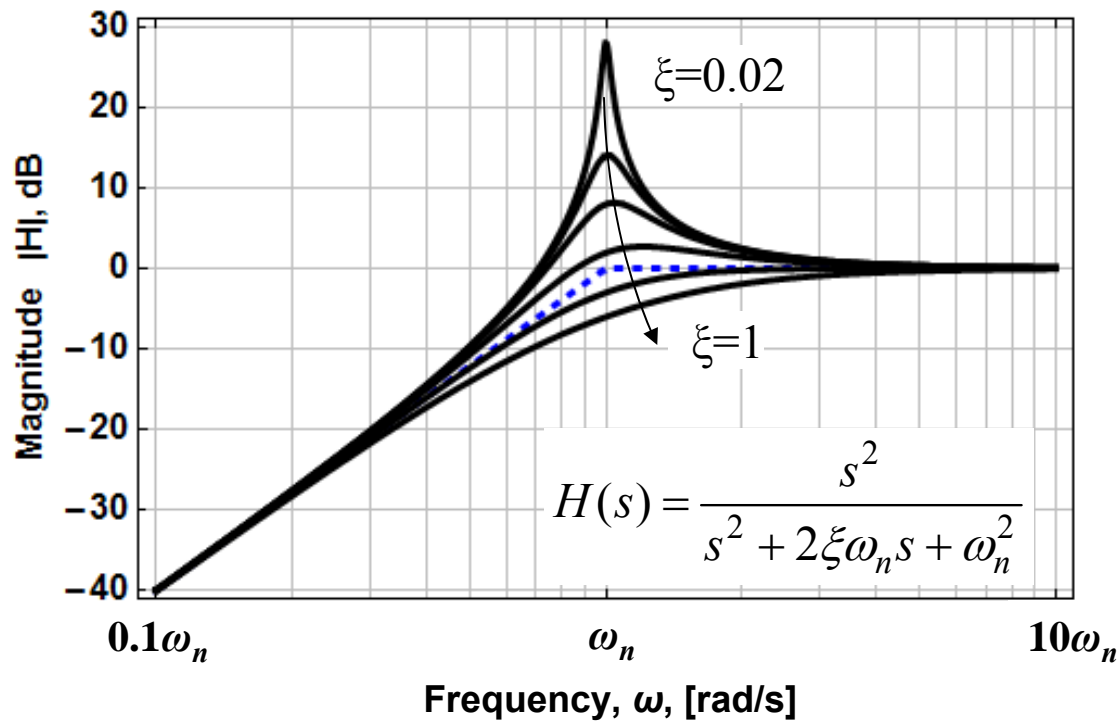


$$|H(j\omega_n)| = 2\xi$$

$$\omega_p = \omega_n \sqrt{1 - 2\xi^2}$$

$$|H(j\omega_p)| = 2\xi \sqrt{1 - \xi^2}$$

General 2nd-order High-Pass



$$|H(j\omega_n)| = \frac{1}{2\xi}$$

$$\omega_p = \frac{\omega_n}{\sqrt{1-2\xi^2}}$$

$$|H(j\omega_p)| = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

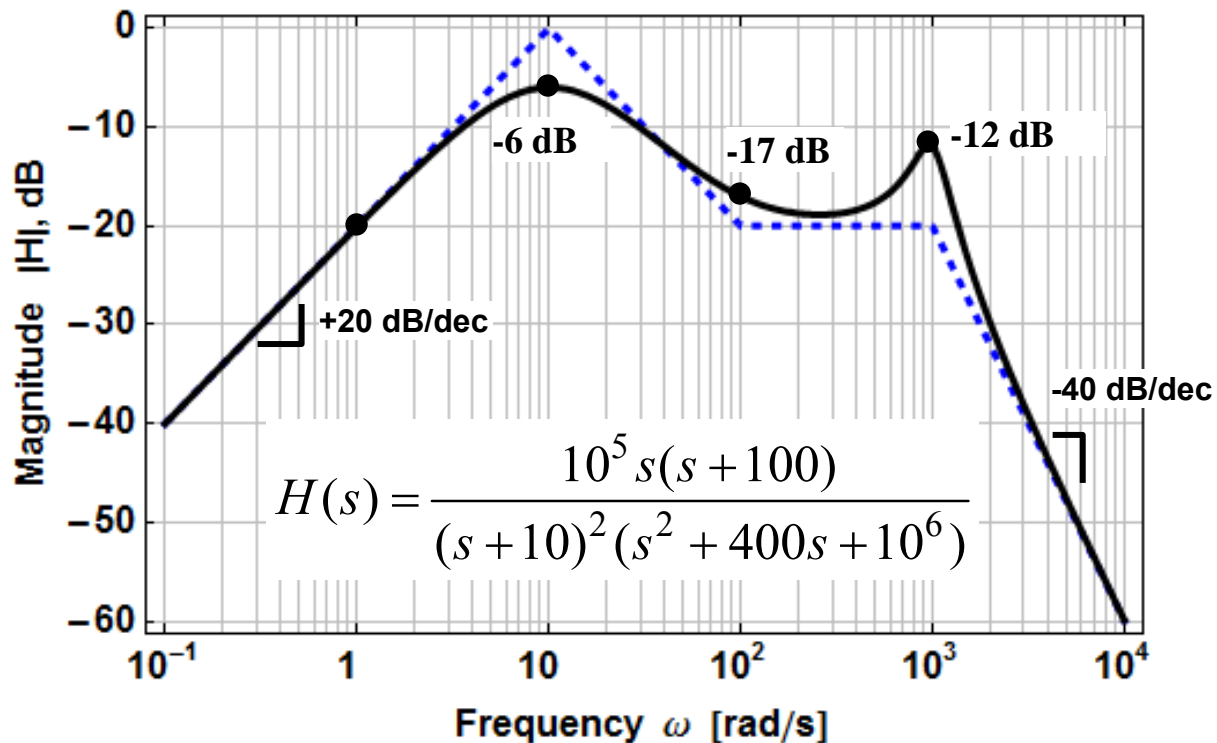
Integrating Example

$$H(s) = \frac{10^5 s(s+100)}{(s+10)^2 (s^2 + 400s + 10^6)}$$

$$\omega_n^2 = 10^6 \Rightarrow \omega_n = 10^3$$

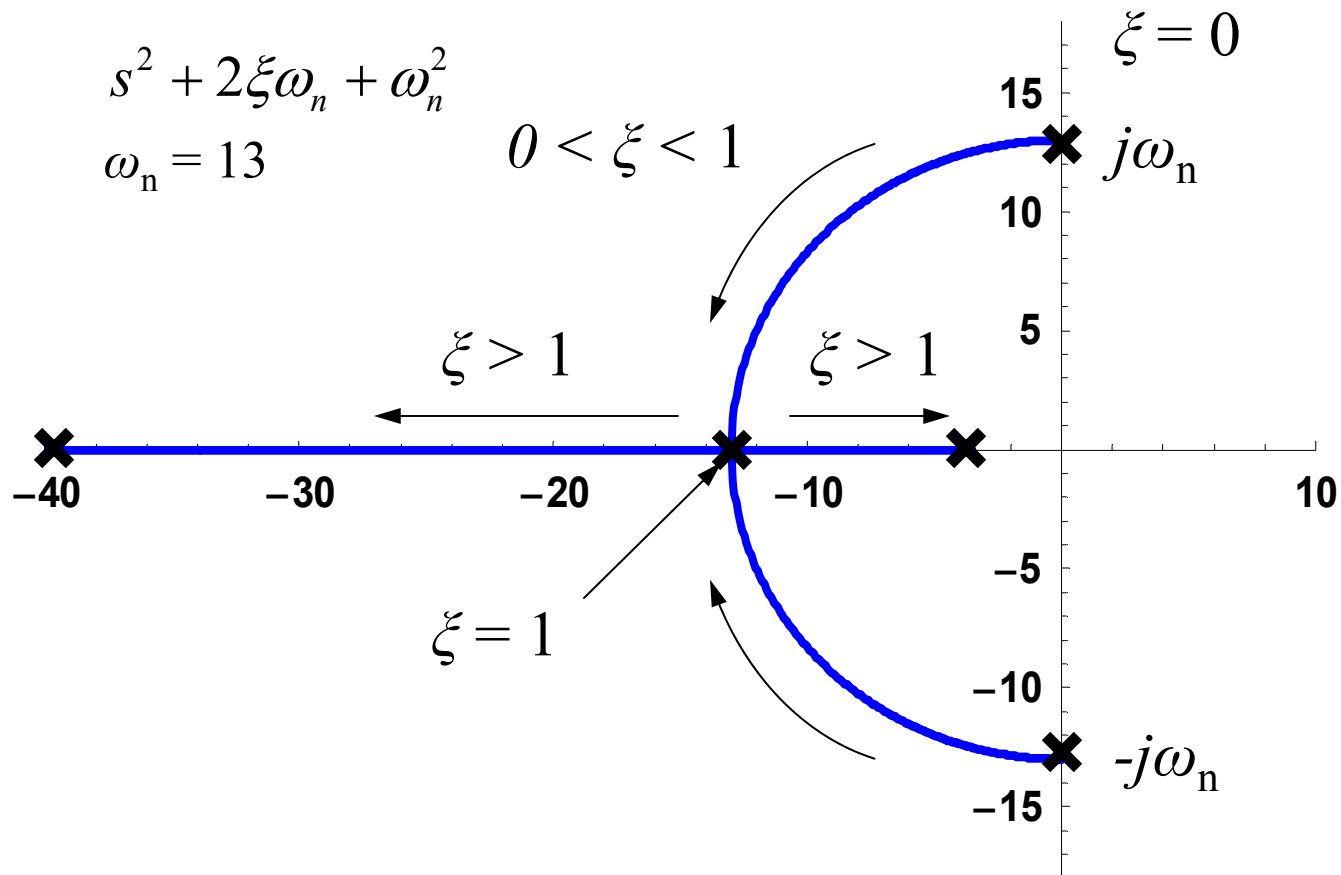
$$2\xi\omega_n = 400 \Rightarrow \xi = 0.2$$

Simple zeros, repeated pole, complex pole pair

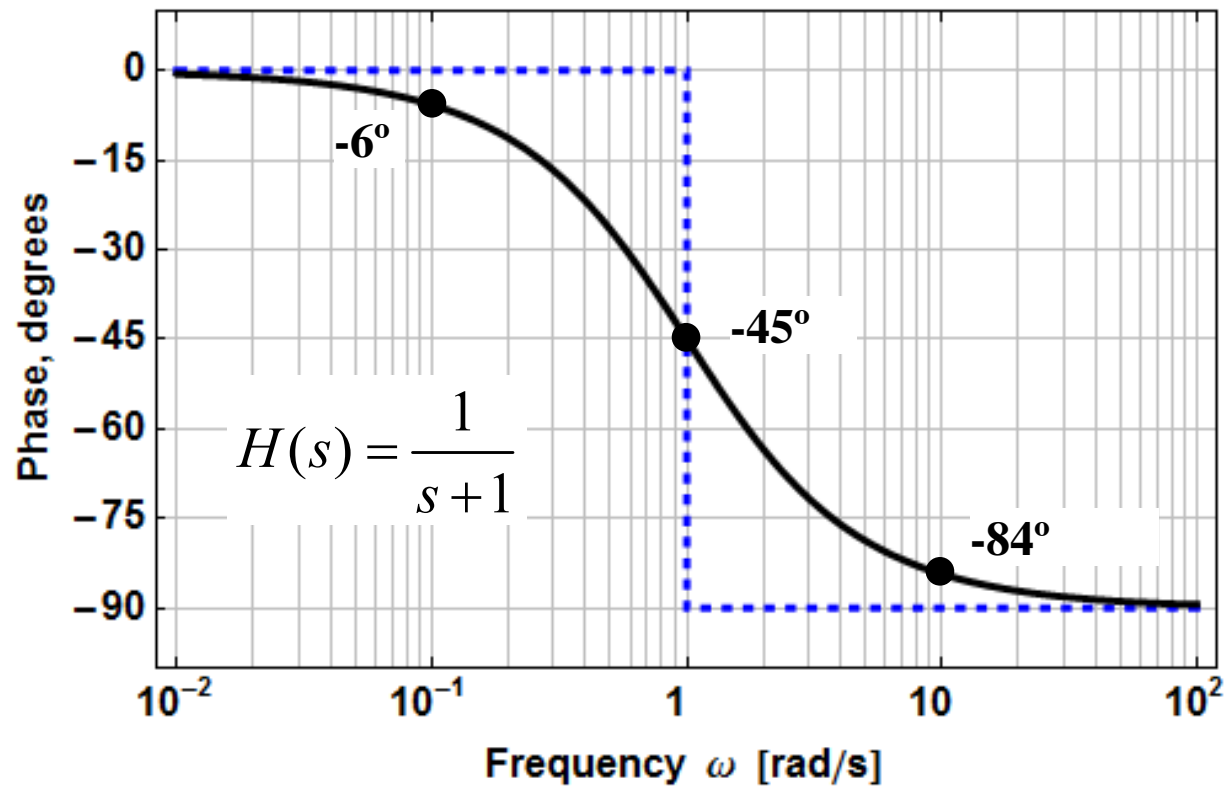


Root Locus Plot

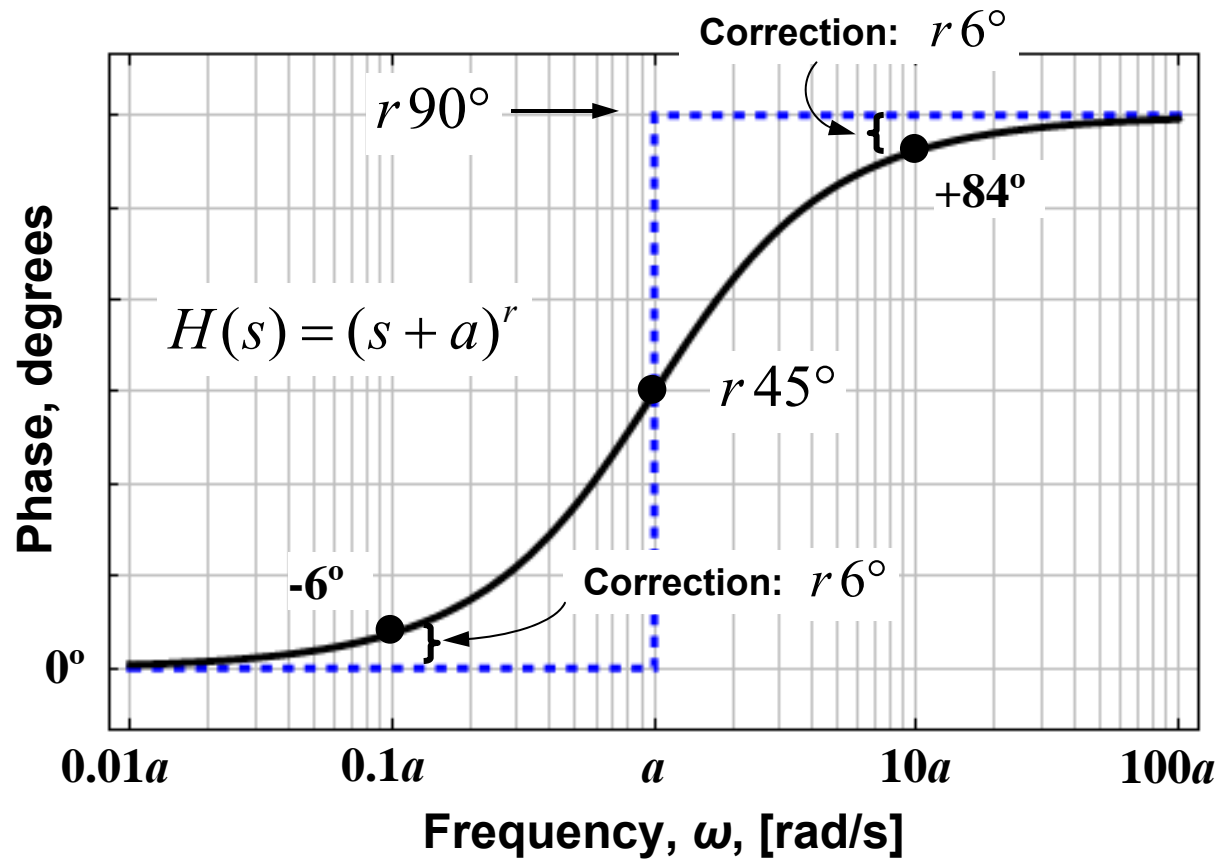
Second-order polynomial



Simple Pole – Phase

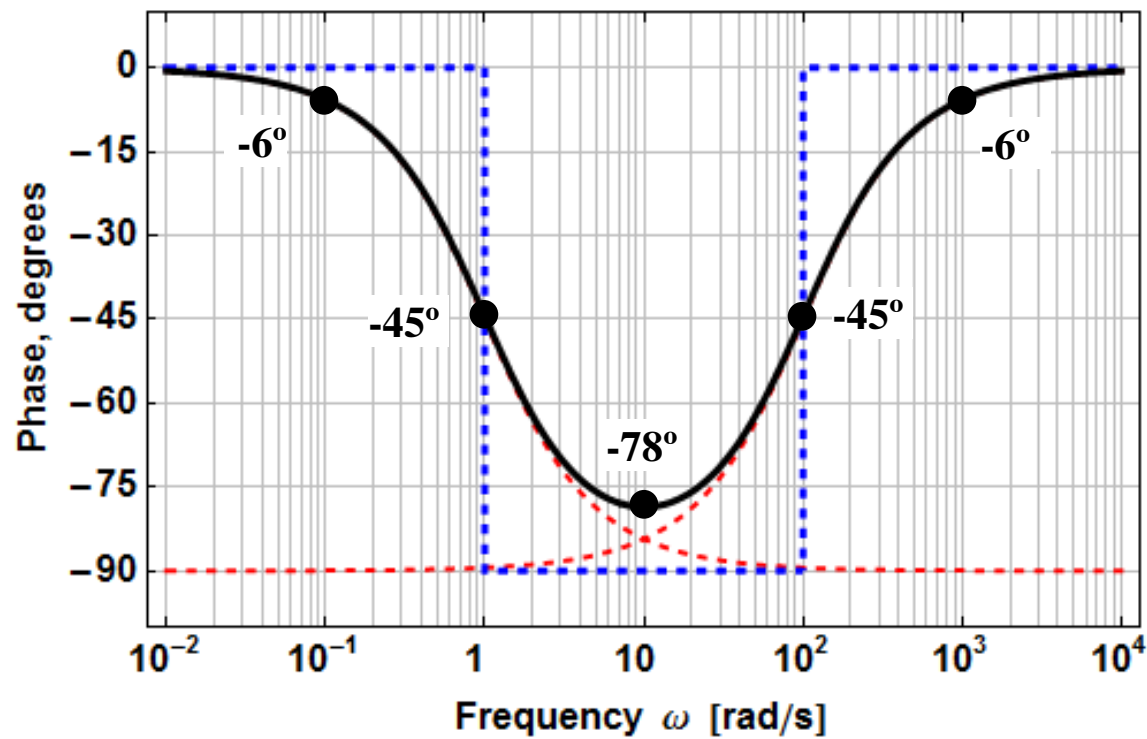


Repeated Zero - Phase



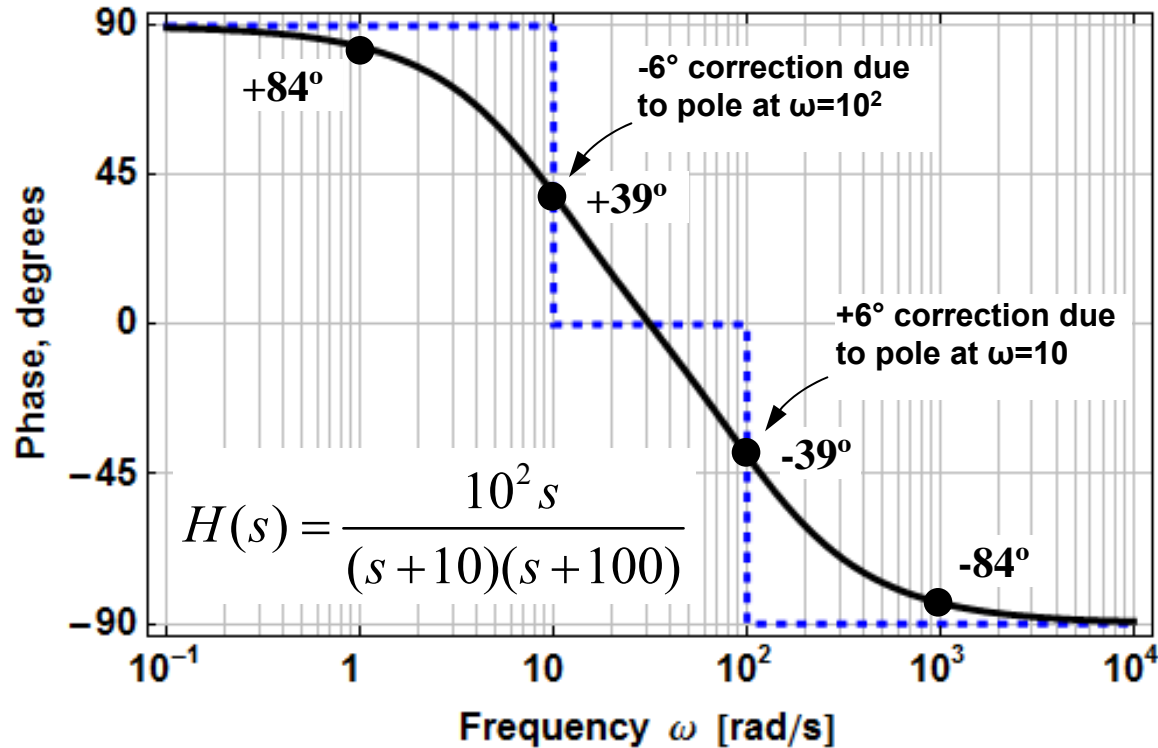
Example with Pole and Zero

$$H(s) = \frac{10(s + 100)}{s + 1}$$



Corrections to Mid-Point of Jumps

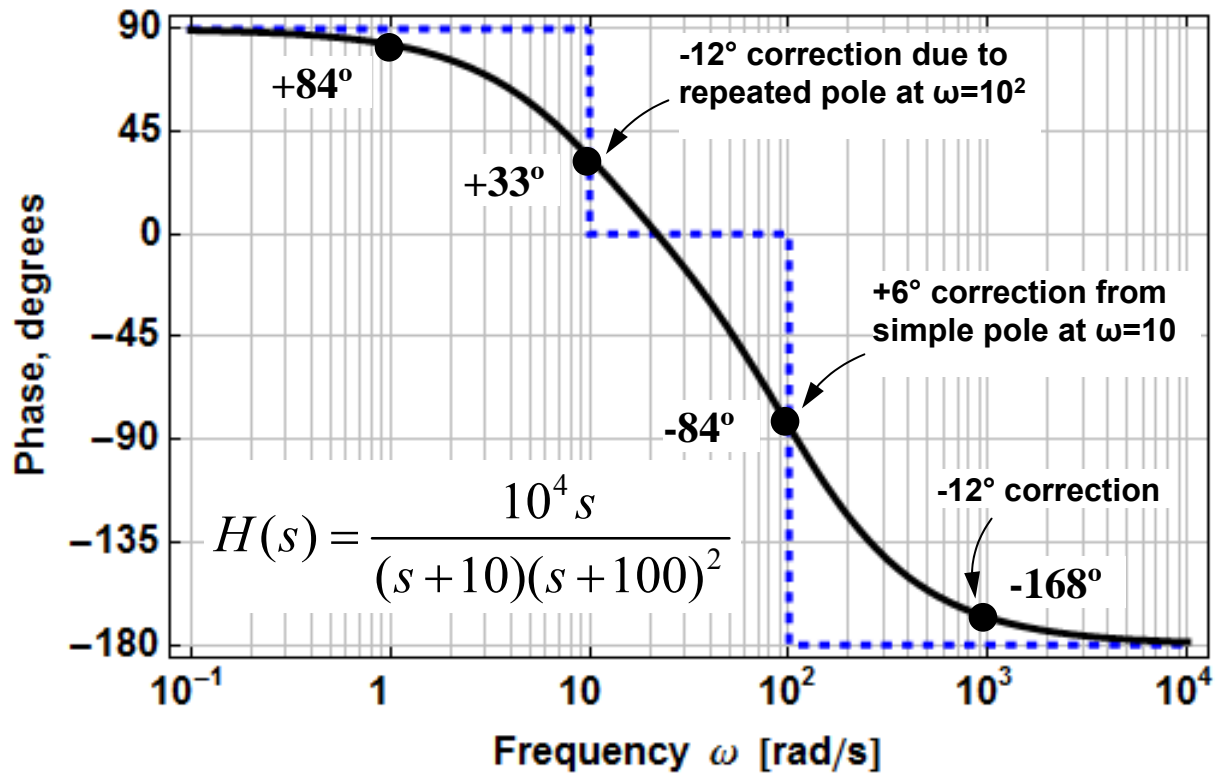
$$H(s) = \frac{10^2 s}{(s+10)(s+100)}$$



Corrections in the Presence of Repeated Root

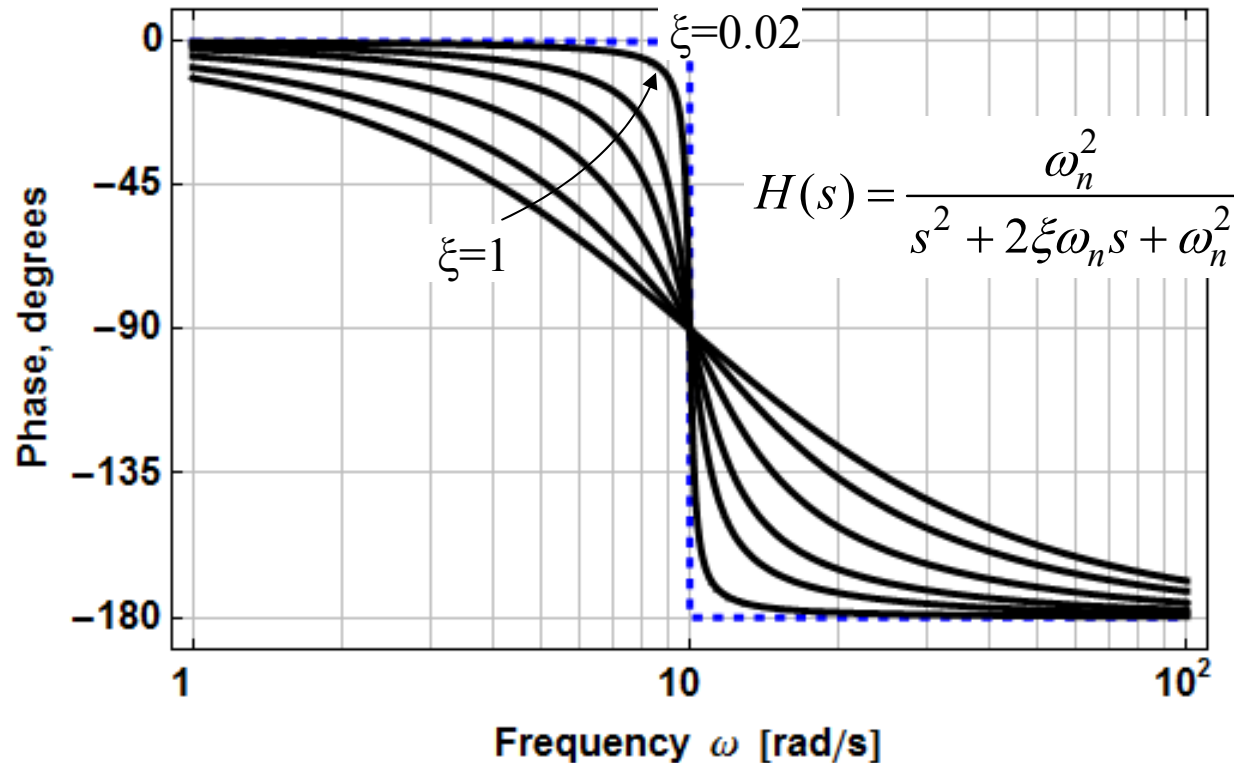


$$H(s) = \frac{10^4 s}{(s+10)(s+100)^2}$$



General 2nd-order Low-Pass

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{2\xi\omega_n\omega}{\omega_n^2 - \omega^2}\right)$$



Integrating Example

