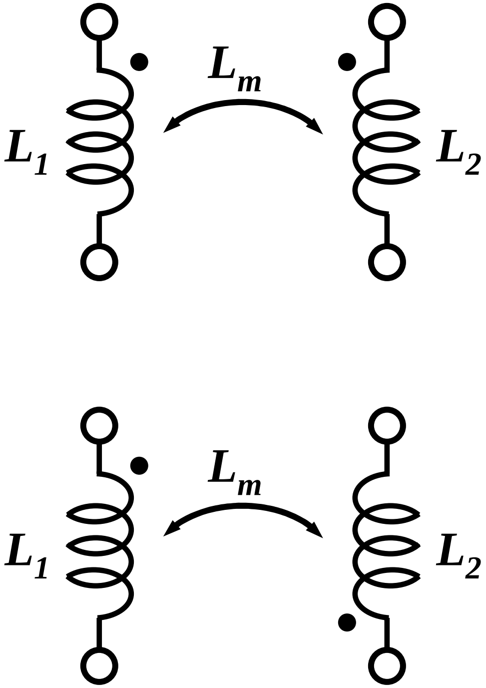
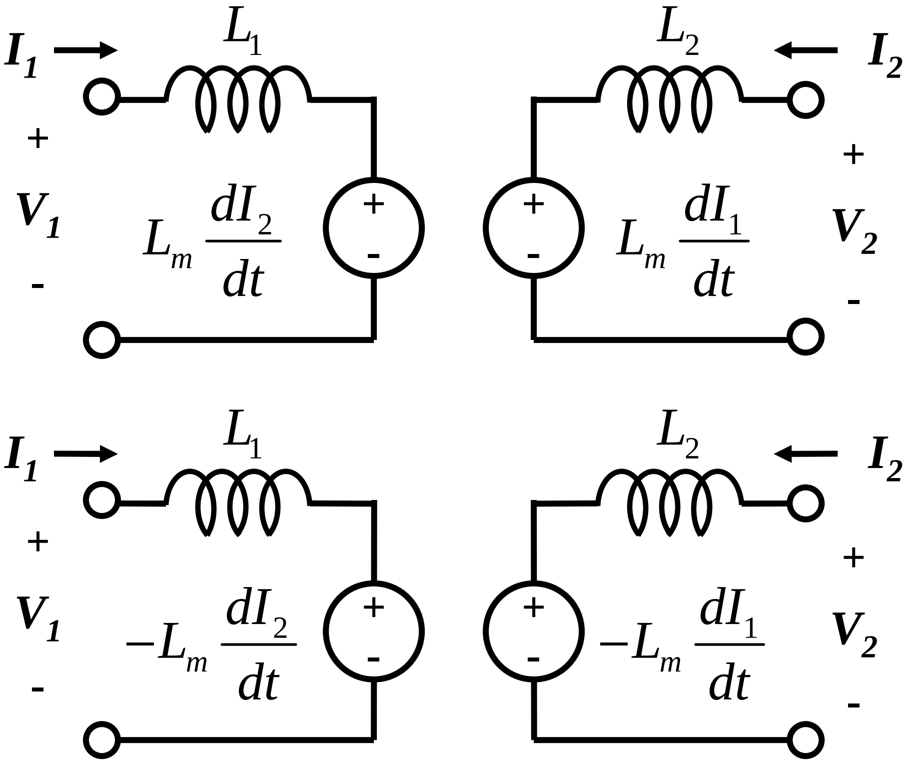
Mutual Inductance





$$V_1 = L_1 \frac{dI_1}{dt} \pm L_m \frac{dI_2}{dt}$$

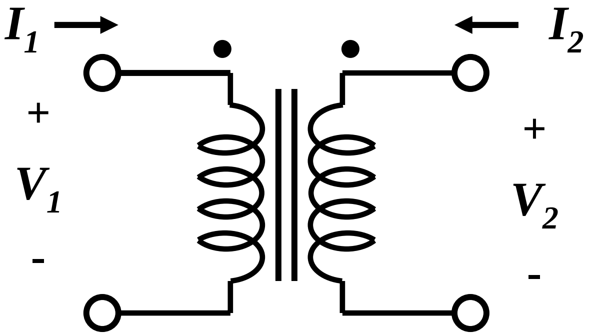
$$V_2 = \pm L_m \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}$$

- **Correct sign for mutual inductance found from Lenz' law and dot convention**
- **Dot convention: current flowing *into* one dot will induce current flow *out* of second dot**

Transformers

A transformer is just a special case where the mutual inductance is made as large as possible by allowing both coils to share the same flux

This is usually achieved by winding them both on a common core of high permeability material (soft iron or ferrite materials)



$$V_1 = j\omega L_1 I_1 + j\omega L_m I_2$$
$$V_2 = j\omega L_m I_1 + j\omega L_2 I_2$$

When there is no flux leakage, the mutual inductance is related to the primary and secondary inductances as

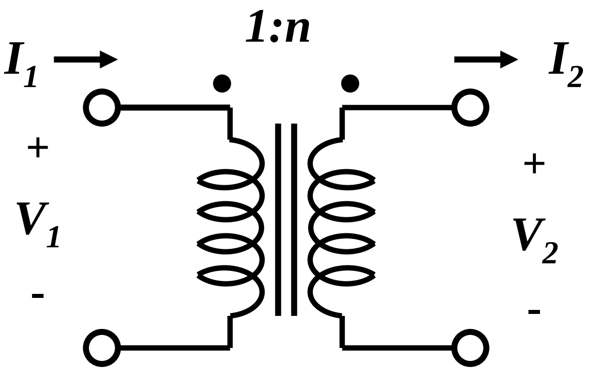
$$L_m = \sqrt{L_1 L_2}$$

For real transformers this can never be quite achieved, so we write

$$L_m = k\sqrt{L_1 L_2} \quad \text{where} \quad 0 < k < 1$$

coefficient of coupling

Ideal Transformer



If both coils share the same flux, then Faraday's law gives:

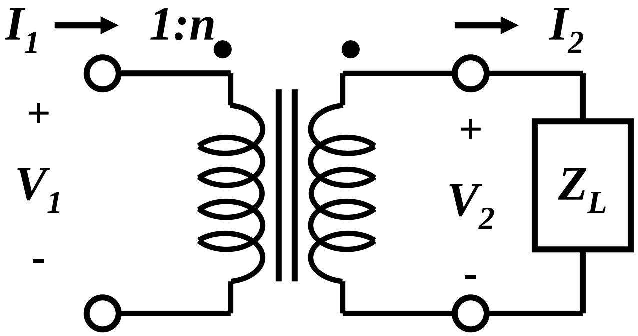
$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1}{n}$$

As the permeability of the core increases, the relationship between the primary and secondary currents approaches a limiting value set by the turns ratio:

$$\frac{I_1}{I_2} \Rightarrow \frac{N_2}{N_1} = n$$

These two relationships define an ***ideal transformer***. This is a fictitous element (note that μ→∞ implies infinite inductancesm so the impedance matrix is infinite) but a real transformer approaches this behavior.

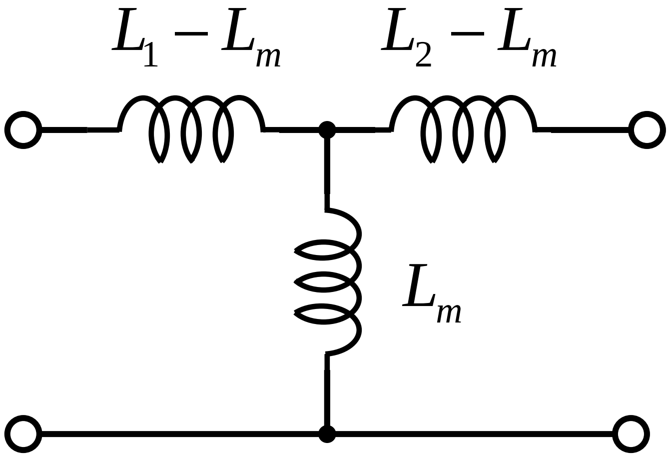
An idea transformer has the following property when one winding is terminated:



$$Z_{in} = \frac{V_1}{I_1} = \frac{(N_1/N_2)V_2}{(N_2/N_1)I_2} = \frac{Z_L}{n^2}$$

Transformer Equivalent Circuit

Using the tee-equivalent for reciprocal networks, we find the followign equivalent circuit for mutual inductances or transformers



$$L_1 \propto N_1$$

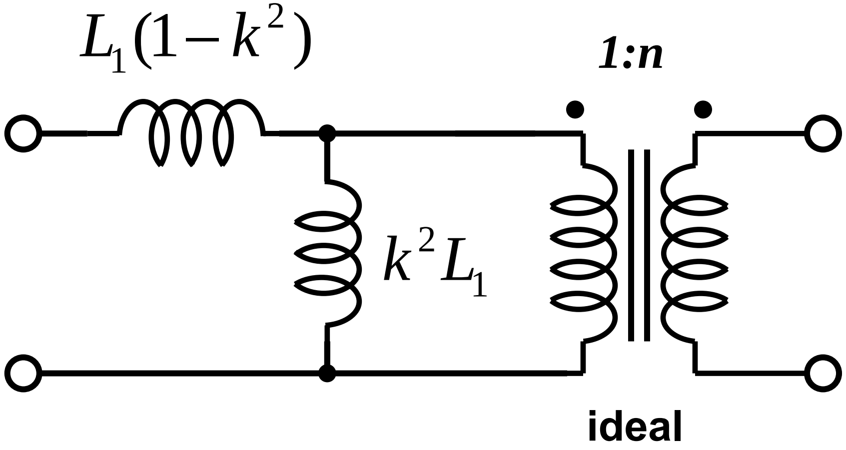
$$L_2 \propto N_2 = n^2 L_1$$

$$L_m = k\sqrt{L_1 L_2} = knL_1$$

This can be cascaded with an ideal 1:1 transformer to simulate the fact that a real transformer has electrically isolated ports

Alternative equivalent circuit

The following is also an identical equivalent that uses an ideal transformer to explicitly incorporate the turns ratio and isolation between ports



$$n = \frac{L_2}{L_m}$$

For good transformers, k is nearly 1, and this model shows why real transformers do not work at DC