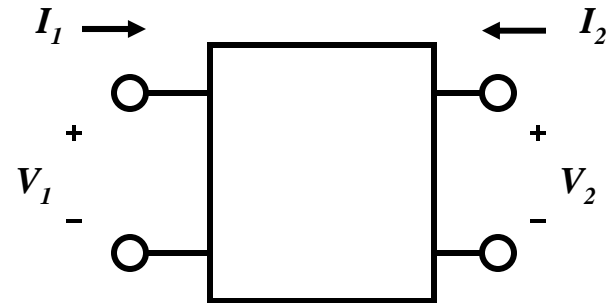


“Black-Box” Description



- No matter how complex the network is inside, the external behavior of the two-port can be completely characterized by the functional relation between V_1 , V_2 , I_1 , and I_2
- If we know this mathematical relationship we do not need to know what is inside
- Simpler equivalent circuits can be constructed to give the same external behavior. This is a generalization of Thevenin & Norton equivalence to multiport networks

Two-Port: Z-parameters



“Open-Circuit” parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

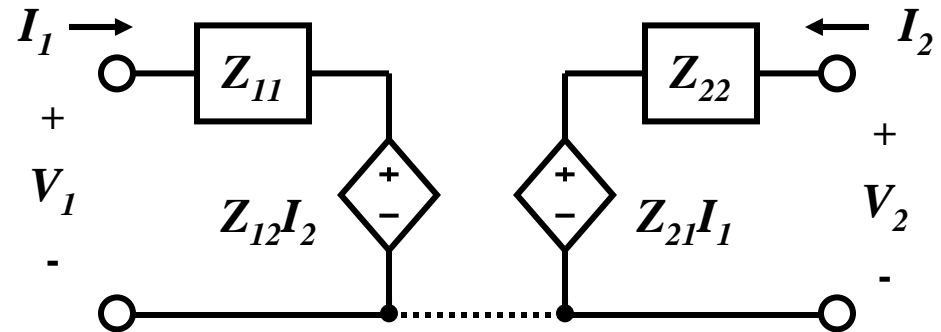
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Reciprocal networks: $Z_{12} = Z_{21}$

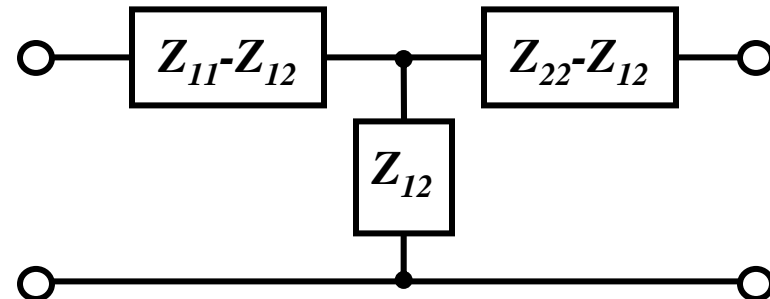
Symmetrical networks: $Z_{11} = Z_{22}$

Lossless networks: $\text{Re}\{Z_{mn}\} = 0$

“Thevenin” equivalent using controlled sources



Equivalent circuit for reciprocal networks



Two-Port: Y-parameters

“Short-Circuit” parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

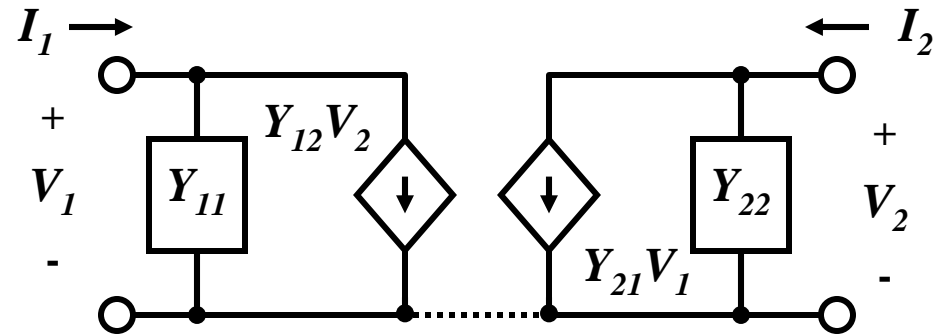
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Reciprocal networks: $Y_{12} = Y_{21}$

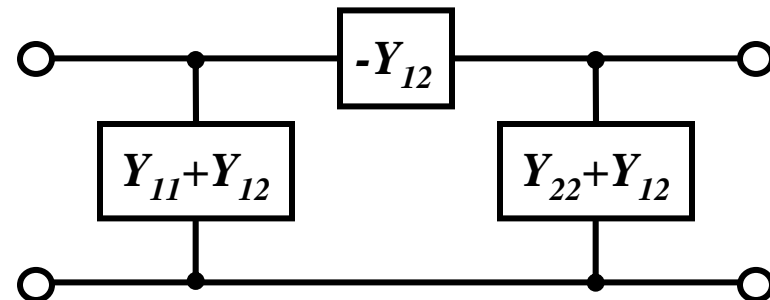
Symmetrical networks: $Y_{11} = Y_{22}$

Lossless networks: $\text{Re}\{Y_{mn}\} = 0$

Norton equivalent circuit



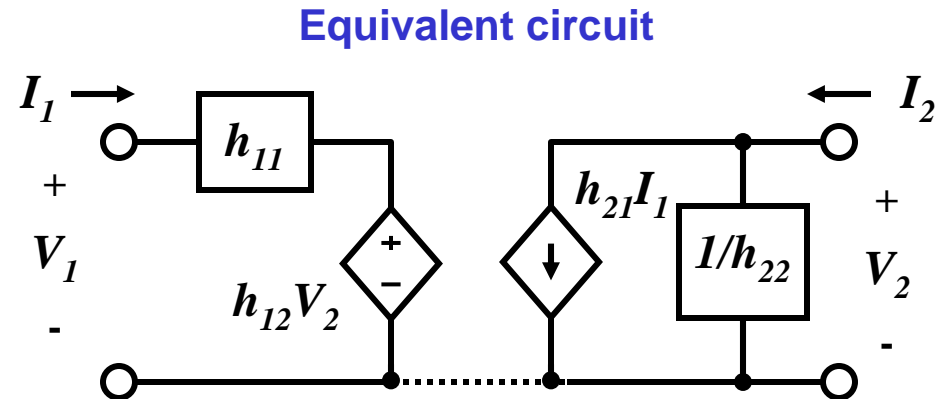
equivalent circuit for reciprocal networks



“Hybrid” parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$



Note that elements of the h matrix have different units!

h_{11} = input impedance [Ω]

h_{21} = "forward current gain" [unitless]

h_{12} = "reverse voltage gain" [unitless]

h_{22} = output admittance [S]

Conversions between parameters



| | <i>Z</i> | <i>Y</i> | <i>h</i> | <i>ABCD</i> |
|-------------|--|--|---|---|
| <i>Z</i> | $\begin{array}{cc} z_{11} & z_{12} \\ z_{21} & z_{22} \end{array}$ | $\begin{array}{cc} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{array}$ | $\begin{array}{cc} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{array}$ | $\begin{array}{cc} \frac{A}{C} & \frac{AD-BC}{C} \\ \frac{1}{C} & \frac{D}{C} \end{array}$ |
| <i>Y</i> | $\begin{array}{cc} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{array}$ | $\begin{array}{cc} y_{11} & y_{12} \\ y_{21} & y_{22} \end{array}$ | $\begin{array}{cc} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{array}$ | $\begin{array}{cc} \frac{D}{B} & \frac{BC-AD}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{array}$ |
| <i>h</i> | $\begin{array}{cc} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{array}$ | $\begin{array}{cc} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{array}$ | $\begin{array}{cc} h_{11} & h_{12} \\ h_{21} & h_{22} \end{array}$ | $\begin{array}{cc} \frac{B}{D} & \frac{AD-BC}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{array}$ |
| <i>ABCD</i> | $\begin{array}{cc} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{array}$ | $\begin{array}{cc} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{array}$ | $\begin{array}{cc} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{array}$ | $\begin{array}{cc} A & B \\ C & D \end{array}$ |

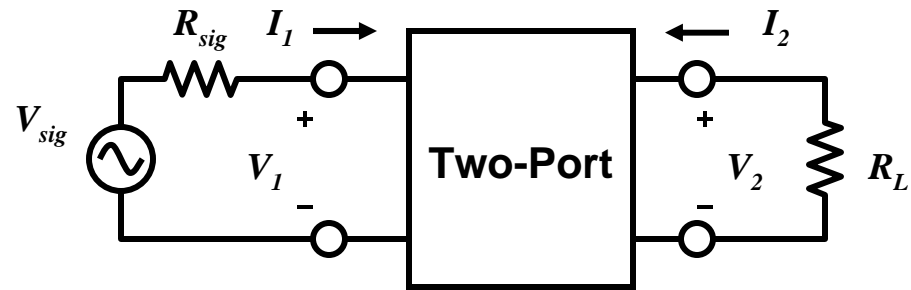
$\Delta z = z_{11}z_{22} - z_{12}z_{21}$
 $\Delta y = y_{11}y_{22} - y_{12}y_{21}$
 $\Delta h = h_{11}h_{22} - h_{12}h_{21}$

Example Using Two-Port Parameters

Start with the Two-Port Description:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$



Add source/load constraints:

$$V_1 = V_{sig} - I_1 R_{sig}$$

$$V_2 = -I_2 R_L = V_{out}$$

$$\curvearrowright I_1 = \frac{V_{sig} - V_1}{R_{sig}}$$

$$\curvearrowright I_2 = \frac{-V_{out}}{R_L}$$

Eliminating I_1 and I_2 gives:

$$\left(1 + \frac{R_{sig}}{Z_{11}}\right)V_1 + \left(\frac{Z_{12}}{R_L} \frac{R_{sig}}{Z_{11}}\right)V_{out} = V_{sig}$$

$$V_1 + \frac{R_{sig}}{Z_{21}} \left(1 + \frac{Z_{22}}{R_L}\right)V_{out} = V_{sig}$$

Then solving for the overall voltage gain gives

$$\frac{V_{out}}{V_{sig}} = \frac{R_L Z_{21}}{(R_{sig} + Z_{11})(R_L + Z_{22}) - Z_{21} Z_{12}}$$