1
Unipolar Space Charge Limited Transport

THE transport in materials where the electrons in the medium contribute to voltage drop in the medium is generally termed as Space Charge Limited Transport (SCL Transport). Introduction to this chapter...

1.1 Dielectric Relaxation

Prior to our discussion of SCL transport, it it important that the reader have a firm understanding of the concept of *dielectric relaxation*. It is well known from basic electromagnetic theory that a conducting material cannot support any free charge that is not compensated by an equal and opposite charge. Therefore, if some free charge is injected into the material, this charge must be neutralized by mobile carriers either from the material boundary or from a charge resevoir. This process, known as dielectric relaxation, occurs within a finite amount of time τ_r, known as the dielectric relaxation time. Since dielectric relaxation is achieved by means of conduction processes, τ_r is small in metals but can be large in lightly doped semiconductors and in insulators.

The magnitude of τ_r can be derived as follows. Consider a homogenous, unipolar conductor of conductivity σ and permitivity ϵ. Assume that initially, some mobile charge is introduced into the material such that at time $t = 0$, the distribution of free charge is $\rho_{free}(\overrightarrow{r}; t = 0)$. Using the following equations from electromagnetic theory,

$$\nabla \cdot \overrightarrow{D} = \rho_{free} \tag{1.1.1}$$

$$\overrightarrow{D} = \epsilon \overrightarrow{E} \tag{1.1.2}$$

$$\overrightarrow{J} = \sigma \overrightarrow{E} \tag{1.1.3}$$

$$\nabla \cdot \overrightarrow{J} = \frac{-d\rho_{free}}{dt} \tag{1.1.4}$$

the time evolution of the mobile charge in the material can be shown to be:

$$\rho_{free}(\overrightarrow{r}; t) = \rho_{free}(\overrightarrow{r}; t = 0) \exp\left[\frac{-t}{(\epsilon/\sigma)}\right] \tag{1.1.5}$$

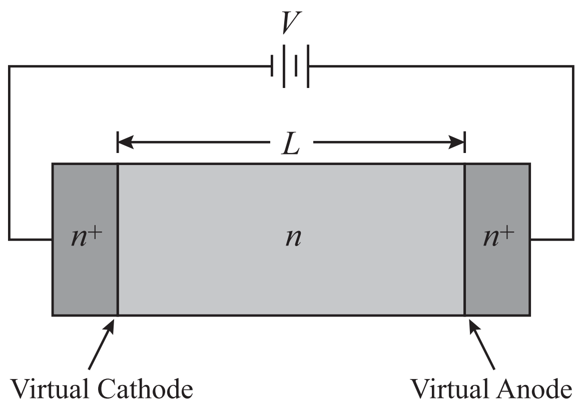

Figure 1.1: Representation of a conductor as an n^+-n-n^+ structure. The cathode and the anode are considerred to be at the n^+-n junctions, since the n^+ regions are considered to be short circuits.

This result shows that uncompensated charge in a uniform conducting material is neutralized within a time $\tau_r = \epsilon/\sigma$.

An interesting result of this phenomena is that when a current J is passed through a uniform conducting material, charge is injected into the material such that the areal charge density in the material increases by an amount $Q_{inj} = \tau_r J$. The injected charge will in general not be uniformly distributed within the material. The distribution will be one for which the Current Continuity Equation and Poisson's Equation can be solved simultaneously and self–consistently. The increase in charge within the material affects the electric field distribution and thereby affects the voltage drop in the medium. The following section will focus on the impact that this injected charge has on the transport properties of the material.

1.2 Current flow in a doped semiconductor

It is useful at this time to reconsider current flow in a conductor in the frame of a charge control analysis. Figure 1.1 shows such a conductor as an n^+-n-n^+ system where the n semiconductor has length L and is doped with N_d donors. This results in an equilibrium electron density $n_0 \simeq N_d$ in the conduction band and an equal number of positively charged ionized donors $N_{d0}^+ = n_0 \simeq N_d$. Our assumption of full donor ionization implies that the donor is shallow and that the Fermi level lies below the donor level. In this system, current is supplied by electrons injected at the cathode and collected at the anode. In our analysis, we assume the cathode to be a resevoir contact, or in other words a contact that is able to supply whatever current is necessary for the solution to the problem as determined in the bulk. We also assume that all currents are

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pure drift currents, and any diffusive component can be neglected. This second assumption is consistent with our neglect of the role of the contacts, since diffusion currents are appreciable only in the area directly adjacent to the contacts. Finally, we assume that all electrons injected into the bar are injected into the conduction band, i.e. none of the injected electrons fill empty donor states. Indeed, injection of charge into the system will cause the electron quasi-fermi level (QFL) to rise above its equilibrium value. However, it can be easily verified using Fermi-Dirac statistics that in the situation which is considerred here, the charge injected into empty donor states is much less than that injected into the conduction band. The system described here is commonly referred to as the trap–free limit.

Any voltage V maintained between the cathode and the anode is accompanied by an electric field within the bar, thus inducing a current J. As stated earlier, this current will result in an increase in the charge concentration within the bar, causing the sample to deviate from space charge neutrality. If we neglect the effects of the injected charge, which is a valid approximation at low current levels, then the electric field within the bar is uniform everywhere with a value $E = V/L$. The current density is proportional to V and is given by $J = qnv = qn_0\mu V/L$, which is a well-known representation of Ohm's Law. As depicted in Figure 1.2(a), the electric field is supported by a charge density Q_a at the anode and $-Q_a$ at the cathode, where $Q_a = \epsilon V/L$. Using the above expression for J, Q_a can be written as $Q_a = \epsilon J/qn_0\mu = J(\epsilon/\sigma) = \tau_r J$. This leads to the very interesting result that the charge injected into the bulk, which we will refer to as Q_{inj}, is equal in magnitude to the charge Q_a induced at the anode. In other words, the entire negative charge that we assumed was located at the cathode is in fact distributed within the bar and has to be resupplied by the contacts every τ_r seconds. Therefore, any current flow in a conductor causes the system to be no longer space charge neutral. As long as the injected charge is much less than the background charge, $Q_{inj} << qn_0L$, the system may at best be considered to be quasi-neutral.

Since the entire charge at the anode is imaged in the bulk, the electric field at the cathode $E(0) = 0$, and the electric field must increase monotonically from the cathode $(x = 0)$ to the anode $(x = L)$. Current continuity thus requires that the injected charge density $n_{inj}(x)$ monotonically increase in magnitude from the anode $(x = L)$ to the cathode $(x = 0)$. In most moderately doped systems and most definitely in metals, the relaxation time is so small $(< 10^{-13}\ \mathrm{secs})$ that the injected charge need be very small to still enable a substantial current flow as determined by the ratio, Q_{inj}/τ_r. In the case of low-level injection, such that $Q_{inj} < qn_0L$, most of the injected charge will be located right near the cathode, and so the transport properties only deviate slighlty from ideal Ohmic behavior. The charge distribution and electric field profile for this case are illustrated in Figure 1.2(b).

An interested reader may wonder how the physical transit time of electrons, τ_t, is related to the current flow. Again going back to Ohm's Law, $J = \sigma E$, we can rewrite the equation as $J = qn_0\mu E$ (neglecting Q_{inj}) or $J = qn_0v$. Setting $v = L/\tau_t$ and $Q_B = qn_0L$ leads to the reassuring result that $J = Q_B/\tau_t$, which tells us that all the electrons in the sample are replaced by the process of current flow every τ_t seconds. Note that the transit time does not determine the time response of the resistor, since the time response of current is via relaxation of electrons and not by their transit.

Now consider the situation in which the current is increased to the point where the injected

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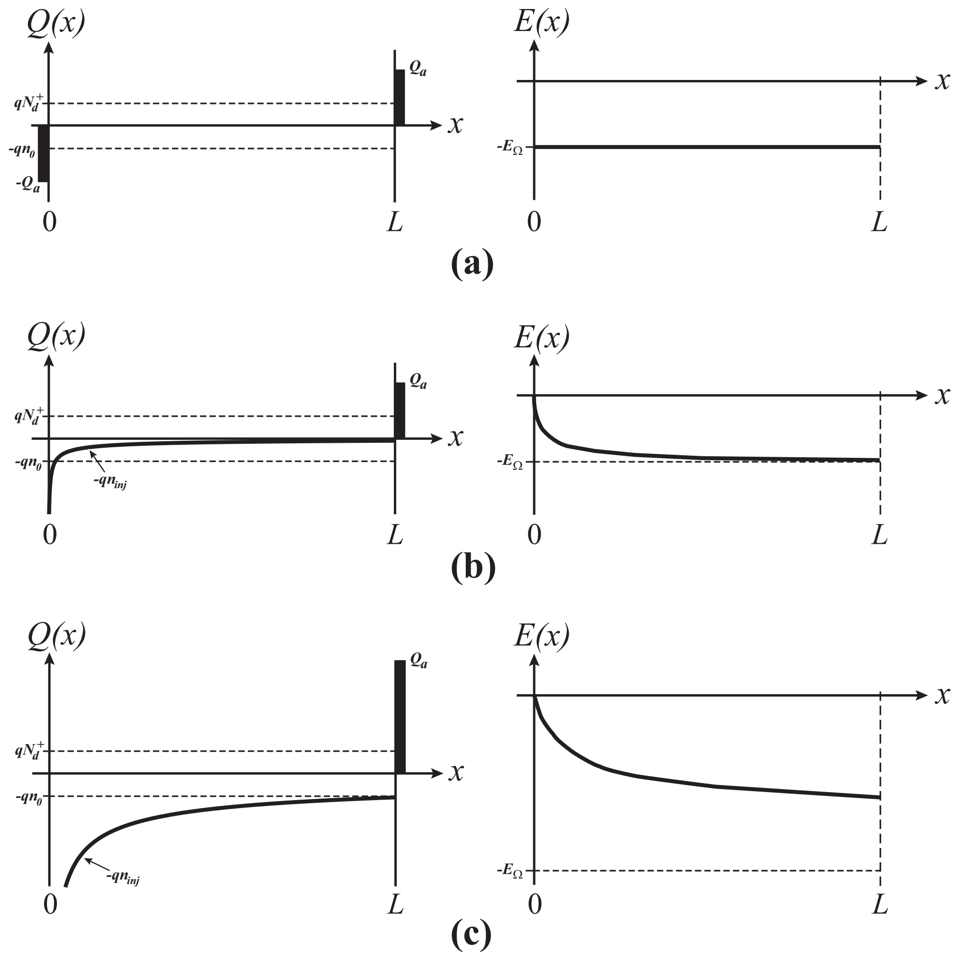

Figure 1.2: Charge distribution and electric field profile of the bar from Figure 1.1 with current J passing through. (a) Ohmic approximation in which Q_{inj} is neglected completely. (b) Low-level injection, $Q_{inj} << qn_0L$, transport characteristics only deviate slightly from Ohm's Law. (c) Current is large, $Q_{inj} >> qn_0L$, transport properties are characterized by Mott-Guerney Law. In each case, the applied voltage is the area underneath the electric field curve, and $E_\Omega = J/qn_0\mu$ is the electric field predicted by Ohm's Law.

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charge density is larger than the thermal charge density in the semiconductor, as illustrated in Figure 1.2(c). Clearly, our assumption of space charge quasi-neutrality is no longer valid. It is now that the nature of transport in the material ceases to be ohmic, and the current voltage relationship is dominated by the space charge in the material. The defining relationship changes from Ohm's Law to the Mott-Guerney Law, which is derived below. The applied voltage at which this occurs, called the cross-over voltage V_x, is approximated as the voltage at which the injected charge is equal to the bulk charge, or $Q_{inj} = qn_0L$. Combining this expression with $Q_{inj} \simeq \epsilon E \simeq \epsilon(V_x/L)$, we find

$$V_x = \frac{qn_0L^2}{\epsilon} \tag{1.2.1}$$

Once the injected charge density is larger than the thermal charge density in the conductor, the current continuity (Equation 1.2.2) and Poissons (Equation 1.2.3) equations shown below have to be simultaneously solved to obtain the current-voltage characteristics of the material.

$$J = qn(x)v(x) \simeq qn_{inj}(x)v(x) \tag{1.2.2}$$

$$\frac{\partial^2 V(x)}{\partial x^2} = \frac{qn_{inj}(x)}{\epsilon} \tag{1.2.3}$$

In Equation 1.2.2, we have assumed that the injected electron density is much greater than the donor density $[n(x) >> N_d$ (or n_0)$]$. These two equations can be combined and rewritten as

$$\frac{\partial^2 V(x)}{\partial x^2} = \frac{J/v(x)}{\epsilon} \tag{1.2.4}$$

The problem here can be readily solved in two limiting cases of the electron velocity versus electric field behavior. In the case that the velocity increases monotonically with electric field or $v = -\mu E$, realizing that $-\partial V(x)/\partial x = E(x)$, and applying the boundary condition $E(0) = 0$, the solution of Equation 1.2.4 is

$$J = (9/8)\epsilon\mu(V^2/L^3) \tag{1.2.5}$$

From the above equations, it is also possible to find expressions for the injected charge density as well as the electric field distribution at a current J:

$$n_{inj}(x) = \left[\frac{\epsilon J}{2q^2\mu_n}\right]^{1/2} x^{-1/2} \tag{1.2.6}$$

$$E(x) = -\left[\frac{2J}{\epsilon\mu_n}\right]^{1/2} x^{1/2} \tag{1.2.7}$$

Additionally, if we ignore the (9/8) prefactor in Equation 1.2.5, it can be shown that the crossover voltage V_x in Equation 1.2.1 corresponds to the point at which Equation 1.2.5 and Ohm's Law intersect.

Equation 1.2.5 is referred to as the Mott–Gurney Law, Child's Law for Solids, or the trap–free square law. In essence, current in a trap–free solid varies as the square of the voltage and

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inversely as the cube of the distance when the velocity-field relationship of the carrier is linear. It should be noted that the maximum electric field in the bar (i.e. the electric field at $x = L$) at the crossover current will depend on the length of the bar, as seen in the equation below.

$$E(J = J_x, x = L) \sim \frac{qn_0L}{\epsilon} \tag{1.2.8}$$

If the electric field in the bar exceeds the saturation field E_{sat}, then the electron velocity saturates. Thus, in order to observe the transition from Ohm's Law to the Mott–Gurney Law, the length of the bar $L < \epsilon E_{sat}/qn_0$. As an example, for silicon doped with 10^{16} donors, in order for the transition from Ohm's Law to the Mott–Gurney Law to occur prior to saturation, the length of the bar $L \leq 600\mathring{A}$. The case when the electron velocity saturates will be dealt with later.

Figure 1.3 displays what we have learned to now. First the current flows obeying Ohm's Law. Once the applied voltage reaches a value V_x, the cross–over voltage, such that the injected charge is larger than the thermal charge, then space charge current flows in accordance with the Mott–Gurney Law. Another interesting physical reason for this cross–over becomes apparent when we look at the physical transit time, τ_t, of carriers across the length L of the conductor. Under the assumptions of constant mobility and uniform electric field, we have $\tau_t = L/v$ or $\tau_t = L/\mu E$. At the cross–over point, the electric field E is approximately given by $E = V_x/L = qn_0L/\epsilon$, which results in $\tau_t = \epsilon/qn_0\mu$, or $\tau_t = \epsilon/\sigma = \tau_r$ the ohmic relaxation time. This states that at the voltage at which the transit time of the carriers becomes shorter than the relaxation time, the injected carriers cannot be relaxed by the thermal carriers before they exit the material. Hence, ohmic conduction will no longer be valid, and space charge limited transport commences.

Calculating an exact solution to the problem involves simultaneously solving the Current Continuity Equation and Poisson's Equation expressed in the following form:

$$J = q[n_{inj}(x) + n_0]\mu_n E(x) \tag{1.2.9}$$

$$\frac{dE(x)}{dx} = \frac{-qn_{inj}(x)}{\epsilon} \tag{1.2.10}$$

Although it is impossible to derive an exact equation which explicitly relates the current J to the voltage V, an exact numerical solution can be obtained and is plotted in Figure 1.3. However, the solution does not provide significant physical insight into the problem and is thus left as an exercise for the reader (see Problem 1.1). Additional insight can be gained by solving the above equations using the Method of Regional Approximation. This method was first applied by Lampert and Mark [1] to the case under consideration here and is described as follows.

Consider the schematic plot of the injected charge distribution n_{inj} vs x illustrated in Figure 1.4. The total mobile charge $n(x)$, also plotted in Figure 1.4, can be expressed as $n(x) = n_{inj}(x) + n_0$. If the current density J is not too high, then there exists a point in the bar, labeled x_1 in Figure 1.4, at which $n_{inj}(x_1) = n_0$. The Regional Approximation assumes that to the left of x_1, the total charge $n(x) = n_{inj}(x) + n_0 \simeq n_{inj}(x)$, and to the right of x_1, $n(x) = n_{inj}(x) + n_0 \simeq n_0$. At very low currents, where the injected charge Q_{inj} is very small, x_1 is very close to the cathode $(x = 0)$, and the solution is very close to Ohm's Law. As J

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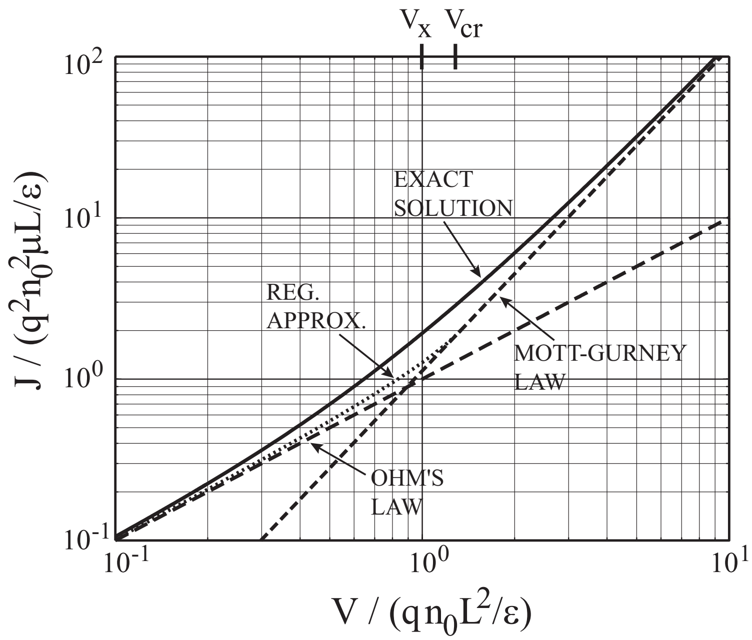

Figure 1.3: Normalized current–voltage characteristics in a trap–free solid.

increases, more charge is injected into the bar, and x_1 moves closer to the anode $(x = L)$. Eventually, a critical current J_{cr} is reached at which $x_1 = L$. For currents $J > J_{cr}$, the results of the Regional Approximation are identical to those of the Mott–Gurney Law, and the current–voltage relationship is characterized by SCL transport.

To solve the equations governing current flow, we split the bar into two regions. The region in which $0 \leq x \leq x_1$ is referred to as Region I, and the region in which $x_1 \leq x \leq L$ is referred to as Region II. The equations needed to be solved in each region are as follows. For Region I $(0 \leq x \leq x_1)$:

$$J = qn_{inj}(x)\mu_n E(x) \tag{1.2.11}$$

$$\frac{dE(x)}{dx} = \frac{-qn_{inj}(x)}{\epsilon} \tag{1.2.12}$$

subject to the boundary conditions that $E(0) = 0$ and $n_{inj}(x_1) = n_0$. For Region II $(x_1 \leq x \leq L)$:

$$J = qn_0 \mu E(x) \tag{1.2.13}$$

$$\frac{dE(x)}{dx} = 0 \tag{1.2.14}$$

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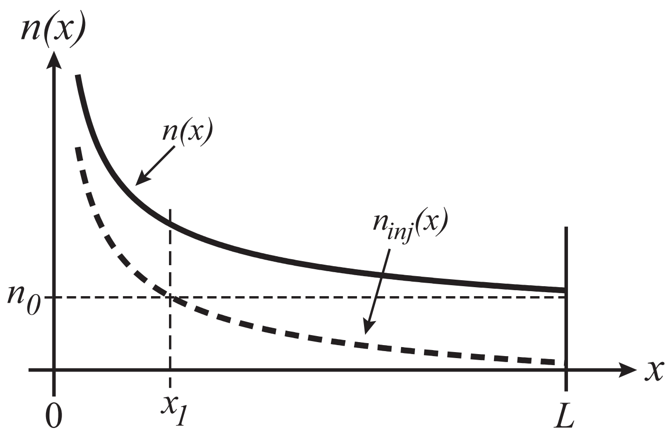

Figure 1.4: Schematic diagram of the mobile charge distribution at relatively low level injection. The total mobile charge $n(x) = n_{inj}(x) + n_0$.

Additionally, in order to be able to join the solutions in Regions I and II, we require that the electric field be continuous across x_1, or $E_I(x_1) = E_{II}(x_1)$.

Beginning with Region I, we can see that the governing equations in this region are identical to those used to derive the Mott-Gurney Law (Equation 1.2.2 and Equation 1.2.3), resulting in the same injected charge distribution and electric field distribution given in Equation 1.2.6 and Equation 1.2.7. Recognizing that $n_{inj} = n_0$ when $x = x_1$, we can substitute these values into Equation 1.2.6 to find an expression for x_1:

$$x_1 = \frac{\epsilon J}{2q^2 {n_0}^2 \mu_n} \tag{1.2.15}$$

From this expression, we can see that x_1 increases linearly with J. We can use Equation 1.2.15 to solve for J_{cr}, since $x_1 = L$ when $J = J_{cr}$.

$$J_{cr} = \frac{2q^2 {n_0}^2 \mu_n L}{\epsilon} \tag{1.2.16}$$

For $J \geq J_{cr}$, Region I fills the entire bar, and the current is given by the Mott–Gurney Law (Equation 1.2.5).

To find the current–voltage relationship when $J < J_{cr}$, we begin by noting that the electric field in each of the two regions is given by:

$$E_I(x) = -\left[\frac{2J}{\epsilon \mu_n}\right]^{1/2} x^{1/2} \qquad x \leq x_1 \tag{1.2.17}$$

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$$E_{II}(x) = E_I(x_1) \qquad x \geq x_1 \tag{1.2.18}$$

We can then integrate the electric field over the length of the bar to find the total voltage supported in the bar.

$$\begin{aligned} V &= -\int_0^L E(x)dx = -\int_0^{x_1} E_I(x)dx - \int_{x_1}^L E_I(x_1)dx \\ &= \frac{2}{3}\left(\frac{2J}{\epsilon\mu_n}\right)^{1/2} {x_1}^{3/2} + \left(\frac{2J}{\epsilon\mu_n}\right)^{1/2} {x_1}^{1/2}\left[L - x_1\right] \end{aligned} \tag{1.2.19}$$

Substituting the value for x_1 obtained in Equation 1.2.15 and rearranging terms, we are left with the following expression for J when $J < J_{cr}$:

$$J - \frac{1}{6}\frac{\epsilon}{q^2 {n_0}^2 \mu_n L} J^2 = q n_0 \mu_n \frac{V}{L} \tag{1.2.20}$$

This solution, which is plotted in Figure 1.3, reduces to Ohm's Law for very small values of J. We can see that for $J < J_{cr}$, the solution given by the Regional Approximation is closer to the exact solution than that obtained by our simplified approach.

The voltage at which $J = J_{cr}$, which we will call V_{cr}, can be found by substituting the value of J_{cr} from Equation 1.2.16 into Equation 1.2.20 and solving for V. V_{cr} is then given by:

$$V_{cr} = \frac{4}{3}\frac{q n_0 L^2}{\epsilon} \tag{1.2.21}$$

The value calculated for V_{cr} only differs from V_x (Equation 1.2.1) by a factor of 4/3. This further verifies our assertion that the transition between ohmic transport and SCL transport occurs when the injected charge is approximately equal to the thermal charge in the material. V_{cr} and V_x are both plotted on the diagram in Figure 1.3.

1.3 SCL Transport With Traps in the Material

The magnitude and nature of the current flow changes markedly when the situation is one in which a dominant trap or set of traps exists in the material. In this case, the equilibrium concentration of mobile carriers is greatly reduced, since many of the electrons in the system now occupy trap states. Additionally, when an external bias is applied, not all of the electrons injected into the material are mobile. While some of the injected electrons occupy states in the conduction band, many of them fill empty trap states and therefore do not contribute to current flow. However, all injected electrons, whether mobile or trapped, contribute to the voltage drop in the material. This is a generalization of the case discussed in Equation 1.2.2 and Equation 1.2.3, where all the negative charge was constituted of mobile electrons. Both ”types” of charge carry equal weight as far as Gauss's Law is concerned, so both contribute to the potential dropped across the film. However, only the injected *free* carriers contribute to the current. In a nutshell,

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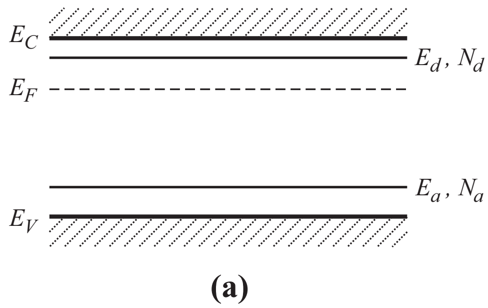

Figure 1.5: Equilibrium band diagram for (a) a material with a shallow donor $(E_F < E_d)$ compensated by an acceptor, and (b) a material with a deep donor $(E_F > E_d)$ compensated by an acceptor.

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understanding the I–V characteristics of a film containing traps boils down to determining the ratio of injected free carriers to injected trapped carriers and their distribution at a given current.

Assume the case we are concerned with, namely an electron injecting contact (the cathode) and an electron capturing contact (the anode) sandwiching a film of interest that contains a much lower equilibrium electron concentration. As in the previous section, we continue to ignore the effects of the contacts, and we still assume that all currents in the system are purely drift currents. Again, we follow the phenomenological analysis offered by Lampert and Mark [1] in order to understand the behavior in the simple case of mobility–dominated transport. The two specific cases that we will consider are those of:

1. A material with a shallow donor which is compensated by an acceptor, and
2. A material with a deep donor which is compensated by an acceptor.

We define a shallow donor as one where the Fermi Level E_F is below the trap level at equilibrium, and the injection level is low enough that this remains the situation under bias. A deep donor is taken to be one in which the Fermi Level is approximately equal to or slightly above the trap level at equilibrium, i.e. pinned by the trap. Equilibrium band diagrams for both of these cases are given in Figure 1.5.

Prior to analyzing this problem, it is important to understand the charge in the system, which is illustrated for the shallow donor case in Figure 1.6 and for the deep donor case in Figure 1.8. For both cases, when the system is in equilibrium, electrons are provided by a donor of density N_d, compensated by an acceptor of density N_a, giving rise to a small mobile charge density n_0. We assume that the acceptor states are fully occupied, and that the number of donor states occupied by electrons when the system is in equilibrium is given by $n_{d0} = N_d - N_{d0}^+$. Charge neutrality at equilibrium requires that the number of ionized donors $N_{d0}^+ = n_0 + N_a$. When a current is passed through the system, both the mobile charge and trapped charge densities increase. The total injected space–charge is the sum of the injected free charge and the injected

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trapped charge, or $n_{inj}(x) = n_{f,inj}(x) + n_{t,inj}(x)$. The total mobile charge is then given by $n(x) = n_0 + n_{f,inj}(x)$, and the total number of donor states which are occupied by electrons is given by $n_d(x) = n_{d0} + n_{t,inj}(x)$.

1.3.1 The Shallow Donor Case

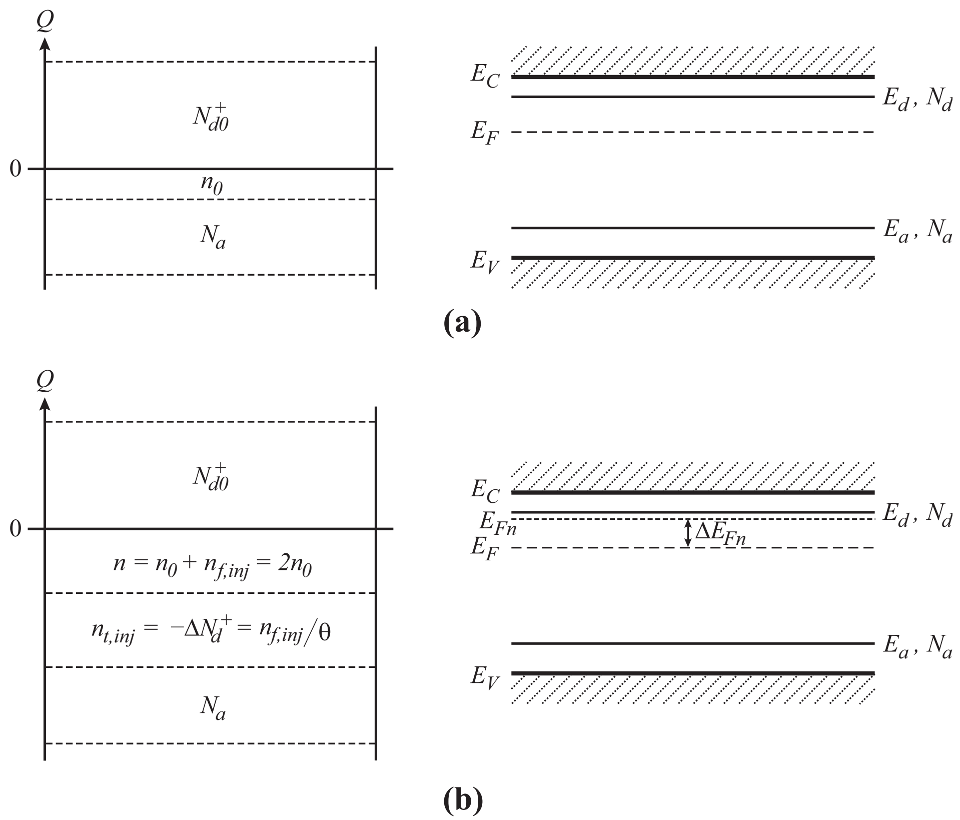

Figure 1.6: The shallow donor case. (a) Charge concentration and band diagram of the system at equilibrium. (b) Charge concentration and band diagram when the injected mobile charge $n_{f,inj} = n_0$. The injected trapped charge is equal in magnitude to the change in density of ionized donors, or $n_{t,inj} = -\Delta N_d^+ = n_{f,inj}/\theta$. The total space–charge in the system $n_{inj} = n_{f,inj} + n_{t,inj}$.

The charge concentration and band diagram for this case are shown schematically in Figure 1.6. A donor trap of density N_d cm^{-3} at an energy level E_d is partially compensated by an acceptor of density N_a at an energy E_a, giving rise to the diagrams in Figure 1.6(a) when the

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system is in thermal equilibrium. When an external bias is applied, charge is injected into the system, which is accompanied by a rise in the electron QFL, as illustrated in Figure 1.6(b). The governing equations for this situation are presented below.

$$n_0 = N_C e^{-(E_C - E_F)/kT} \tag{1.3.1}$$

$$n(x) = N_C e^{-[E_C - E_{Fn}(x)]/kT} \tag{1.3.2}$$

$$n_{d0} = N_d e^{-(E_d - E_F)/kT} \tag{1.3.3}$$

$$n_d(x) = N_d e^{-[E_d - E_{Fn}(x)]/kT} \tag{1.3.4}$$

$$n_{f,inj}(x) = n(x) - n_0 \tag{1.3.5}$$

$$n_{t,inj}(x) = n_d(x) - n_{d0} \tag{1.3.6}$$

Here E_F is the equilibrium fermi level, $E_{Fn}(x)$ is the electron QFL under injection, and E_d is the trap energy. Because of the non–uniform charge distribution within the material under injection, the electron QFL is a function of position x. Following Lampert and Mark [1], we can define a factor $\theta = n(x)/n_d(x)$ which is readily seen from Equation 1.3.2 and Equation 1.3.4 to be

$$\theta = \frac{n(x)}{n_d(x)} = \frac{N_C}{N_d} e^{-(E_C - E_d)/kT} \tag{1.3.7}$$

We can see that θ is independent of position x and remains constant under all bias conditions. This gives the interesting result that regardless of the level of injection, the ratio of mobile charge to trapped charge at every point x within the material remains constant, or

$$\theta = \frac{n(x)}{n_d(x)} = \frac{n_0}{n_{d0}} = \frac{n_{f,inj}(x)}{n_{t,inj}(x)} \tag{1.3.8}$$

To obtain the current–voltage relationship with a shallow dominant donor, we must again solve the Current Continuity and Poisson's Equations subject to the appropriate boundary conditions. For low level injection, we still expect ohmic behavior. When the injection level is high, such that the injected mobile charge $n_{f,inj} > n_0$, the equations needed to be solved are:

$$J = qn(x)\mu E(x) \simeq qn_{f,inj}(x)\mu E(x) \tag{1.3.9}$$

$$\frac{dE(x)}{dx} = \frac{-q\left[n_{f,inj}(x) + n_{t,inj}(x)\right]}{\epsilon} = \frac{-qn_{f,inj}(x)}{\epsilon}\left[1 + 1/\theta\right] \tag{1.3.10}$$

The solution to these equations results in the following current–voltage characteristics:

$$J = \left(\frac{\theta}{\theta + 1}\right)(9/8)\epsilon\mu\frac{V^2}{L^3} \tag{1.3.11}$$

It is instructive to examine this solution in the limits where $\theta >> 1$ and $\theta << 1$. When $\theta >> 1$, or $n_0 >> n_{d0}$, Equation 1.3.11 reduces to Equation 1.2.5, the trap–free limit. Since in this case $n_{f,inj}(x) >> n_{t,inj}(x)$, the traps do not have a significant qualitative impact on the current–voltage characteristics. Only the magnitude of the current changes as a result of the reduction in n_0 due to the pressence of the compensating acceptor.

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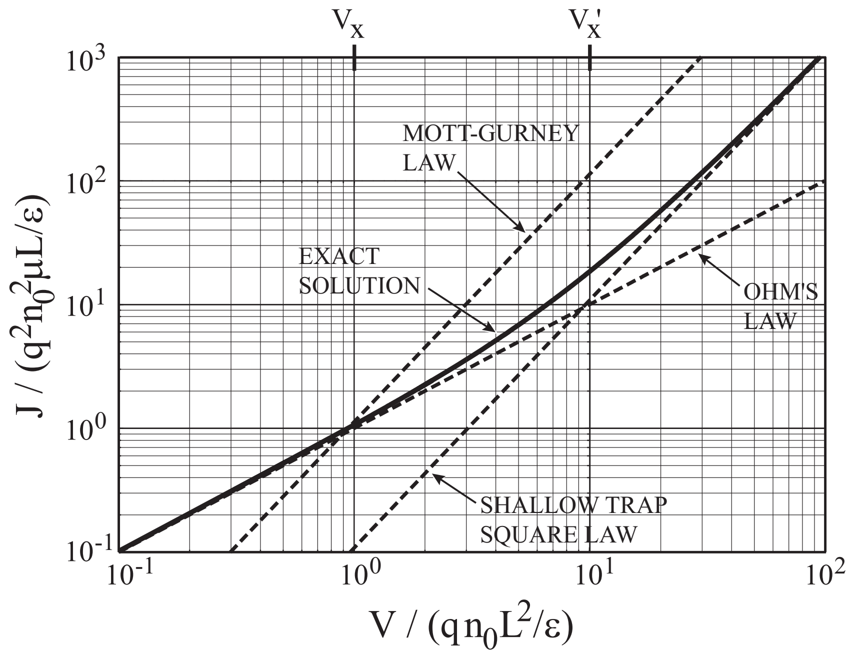

Figure 1.7: Normalized current–voltage characteristics for the shallow trap case with $\theta = 1/10$.

When $\theta << 1$, or $n_0 << n_{d0}$ and $n_{f,inj}(x) << n_{t,inj}(x)$, then the current–voltage characteristics are dominated by the trap. In this case, Equation 1.3.11 becomes

$$J = \theta \cdot (9/8)\epsilon\mu\frac{V^2}{L^3} \quad (1.3.12)$$

or the current is suppressed from the trap–free case by a factor of θ. Furthermore the cross–over voltage, V_x', can also be derived to be increased from the trap–free voltage, V_x, to $V_x' = V_x/\theta$. This result is shown in Figure 1.7 and demonstrates the effectiveness of a material with traps for isolation purposes, or as a means of suppressing currents at larger voltages.

In calculating the cross–over voltage V_x', we have continued with our assertion that the transition between ohmic transport and SCL transport occurs approximately when the injected free charge is equal in magnitude to the thermal charge, or $\bar{n}_{f,inj} = n_0$. Here $\bar{n}_{f,inj}$ is taken to be the average value of the injected free space–charge. This treatment is consistent with the expressions used in Equation 1.3.9 and Equation 1.3.10 for calculating the current characteristics. At the cross–over voltage V_x', $\bar{n} = 2n_o$, and the change in the electron QFL ΔE_{Fn} can be obtained by taking the ratio of Equation 1.3.2 and Equation 1.3.1. The resulting equation,

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$2 = e^{(E_{Fn}-E_F)/kt} = e^{\Delta E_{Fn}/kt}$, gives the useful relationship that *to double the electron density, the quasi fermi level has to rise by* $kT\ln(2)$ *or* $\sim 0.7kT$.

We conclude this section with a discussion of our parameter θ. We saw that when $\theta >> 1$, the presence of the donor trap has very little impact on the current–voltage characteristics. From Equation 1.3.7, we can see that in order for $\theta >> 1$ and for $E_F < E_d$, it is necessary that $N_d << N_C$ and that the donor be shallow (i.e. $E_C - E_d < kT$). The conditions required for $\theta << 1$ are not quite as simple. From Equation 1.3.7, it is clear that $\theta << 1$ when $N_d > N_C$. However, when this condition is met, if there is no compensating acceptor, then the Fermi Level $E_F > E_d$, and so the assumption of a shallow donor is no longer valid. In order for the Fermi Level to be below the donor level, it is necessary that a significant number of electrons in the system be compensated by acceptor states. The number of acceptors N_a required to ensure that $E_d - E_F \geq kT$ for all voltages $V \leq V_x^{'}$ is given by the following equation:

$$N_a \gtrsim N_d \left(1 - \frac{1}{2\mathrm{e}}\right) - \frac{N_C}{2\mathrm{e}} \exp\left[\frac{-(E_C - E_d)}{kT}\right] \tag{1.3.13}$$

The derivation of this equation is left to the reader in Problem 1.2. In short, in order to achieve the conditions that lead to the current–voltage characteristics of Figure 1.7, it is necessary that the system be heavily doped with shallow donors ($N_d > N_C$) *and* be heavily compensated with acceptors.

1.3.2 The Deep Donor Case

To describe the deep donor case, we begin with a qualitative explanation which is then followed by a more rigorous analysis in which the Method of Regional Approximation is again utilized. In our qualitative explanation, we do not account for the non–uniform charge distribution across the material. Instead, we generally assume that the charge density is everywhere equal to its average value. While this treatment is physically inaccurate, it provides significant physical insight into the problem, and the result it produces is qualitatively correct. The charge distribution across the bar is later accounted for when the Regional Approximation is applied.

The charge concentration and band diagram for the deep donor case are illustrated qualitatively in Figure 1.8. A deep donor trap of density N_d cm^{-3} at an energy level E_d is partially compensated by an acceptor of density N_a at an energy E_a, giving rise to the diagrams in Figure 1.8(a) when the system is in thermal equilibrium. Charge neutrality at equilibrium requires that $N_{d0}^+ = n_0 + N_a$. The fact that the Fermi Level is higher than the donor level implies that less than half of the donor states are ionized. Additionally, of the donors that are ionized at equilibrium, only a fraction of the electrons end up in the conduction band; many of the electrons instead fill acceptor states. As a result, when the system is in equilibrium, n_0 is very small, and the number of empty donor states is much larger than the number of electrons in the conduction

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band, or $N^+_{do} >> n_0$. The governing equations for the deep donor case are as follows:

$$
\begin{aligned}
n_0 &= N_C e^{-(E_C - E_F)/kT} && (1.3.14)\\
n(x) &= N_C e^{-[E_C - E_{Fn}(x)]/kT} = n_0 \left[\mathrm{e}^{\Delta E_{Fn}(x)/kT}\right] && (1.3.15)\\
N^+_{d0} &= N_d e^{-(E_F - E_d)/kT} = n_0 + N_a && (1.3.16)\\
N^+_d(x) &= N_d e^{-[E_{Fn}(x) - E_d]/kT} = N^+_{d0} \left[\mathrm{e}^{-\Delta E_{Fn}(x)/kT}\right] && (1.3.17)\\
n_{f,inj}(x) &= n(x) - n_0 && (1.3.18)\\
n_{t,inj}(x) &= N^+_{d0} - N^+_d = N^+_{d0}\left[1 - \mathrm{e}^{-\Delta E_{Fn}(x)/kT}\right] && (1.3.19)
\end{aligned}
$$

It is useful to define a factor A as the ratio between N^+_{do} and n_0

$$A = \frac{N^+_{do}}{n_0} = 1 + \frac{N_a}{n_0} \tag{1.3.20}$$

where Equation 1.3.16 has been applied. It can be seen that A^{-1} is the fraction of ionized donor electrons which end up in the conduction band. Note that A is largely determined by the density of acceptor states N_a. In the absence of any acceptors, $A = 1$. When a compensating acceptor is present, A can be very large, since typically $N_a >> n_0$. However, in order for our analysis to be valid, $N_a \lesssim N_d/2$, since otherwise $E_F < E_d$. In the remainder of our discussion, we assume that $A >> 1$.

When a bias is applied, charge is injected into the bar, and so the concentration of both mobile charge and trapped charge increases. The current flow in the system remains ohmic approximately until the injected mobile charge density $n_{f,inj} = n_0$. This we recognize causes the Fermi Level to move upward by $0.7kT$. This results in all the donor states that were empty at equilibrium N^+_{d0} to be filled. Thus the total injected trapped charge will be $n_{t,inj} = N^+_{d0}$, and the average space–charge in the material is given by $n_{inj} = n_{t,inj} + n_{f,inj} = N^+_{d0} + n_0 \simeq N^+_{d0}$, where the condition that $N^+_{d0} >> n_0$ has been applied. The resulting charge distribution in the system is shown in Figure 1.8(b). The voltage supported by this charge, called the trap–filled limit voltage, is given by

$$V_{TFL} = \frac{qN^+_{d0}L^2}{2\epsilon} = \frac{qn_0L^2}{\epsilon}\left(\frac{A}{2}\right) \tag{1.3.21}$$

Notice that the trap–filled limit voltage $V_{TFL} = (A/2)V_x$. In other words, for the deep donor case, the voltage at which ohmic conduction ceases is larger than that of the trap–free case by a factor $A/2$.

When the voltage is increased above V_{TFL}, all injected charge is mobile charge (since all the traps are full), and so the current rises at a much higher rate, which can be mistakenly interpreted as breakdown. This rise continues until the injected free charge $n_{f,inj} \simeq N^+_{d0}$, which is the condition illustated in Figure 1.8(c). The voltage supported across the material when this occurs, which we will call V'_{TFL}, is approximately given by

$$V'_{TFL} \simeq \frac{qN^+_{d0}L^2}{\epsilon} = 2 \cdot V_{TFL} \tag{1.3.22}$$

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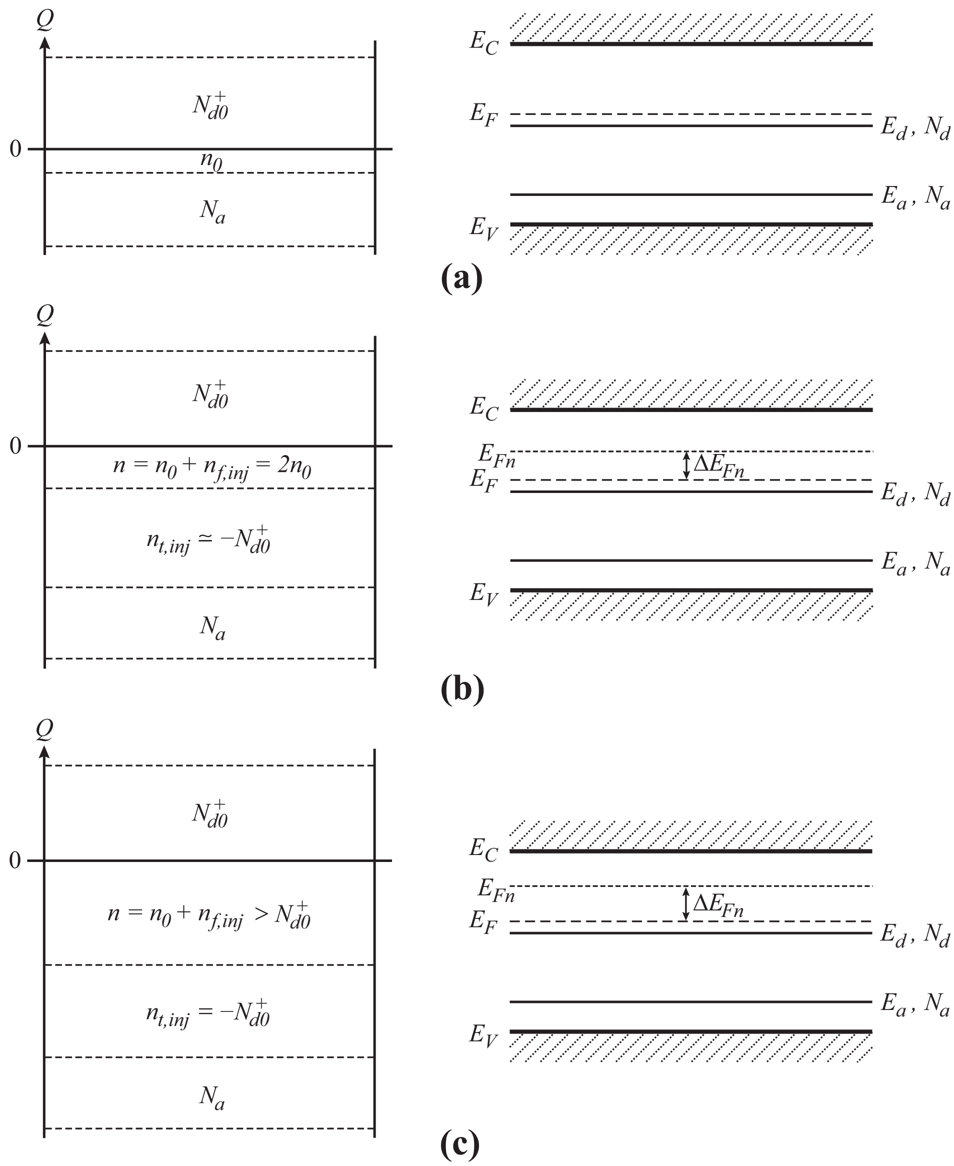

Figure 1.8: The deep donor case. (a) Charge concentration and band diagram of the system at equilibrium. (b) Charge concentration and band diagram when the injected mobile charge $n_{f,inj} = n_0$. When the Fermi Level rises by $0.7kT$, all of the donor states are filled, so the injected trapped charge $n_{t,inj} = N_{d0}^{+}$. The total space–charge in the system $n_{inj} = n_{f,inj} + n_{t,inj} = n_0 + N_{d0}^{+} \simeq N_{d0}^{+}$. (c) Charge concentration and band diagram when the injected mobile charge is greater than the injected trapped charge, or $n_{f,inj} > N_{d0}^{+}$.

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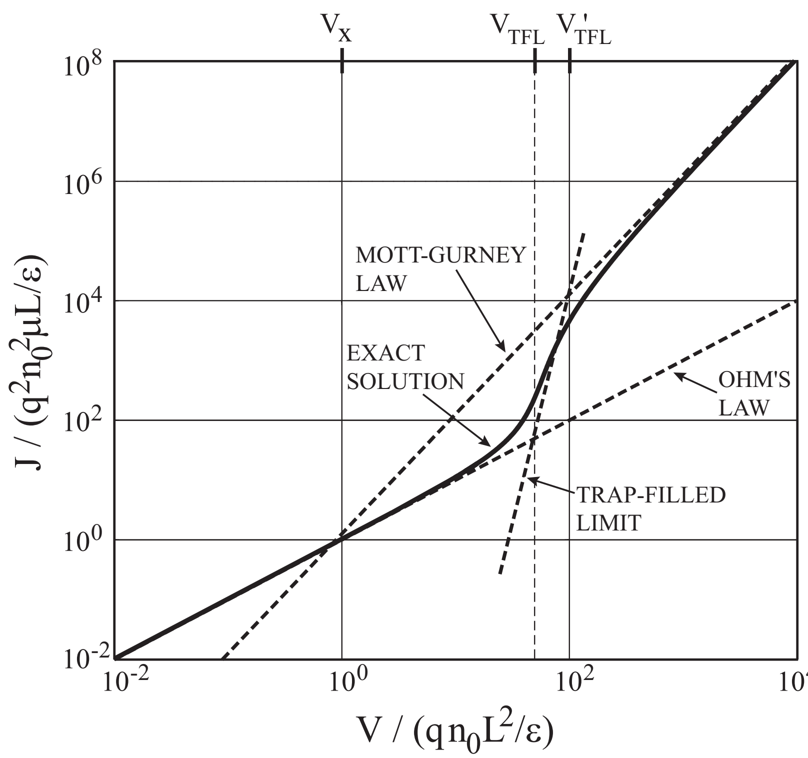

Figure 1.9: Normalized current–voltage characteristics for the deep trap case with $A = 10^{\,2}$.

Once $n_{f,inj} > N_{d0}^{+}$, or equivalently $V > V_{TFL}'$, the injected space–charge in the material is predominantly mobile electrons. Hence, the injected trapped charge in the system is now less than the injected mobile charge and can therefore be neglected. The governing equations for these conditions are identical to those of the trap–free case (Equation 1.2.2 and Equation 1.2.3), so the current therefore follows the trap–free Mott–Gurney curve.

The results obtained for the deep donor case are shown in Figure 1.9. For small applied biases, the current obeys Ohm's Law. Once the voltage exceeds the trap–filled limit voltage V_{TFL}, the current rises rapidly. This rise continues until the voltage $V = V_{TFL}'$, at which point the curve meets the trap–free SCL curve. For voltages exceeding V_{TFL}', the current follows the Mott–Gurney curve. By comparing our results with the exact numerical solution, which is also plotted in Figure 1.9, we can see that our relatively simplistic approach yields a remarkably accurate solution.

In order to gain a more thorough understanding of the deep donor case, and for a more accurate treatment of the non–uniform charge distribution within the material, we now apply the Method

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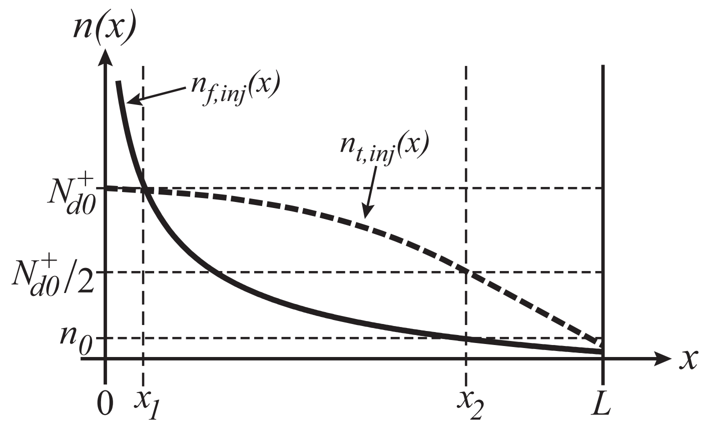

Figure 1.10: Schematic diagram of the injected charge distribution for the deep donor case at relatively low level injection.

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of Regional Approximation. Consider the schematic diagram of the charge distribution within the bar illustrated in Figure 1.10. In this case, we split the bar up into three distinct regions whose boundaries are defined by the points x_1 and x_2. x_1 is defined as the point at which the injected mobile charge $n_{f,inj} = N_{d0}^+$, and x_2 is defined as the point at which $n_{f,inj} = n_0$. Note that at x_2, since $n_{f,inj} = n_0$, the electron QFL has risen $0.7kT$ above its equilibrium value, and so by applying Equation 1.3.19, we find the injected trapped charge at this point to be $n_{t,inj}(x_2) = N_{d0}^+/2$. The region in which $0 \leq x \leq x_1$ is referred to as Region I, the region in which $x_1 \leq x \leq x_2$ is referred to as Region II, and the region in which $x_2 \leq x \leq L$ is referred to as Region III.

In Region I, the injected mobile charge $n_{f,inj}$ is much larger than either the thermal charge n_0 or the injected trapped charge $n_{t,inj}$, and so the total mobile charge and space–charge in this region are both assumed to be $n_{f,inj}$. The equations governing current flow in this region are therefore

$$J = qn_{f,inj}(x)\mu_n E(x) \tag{1.3.23}$$

$$\frac{dE(x)}{dx} = \frac{-qn_{f,inj}(x)}{\epsilon} \tag{1.3.24}$$

The conditions for Region I are the same as those for trap–free SCL transport, and so the solution to the governing equations in this region is just the Mott–Gurney Law (Equation 1.2.5). Also from Equation 1.3.23 and Equation 1.3.24, the injected mobile charge density and electric field distribution in Region I at a current J are given by:

$$n_{f,inj}(x) = \left[\frac{\epsilon J}{2q^2\mu_n}\right]^{1/2} x^{-1/2} \tag{1.3.25}$$

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$$E(x) = -\left[\frac{2J}{\epsilon\mu_n}\right]^{1/2} x^{1/2} \tag{1.3.26}$$

In Region II, the injected space–charge is predominantly trapped charge. $n_{t,inj}$ is very close to N_{do}^{+} everywhere, so we assume that the space–charge throughout this region is equal to N_{do}^{+}. However, $n_{f,inj}$ is still larger than n_0 everywhere, so we assume the mobile charge density to be $n_{f,inj}$. The equations needed to be satisfied in this region are

$$J = qn_{f,inj}(x)\mu_n E(x) \tag{1.3.27}$$

$$\frac{dE(x)}{dx} = \frac{-qN_{do}^{+}}{\epsilon} \tag{1.3.28}$$

Region II corresponds to the trap–filled limit.

In Region III, the injected space–charge is small and is taken to be zero, and the mobile charge density is assumed to be n_0, since in this region $n_{f,inj} < n_0$. The Current Continuity and Poisson's Equations in this region are given by

$$J = qn_0\mu_n E(x) \tag{1.3.29}$$

$$\frac{dE(x)}{dx} = 0 \tag{1.3.30}$$

The solution to these equations is given by Ohm's Law, or $J = qn_0\mu_n V/L$.

As before, x_1 and x_2 will both be functions of the current J. As J increases, the injected mobile charge $n_{f,inj}$ increases everywhere, and so x_1 and x_2 both move closer to L. At low level injection, x_1 and x_2 are both very small, so Region III fills most of the bar, and the current–voltage characteristics are very close to Ohm's Law. As J increases, x_1 and x_2 move closer to L, until at some critical current $J_{cr,1}$, $x_2 = L$. The voltage at which $J = J_{cr,1}$ will be referred to as $V_{cr,1}$. When $V = V_{cr,1}$, Region II fills most of the bar, and the current rises sharply with voltage. If the current is raised even farther, eventually a second critical current $J_{cr,2}$ will be reached at which $x_1 = L$. The voltage corresponding to $J_{cr,2}$ is termed $V_{cr,2}$. Above $J_{cr,2}$, Region I fills the entire bar, and the current follows the Mott–Gurney curve.

To find $J_{cr,2}$ and $J_{cr,1}$, we solve for x_1 and x_2 in terms of J and then find the values of J corresponding to $x_1 = L$ and $x_2 = L$. Recalling that $x = x_1$ is defined as the point at which $n_{f,inj} = N_{do}^{+}$, we can substitute these values into Equation 1.3.25 to find an expression for x_1:

$$x_1 = \frac{\epsilon J}{2q^2\left(N_{d0}^{+}\right)^2\mu_n} = \frac{\epsilon J}{2A^2q^2{n_0}^2\mu_n} \tag{1.3.31}$$

Inserting this value into Equation 1.3.26, the electric field at x_1 is given by:

$$E_I(x_1) = -\left[\frac{J}{8\epsilon\mu_n}\right]^{1/2}\left[\frac{\epsilon J}{2A^2q^2{n_0}^2\mu_n}\right]^{1/2} = -\frac{J}{4Aqn_0\mu_n} \tag{1.3.32}$$

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Solving for x_2 in terms of J is a bit more complicated. We begin by solving Equation 1.3.28 to find an expression for the electric field in Region II as a function of x:

$$E_{II}(x) = -\frac{qN_{do}^{+}}{\epsilon}x - C \tag{1.3.33}$$

C is solved for by setting $E_{II}(x_1) = E_I(x_1)$. Using the expressions in Equation 1.3.31, Equation 1.3.32 and Equation 1.3.33,

$$C = \frac{J}{2Aqn_0\mu_n} \tag{1.3.34}$$

We can then combine Equation 1.3.27, Equation 1.3.33, and Equation 1.3.34 into the following expression:

$$-E_{II}(x) = \frac{qAn_0}{\epsilon}x + \frac{J}{2Aqn_0\mu_n} = \frac{J}{q\mu_n n_{f,inj}(x)} \tag{1.3.35}$$

Recalling that $n_{f,inj} = n_0$ when $x = x_2$ and inserting these expressions into Equation 1.3.35, we can establish a relationship between x_2 and J

$$\begin{aligned} \frac{qAn_0}{\epsilon}x_2 &= \frac{J}{qn_0\mu_n} - \frac{J}{2Aqn_0\mu_n} \\ x_2 &= \frac{\epsilon J}{Aq^2{n_0}^2\mu_n}\left[1 - \frac{1}{2A}\right] \simeq \frac{\epsilon J}{Aq^2{n_0}^2\mu_n} \end{aligned} \tag{1.3.36}$$

where our assumption that A is very large has been applied.

Now that we have solved for x_1 and x_2 in terms of J, we are able to write expressions for our critical currents $J_{cr,1}$ and $J_{cr,2}$. From Equation 1.3.36 with $x_2 = L$ and Equation 1.3.31 with $x_1 = L$, we find

$$J_{cr,1} = \frac{Aq^2{n_0}^2\mu_n L}{\epsilon} \tag{1.3.37}$$

$$J_{cr,2} = \frac{2A^2q^2{n_0}^2\mu_n L}{\epsilon} \tag{1.3.38}$$

It is also useful to examine the ratio of x_1 to x_2. From Equation 1.3.31 and Equation 1.3.36:

$$\frac{x_1}{x_2} = \frac{1}{2A} << 1 \tag{1.3.39}$$

From this relationship, we can see that when $x_2 = L$, $x_1 = L/2A << L$. What this means is that when $x_2 = L$, x_1 is very small, and so Region II covers almost the entire bar. As an example, when $A = 100$, Region II covers 99.5% of the bar when $J = J_{cr,1}$.

The most direct way to calculate the voltage supported across the bar for a given set of injection conditions is to integrate over the electric field within the bar. To determine the critical voltage $V_{cr,1}$, we perform this integral with $x_2 = L$, and to determine $V_{cr,2}$, we perform this integral with $x_1 = L$. When $x_2 = L$, Region II covers most of the bar, so the space–charge everywhere is equal to N_{d0}^{+}, giving rise to an electric field

$$E(x) = -\frac{Aqn_0}{\epsilon}x \qquad\qquad x_2 = L \tag{1.3.40}$$

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everywhere within the bar. When $x_1 = L$, Region I covers the entire bar, and so the electric field everywhere within the bar is given by

$$E(x) = -\frac{2Aqn_0 L^{1/2}}{\epsilon} x^{1/2} \qquad x_1 = L \tag{1.3.41}$$

Integrating each of these fields over the length of the bar, we find:

$$V_{cr,1} = \frac{Aqn_0}{\epsilon} \int_0^L x dx = \frac{Aqn_0 L^2}{2\epsilon} \tag{1.3.42}$$

$$V_{cr,2} = \frac{2Aqn_0 L^{1/2}}{\epsilon} \int_0^L x^{1/2} dx = \frac{4Aqn_0 L^2}{3\epsilon} \tag{1.3.43}$$

We can see that the critical voltages calculated using the regional approximation agree very closely with those calculated using our more basic analysis. In fact $V_{cr,1}$ is exactly equal to the trap–filled limit voltage V_{TFL}, and $V_{cr,2}$ only differs from V'_{TFL} by a factor of 4/3.

Finally, we draw our attention to the steep rise in the current that occurs when all of the deep donor traps are filled. To estimate the rise in current, we examine the ratios $V_{cr,2}/V_{cr,1}$ and $J_{cr,2}/J_{cr,1}$:

$$\frac{V_{cr,2}}{V_{cr,1}} = \frac{8}{3} \tag{1.3.44}$$

$$\frac{J_{cr,2}}{J_{cr,1}} = 2A \tag{1.3.45}$$

We can see that when the voltage is increased from $V_{cr,1}$ to $V_{cr,2}$, corresponding to a factor of 8/3 increase in the voltage, the current increases by a factor $2A$. This result, which is depicted by the line labelled 'TRAP–FILLED LIMIT' in Figure 1.9, shows why we see such a steep increase in the current once the voltage V exceeds $V_{cr,1}$.

References

[1] M. A. Lampert and P. Mark. Current Injection in Solids. Academic Press, Inc., New York, NY, 1970.

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