Parametric Oscillation in Nonlinear Dipole Arrays

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Abstract— **An experimental investigation of large nonlinear dipole arrays is presented, using a diode-loaded dipole placed between two parallel conducting plates. Using this topology we were able to demonstrate enhanced conversion efficiency into the second harmonic and obtain spontaneous parametric oscillation for certain array geometries. Parametric oscillation was achieved by strongly driving the antenna at a single pump frequency and adjusting the cavity length for resonance at a subharmonic. A pump threshold for parametric down-conversion was observed, below which no sub-harmonics were produced. These results suggest using quasi-optical techniques for frequency multiplication and parametric up-conversion or amplification.**

I. Introduction

Antenna arrays provide a convenient means for combining the output power from a collection of semiconductor devices. Recent efforts using this "quasi-optical" power-combining approach have centered around the development of oscillator and amplifier arrays [1]–[4]. Mutual coupling between the array elements plays an important role in the operation of these components. In the former case, mutual coupling between planar antenna elements was exploited to achieve mutual synchronization of the oscillators with the proper phasing [1]. In the amplifier grid [4], mutual coupling increases the radiation resistance for easier matching, and is responsible for the large observed operating bandwidth.

The measurement and modelling of mutual coupling in large arrays is greatly simplified by image theory. In a large linear array, the influence of mutual coupling on the central elements of the array is equivalent to the response of a single radiator in a planar Fabry-Perot cavity. The relative phasing along the array is reproduced by properly choosing the complex reflectivity of the cavity walls. When the cavity walls are infinite and perfectly conducting planes, an infinite array with uniform excitation and 180° progressive phase shift is simulated. Experimentally, unavoidable losses in the cavity result in the simulation of a finite array with tapered excitation. Stated another way, a linear array of coherently-driven antennas is inherently a frequency-selective structure, with a Q-factor that depends on the number of array elements. Interestingly, the cavity mode structure is completely reproduced by the mutual coupling between array elements. This fact can be exploited by introducing a nonlinear device into the array elements, such as a p-n diode. The spectral content of the dipole radiation can then be manipulated by the element spacing and antenna design.

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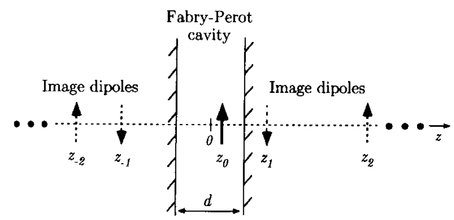

Fig. 1. Geometry for the dipole in a parallel-plate cavity, and indexing for the equivalent image array. Arrows denote the direction of current flow in the dipoles.

An array of such coherently-pumped nonlinear antennas could have practical value for millimeter-wave frequency-conversion. If a suitable device is used and there is sufficient pump power, nonlinear antenna arrays will also exhibit parametric effects which can be exploited for both low-noise amplification and frequency up-conversion with gain [5]. In the following we examine these possibilities experimentally, using a nonlinear dipole antenna in a planar Fabry-Perot cavity to simulate the response of a driven nonlinear array. Although the dipole antenna was chosen because of its analytical and experimental simplicity, a practical implementation of the frequency conversion array would most likely function with planar antennas on a dielectric substrate.

II. Dipole-Cavity Resonance

We first consider the behavior of the dipole driving-point impedance in the presence of the cavity. Fig. 1 describes the geometry of the problem. The mutual coupling that is introduced by the image elements can either enhance or inhibit emission at selected frequencies. For example, if the dipole is situated at the null of the standing cavity field, the radiation destructively interferes with image emission and the radiation resistance will decrease. Likewise, the radiation resistance is larger if the dipole is situated at the antinode of the cavity field, where there is constructive interference with image emission. The cavity length and dipole placement can therefore be tuned to selectively enhance emission at a certain frequency.

A perfect cavity with no conduction or diffraction losses would simulate an infinite linear array with uniform excitation; a lossy cavity can be represented by an array of $2N + 1$ elements with a tapered excitation. The size of the array can be related to the Q-factor approximately as follows. The transient response of the cavity will exhibit a time constant, or decay rate given by

$$\tau = \frac{Q}{\omega_0} \tag{1}$$

This transient response will die out after a time $\sim 3\tau$, corresponding to the delay associated with the interaction of the dipole and the outermost images,

$$2N\frac{d}{c} \approx 3\tau \quad (2)$$

where c is the speed of light in free-space, and the factor of 2 accounts for the round-trip delay. Combining (1) and (2) gives

$$2N \approx \frac{Q}{2d/\lambda} \quad (3)$$

For a half-wavelength cavity, $2N \approx Q$; that is, the cavity Q-factor is approximately the same as the number of array elements. For large arrays we can take the excitation currents to be approximately

$$I_n = I_0(-1)^n e^{-3|n|/N} \qquad n = -N \ldots N \quad (4)$$

For a given array size, the driving-point impedance can be expressed as a summation of the dipole self-impedance and contributions from mutual coupling between the physical dipole and images. The location of each dipole along the z-axis (of Fig. 1) is given by

$$z_n = nd + (-1)^n z_0 \quad (5)$$

where d is the cavity length and z_0 is the location of the physical dipole relative to the cavity center. The driving point impedance is then

$$Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}} = \sum_{n=-N}^{N} \left(\frac{I_n}{I_0}\right) Z_{n0} \quad (6)$$

where Z_{00} is the self-impedance of the (physical) dipole, and Z_{n0} is the mutual impedance between the dipole and its nth image separated by a distance $(z_n - z_0)$. Dipole-cavity resonance occurs when $X_{\text{in}}(f) = 0$, which for large arrays (high-Q cavity) will occur in the vicinity of $d \approx \lambda/2$ or multiples thereof. For thin, center-fed dipoles of length l and radius a, a simple expression for both the self and mutual impedance can be found using the induced EMF method [6],

$$Z_{n0} = j\frac{\eta}{4\pi}\int_{-l/2}^{l/2} \frac{\sin k(l/2 - |\chi|)}{\sin^2(kl/2)} \times \left[\frac{e^{-jkR_{-1}}}{R_{-1}} + \frac{e^{-jkR_1}}{R_1} - 2\cos(kl/2)\frac{e^{-jkR_0}}{R_0}\right] d\chi \quad (7)$$

where $R_m = \sqrt{(|z_n - z_0| + a)^2 + (\chi + ml/2)^2}$

Equations (6) and (7) provide a simple means for investigating the influence of mutual coupling on the dipole impedance as a function of array spacing and frequency. It should be noted that many other published techniques are well suited to calculate driving point impedances in an infinite array environment, but the present method is more convenient for investigating both the real and imaginary impedances of a *finite* array with *tapered excitation*. It can also be used to check the validity of our experimental methods, since the theory behind the equations is well accepted. As an example, typical measurements of radiation resistance versus location within a planar Fabry-Perot cavity are shown in Fig. 2, for two different cavity lengths. These measurements were made at 11.4 GHz with an HP8720 network analyzer, using an unbalanced 2.7 cm dipole, center-fed from a semi-rigid coaxial cable, and with square copper sheets for the cavity measuring roughly 30 cm on a side. The theoretical result from (6) is also shown for comparison. Both curves have been normalized to the respective impedances measured without a cavity, and show very good correlation in the behavior of impedance with location. Note that the extrapolated radiation resistance does not go to exactly zero at the cavity walls because of the finite Q of the cavity.

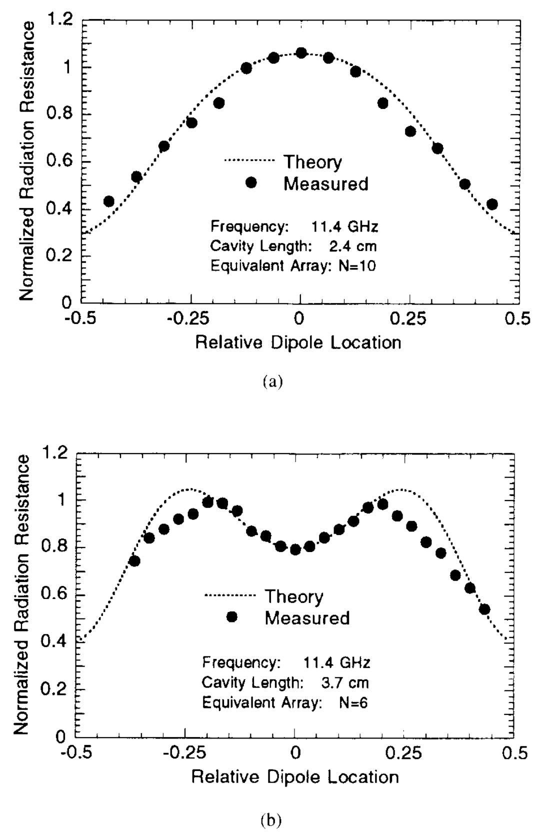

Fig. 2. Two measurements of dipole driving point resistance versus placement within a cavity, showing good agreement with the simple theory of (6). Cavity supports (a) only the lowest-order mode, and (b) the first two modes.

III. Cavity-Controlled Emission from Nonlinear Dipoles

The influence of mutual coupling on the spectral emission of a nonlinear array element was explored experimentally by embedding a p-n diode (the base-emitter junction of an NE6448 microwave BJT, with $f_\tau \approx 10$ GHz) at the feedpoint of the dipole used in the previous experiments. This particular device was chosen for convenience and not because of any desirable properties for this application. The experimental

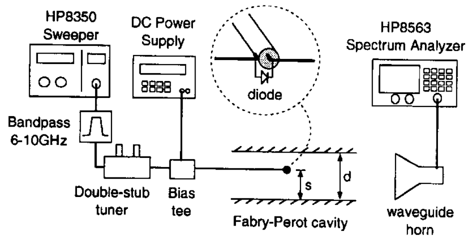

Fig. 3. Illustration of the experimental setup for observing the nonlinear dipole-cavity response. The HP8350 source plug-in was capable of delivering nearly 1 Watt at 8 GHz. The double-stub tuner was initially adjusted for a good match at small-signal drive levels using an HP8720 network analyzer.

setup is shown in Fig. 3. The antenna was driven by an HP8350 sweeper, typically at frequencies between 8 and 9 GHz. The output of the HP8350 was fed through a bandpass filter with a bandwidth of 4 GHz centered at 8 GHz, and out-of-band rejection of 60 dB. The filter removes harmonic distortion in the sweeper output as well as isolating the sweeper from the harmonics generated in the antenna. The spectral response of the antenna was observed using a waveguide horn and an HP8563 spectrum analyzer, with the horn at the broadside position to the image array.

Since the antenna contains a nonlinear device, it was not possible to achieve a good impedance match to the HP8350 over a wide range of pump power and bias levels. A nominal effort was made using a double-stub tuner, adjusted for a good match under small-signal conditions and a reverse bias of -1 Volt. This means that the observed output power and conversion efficiency will not be optimal, however we are most interested in the qualitative behavior of the system in this paper. A practical array would of course include large-signal matching networks and a more suitable nonlinear device.

For the isolated active dipole (no cavity), the measured frequency response is given in Fig. 4a. The available power from the sweeper for these measurements was 18 dBm. The absolute power in each spectral component is small as a result of our detection scheme—the array is simulating an end-fire array, while we are detecting radiation at the broadside position—however, the relative power is of most interest. In fig. 4a a significant second harmonic is generated by the antenna radiating into free space, but the relative power in the second harmonic can be changed dramatically by placing the element within a planar Fabry-Perot cavity. To increase the conversion efficiency into the second harmonic the cavity is tuned so that the fundamental mode is at the second harmonic. The pump frequency is therefore below cut-off, meaning that the radiation resistance is very small and little power will be radiated at this frequency. Fig. 4b shows the emission spectrum for this case, clearly indicating a significant increase ($\sim$16 dB) in the second harmonic content and corresponding suppression of the fundamental. This suggests that a nonlinear array with $\sim\lambda/4$ spacing driven with a 180° progressive phase shift (end-fire array) could be used for quasi-optical frequency multiplication.

Conversely, the second harmonic can be suppressed when the cavity supports the fundamental and the dipole location is

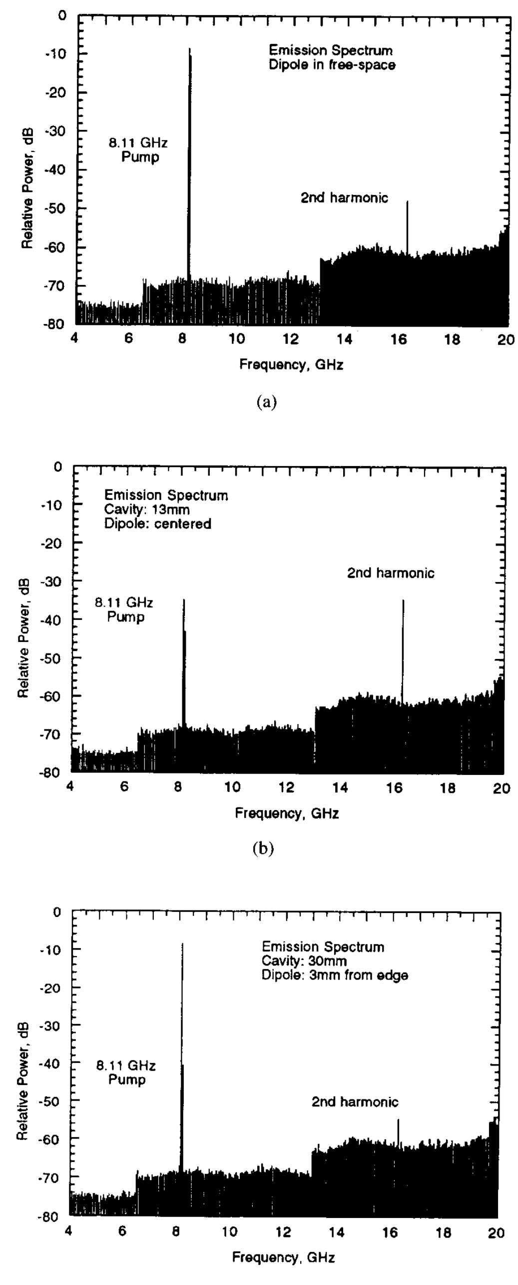

Fig. 4. Harmonic response under small-signal conditions. (a) Measured dipole emission in free-space (no cavity present). (b) Measured response with cavity length below cutoff for the pump signal, showing increased 2nd-harmonic content. (c) Dipole position adjusted to suppress 2nd harmonic.

adjusted off-center (non-uniform array spacing). This was verified experimentally as shown in Fig. 4c, and can be understood qualitatively by considering the behavior of the driving-point impedance at the fundamental and second-harmonic as in Fig. 2. It is important to note that the difference in the measurements of Figs. 4b and 4c are due entirely to the change

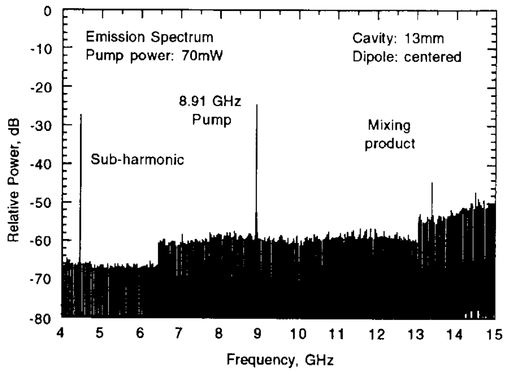

Fig. 5. Dipole emission under large-signal conditions, showing spontaneous parametric down-conversion (and new mixing products) with cavity adjusted for resonance at the sub-harmonic.

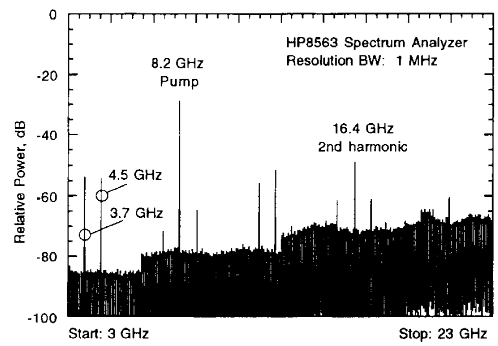

Fig. 6. Measured emission spectra under similar conditions to that of Fig. 5, but showing non-degenerate down-conversion due to a different cavity spacing and dipole placement.

in dipole impedance as a result of the mutual coupling, and are not affected by the detection scheme. This can be shown approximately by noting that the array factor for the equivalent image array is given by

$$AF(\theta) = \sum_{n=-N}^{N} I_n e^{jk_0(z_n - z_0)\cos\theta} \tag{8}$$

where the angle θ is measured from the z-axis of Fig. 1, and the I_n are given by (4). Our detector is located at $\theta = \pi/2$, where the array factor (and hence the detected power on the spectrum analyzer) is independent of the dipole location, z_0, and cavity length, d.

IV. Spontaneous Parametric Oscillation

For certain values of the cavity spacing and pump power, we also observed dipole emission at the *sub-harmonic* and corresponding mixing products, as shown in Fig. 5. This is a result of spontaneous parametric down-conversion in our setup. When a nonlinear reactance is modulated at a pump frequency ω_p, it can easily be shown [7] that a negative resistance is induced at the pump subharmonic, $\omega_p/2$. This is the basis for degenerate parametric amplification [5]. If the embedding circuit satisfies the oscillation conditions at this frequency, spontaneous down-conversion will occur. Interestingly, we consistently observed this phenomenon for cavity spacings below cutoff for both the fundamental and sub-harmonic frequencies. In some cases we were also able to observe non-degenerate parametric oscillation, as shown in Fig. 6. In this case one of the lower frequency components can be thought of as the "idler" signal in parametric amplifier terminology [5]. At no time did we observe parametric oscillation without the cavity present.

The dipole-cavity impedance from (6) can be represented as a simple RLC circuit for a narrow range of frequencies, in parallel with the diode impedance and any parasitic feed reactance. There are two conditions for oscillation at the sub-harmonic: the *total* reactance must be zero, and the pump-induced negative resistance must overcome the losses in the circuit, which includes the radiation resistance as well as parasitic losses in the diode and feed network. The bandpass filter in our setup ensures that the predominant losses in the system for frequencies other than the fundamental are due to the dipole-cavity interaction. Since the radiation resistance at the sub-harmonic is very small (but not zero!) below the cutoff frequency of the cavity, the induced negative resistance must also be small to explain the observed oscillations. This is a result of using a device with a small dynamic change in reactance—a varactor diode with a larger variation in capacitance with reverse bias and a higher cutoff frequency would result in a larger induced negative resistance for a given pump power. In any case, the measurements clearly indicate that energy is being delivered to the small radiation resistance, since radiation is detected. In principle, spontaneous parametric oscillation should also be observed for cavity spacings for which the sub-harmonic is above cut-off, provided the oscillation conditions are again satisfied. This was not possible in our particular setup because of excessive cavity losses at the sub-harmonic and the small induced negative resistance.

The driven parametric device can be considered as providing gain, with positive feedback supplied by the dipole-cavity interaction. The magnitude of the induced negative resistance is proportional to the pump power, suggesting that there should be a threshold for parametric down-conversion. This was verified by measurement of the sub-harmonic power versus pump power, shown in Fig. 7. For pump powers below threshold the subharmonic component of the dipole response is indistinguishable from the noise floor. At approximately 35 mW, a sharp increase in sub-harmonic output is observed. As pump power continues to increase, the slope of the curve changes due to the change in diode impedance. (Note: the "staircase" pattern in the data is a result of using a 0.2 dBm increment in pump power, rounding the received power to the nearest 0.5 dBm, and subsequently converting to a linear format). When the pump power is very large, the negative resistance saturates and sub-harmonic power levels off. The maximum power could be slightly increased by adjusting the diode bias and double-stub tuner for a better impedance match under large-signal conditions. Also shown in the inset of Fig. 7 is the range of frequencies over which sub-harmonic

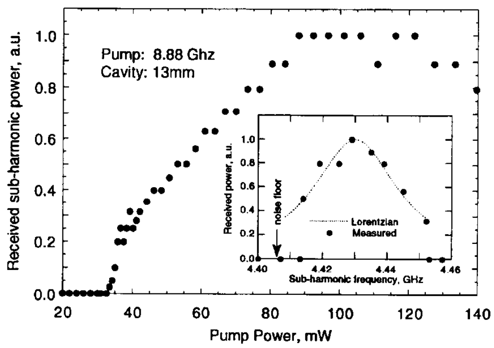

Fig. 7. Sub-harmonic power generated versus pump power. Above at threshold of approximately 35 mW, sub-harmonic power increases rapidly, then falls off as diode impedance changes. Inset shows the range of frequencies over which parametric oscillation was observed, and corresponding variation in output power.

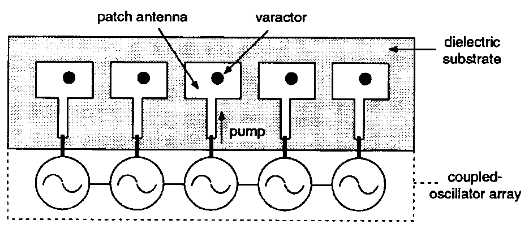

Fig. 8. Illustration of a posible implemetation of a planar frequency multiplier/parametric amplifier array. An array of patch antennas with varactor diodes embedded in them is driven by a local coupled-oscillator array. Antenna spacing, diode placement, and oscillator phasing determines the emission spectra.

generation was observed, with the corresponding variation in output power. The data compares well with the expected Lorentzian behavior of a single-tuned resonant cavity.

V. Discussion and Conclusion

The measurements have shown that mutual coupling between the elements of a nonlinear array can have a strong influence on the spectral content of the array output. To the extent that mutual coupling can be reproducibly controlled in a practical array, it may be possible to realize high power frequency converters and parametric amplifiers using large arrays of nonlinear devices, by adjusting the mutual coupling to provide a good impedance match and the desired frequency response. In this approach, each array element would have its own pump source, with all the pump sources operating coherently. This could be achieved in several ways, such as through a traditional array feed network, injecting the pump source quasi-optically, or using a coupled-oscillator array as shown in Fig. 8. In the latter case, coupling between the oscillators allows them to synchronize through the phenomenon of injection-locking [9].

Although the experiments presented in this paper simulated an end-fire array, the results should extend to broadside arrays as well. Conceptually, this is equivalent to placing our dipole between two magnetic conducting planes, which has a similar mode structure to the electric conducting cavity. Either case can be realized in a planar array by controlling the relative phasing of the pump sources. In the case of a coupled-oscillator pump array (Fig. 8), the relative phasing could be adjusted using recently developed scanning techniques [10].

The observed spontaneous down-conversion could provide a useful technique for frequency division or generating sub-harmonic mixing products. However, we consider the observation of parametric oscillations to be significant only as an indication that other parametric effects could be exploited in arrays. These include negative resistance parametric amplification and parametric up-conversion [5], where the regime of parametric oscillations would necessarily be avoided. Such possibilities are intriguing — parametric amplifiers have not historically been used as power amplifiers, despite low-noise and high efficiency, because all the power in the system would converge on a single small-junction diode. Since the junction area of the device must be kept small for impedance matching and reduced parasitics, there are fundamental limitations on the power-handling capacity of a single device amplifier. By combining a large number of parametric amplifiers in a quasi-optical array, we can take advantage of this high efficiency and high output powers while still keeping the power requirements of a single device to a reasonable level. Since the signals are coupled to free-space, there is very little parasitic loss in the system, which is important for noise reduction.

Additionally, the degenerate negative-resistance parametric amplifier is known to possess a *phase-sensitive gain* [5]. When the pump signal is fully coherent with the input frequency, the maximum gain of the amplifier can be as much as 6 dB greater than for a non-coherent pump source. Therefore if the pump phase can be made to track the incoming signal phase, the signal-to-noise ratio can be improved by up to 6 dB since non-coherent noise would receive 6 dB less gain. This same principle has recently been demonstrated in parametric laser systems [8], and could be achieved by making the degenerate parametric amplifier part of a phase-locked loop.

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