Oscillator Array Dynamics with Broadband *N*-Port Coupling Networks

Robert A. York, *Member, IEEE*, Peter Liao, *Student Member, IEEE*, and Jonathan J. Lynch

***Abstract*— This paper considers the analysis of an oscillator array with an arbitrary coupling network, described in terms of *N*-port circuit parameters. A Kurokawa analysis is used to transform the frequency domain network description into a set of equations for the oscillator amplitude and phase dynamics. The results reduce to previous work with "loosely" coupled Van der Pol oscillators, provided that the coupling network satisfies a broadband condition: the *Q*-factor of the coupling network must be much smaller than that of the oscillator. The theory is verified using a new coupling structure and a six-element patch oscillator array operating at 4 GHz, which produced a 70° scanning range using a phase-shifterless technique.**

I. Introduction

QUASI-OPTICAL oscillator arrays offer a potentially useful technique for producing higher powers at millimeter-wave frequencies with better efficiency than is possible using conventional power-combining techniques [1], [2]. Several types of arrays have been developed: the coupled-oscillator array [3]–[6], the oscillator grid [7]–[9], and the periodic spatial combiner [10], [11]. This paper focuses on the coupled oscillator array, where a set of antenna-loaded, single-device oscillators are fabricated in an array and coupled together, permitting synchronization through mutual injection locking. The desired phase relationship is preserved through control of the free-running frequency distribution. Arrays of this type have been demonstrated with both two- and three terminal devices, relying on mutual coupling between patch antennas to provide the mutual synchronization [5], [13].

Although successful in proving the power-combining concept, arrays exploiting radiative coupling between antennas would not be useful in practice, because the interactions are typically weak and difficult to control and/or predict accurately. Weakly coupled oscillators require tighter tolerances in the fabrication of the oscillators to ensure mutual locking with the proper phase relationships. On the other hand, loose coupling does simplify the modeling of array dynamics—it was found that a simple set of differential equations for the phase dynamics would adequately predict the mutual synchronization, mode stability, and steady-state phase distribution in the system [13]. The theory is a generalization of Adler's equation for injection locking [14], [15], with coupling between the oscillators described by a set of complex (magnitude and phase) coupling parameters. These parameters could be determined experimentally [17], and were assumed constant (independent of frequency).

A more attractive approach is to couple the oscillators together with a transmission-line network, which can be designed to provide the appropriate coupling strength and coupling phase. However, this complicates the analysis of the array dynamics in several ways. When the oscillators are strongly coupled, the amplitude dynamics (which were ignored in the previous analyses) become important. The electrical characteristics of the coupling circuit will vary with frequency, which must also be taken into account. Furthermore, as the coupling strength is increased, the coupling network itself will perturb the oscillator frequencies and consequently perturb the steady-state phase relationships. In the limit of very strong coupling, a bifurcation point is reached above which multi-moding problems are encountered.

Some work has been published which concerns determining the stable mode of operation in a multi-mode system [11], [19]. In this paper we derive a set of equations describing the amplitude and phase dynamics in a coupled-oscillator array with an arbitrary N-port coupling network. We assume that either the coupling is loose enough, or the coupling network is designed in such a way as to avoid the possibility of multiple frequency operation (multi-moding). Following the work of [12], the coupling network is represented by commonly used N-port network parameters such as Y- or Z-parameters. A Kurokawa technique is used to convert the frequency-dependent network description into dynamic rate equations [15]. The resulting equations reduce to our previous analyses [13], and those of Stephan and Morgan, only if the coupling network satisfies a broadband condition. This analysis not only allows us to calculate coupling parameters for arbitrary coupling networks, but also indicates the range of validity and limitations of our previous work, which is of great help in designing new types of coupling networks to realize the previously reported beam-scanning concept [18]. The results apply to the periodic spatial combiners as well. The theory will be illustrated by application to a simple coupling networks, with experimental verification using a low-power linear oscillator chain.

II. Amplitude and Phase Dynamics

The system under consideration is illustrated in Fig. 1. A set of N nearly sinusoidal oscillator circuits is coupled through a linear N-port network. Following [12], the coupling

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The authors are with the Department of Electrical Engineering, University of California at Santa Barbara, Santa Barbara, CA 93106 USA.

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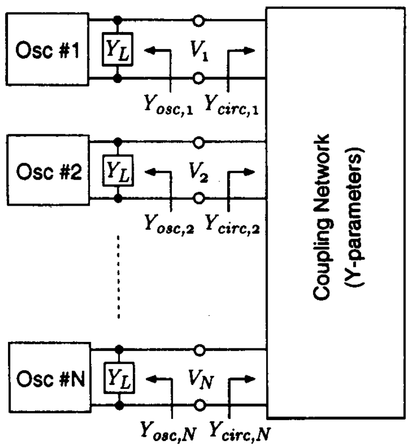

Fig. 1. Oscillator array with coupling network described by Y-parameters

network is described in terms of Y-parameters, so that the input admittance at port i is

$$Y_{\text{circ},i} = \sum_{j=1}^{N} Y_{ij} \frac{V_j}{V_i} \tag{1}$$

where V_i is the terminal voltage at port i. Although Y-parameters were chosen here, the Z-parameter formulation may be more appropriate for certain types of oscillators; the results of the analysis will be the same in either case.

Each oscillator circuit, described by $Y_{\text{osc},i}$ for the i th oscillator, contains an active device and embedding network such that when no coupling circuit is present (i.e.$Y_{ij} = 0$ for all i and j) the system becomes a set of isolated, incoherent sinusoidal oscillators, where the ith oscillator is described by a free-running frequency ω_i and free-running amplitude α_i. A key point is that the load network is distributed among the oscillators and hence separated from the coupling circuit. In a quasi-optical array, the load of each oscillator is essentially the radiation resistance of a planar antenna (or the unit cell of a grid), and hence the load is distributed over the entire array.

For nonzero terminal voltages, the condition for oscillation at a frequency ω is

$$Y_{\text{osc},i}(\omega, V_i) + Y_{\text{circ},i}(\omega, \overline{V}) = 0 \qquad i = 1, 2, \ldots N \tag{2}$$

where $\overline{V}$ denotes a vector containing all of the terminal voltages, V_i. For convenience, the total admittance at the ith port will be defined as $Y_i(\omega, \overline{V}) \equiv Y_{\text{osc},i}(\omega, V_i) + Y_{\text{circ},i}(\omega, \overline{V})$, so (2) is written as $Y_i(\omega, \overline{V}) = 0$. These equations can also be expected to hold when the frequency, ω, varies in time, provided that the variation is slow relative to ω_i. Kurokawa first showed that such equations can be converted to nonlinear rate equations in the amplitude and phase variables [15]. We anticipate nearly sinusoidal oscillations of the form

$$V_i = A_i(t) e^{j[\omega_i t + \phi_i(t)]} = A_i(t) e^{j\theta_i(t)} \tag{3}$$

where the amplitude and phase are allowed to be slowly varying functions of time, and θ_i is the instantaneous phase of the ith oscillator. Considering the time derivative of V_i gives

$$\frac{dV_i}{dt} = j\left[\omega_i + \frac{d\phi_i}{dt} - j\frac{1}{A_i}\frac{dA_i}{dt}\right] V_i. \tag{4}$$

Comparing (4) with the result from Fourier theory, $dV/dt \to j\omega V$, Kurokawa concluded that the expression in brackets must be the time-domain representation of the instantaneous frequency. Using this expression for the frequency, ω, and the slowly-varying approximations

$$\frac{d\phi_i}{dt} \ll \omega_i \qquad \frac{1}{A_i}\frac{dA_i}{dt} \ll \omega_i \tag{5}$$

allows (2) to be expanded in a Taylor series about the free-running frequencies, giving

$$Y_i(\omega_i, \overline{V}) + \left(\frac{d\phi_i}{dt} - j\frac{1}{A_i}\frac{dA_i}{dt}\right)\frac{\partial Y_i}{\partial \omega}\bigg|_{\omega_i} + \cdots = 0 \tag{6}$$

which can be solved for the desired set of equations describing the amplitude and phase dynamics. Using the substitution $\theta_i = \omega_i t + \phi_i$ in (6) leads to

$$\begin{aligned} \frac{dA_i}{dt} &= A_i \quad \text{Im}\{F_i(\overline{A}, \overline{\theta})\} \\ \frac{d\theta_i}{dt} &= \omega_i - \text{Re}\{F_i(\overline{A}, \overline{\theta})\} \qquad i = 1, 2, \ldots N \end{aligned} \tag{7}$$

where

$$F_i(\overline{A}, \overline{\theta})) = \frac{Y_i}{\partial Y_i / \partial \omega}\bigg|_{\omega_i}.$$

Given the Y-parameters of the oscillators and coupling networks, and first derivatives with frequency, (7) describes the amplitude and phase dynamics for the coupled-oscillator system. According to (5), (7) is valid provided that the oscillator and coupling circuits satisfy the condition

$$|F_i(\overline{A}, \overline{\theta})| \ll \omega_i. \tag{8}$$

Note that the total admittance Y_i depends nonlinearly on the amplitude and phase variables through (1), and hence (7) represents a complicated set of coupled nonlinear differential equations. We are most interested in steady-state solutions to (7) where all oscillators are synchronized to a common frequency, ω, which occurs when

$$\frac{dA_i}{dt} = 0 \quad \text{and} \quad \frac{d\theta_i}{dt} = \omega \qquad i = 1 \cdots N. \tag{9}$$

Using (7) and (9), the steady-state solutions must satisfy

$$F_i(\overline{A}, \overline{\theta}) = \omega_i - \omega \qquad i = 1 \cdots N. \tag{10}$$

Let $(\hat{\overline{A}}, \hat{\overline{\theta}})$ be a solution vector to (10). The stability of these solutions can be investigated by expanding (7) around them

$$\begin{aligned} \frac{d\hat{A}_i}{dt} + \frac{d\rho_i}{dt} &= (\hat{A}_i + \rho_i) \, \text{Im}\left\{F_i(\hat{\overline{A}} + \overline{\rho}, \hat{\overline{\theta}} + \overline{\delta})\right\} \\ \frac{d\hat{\theta}_i}{dt} + \frac{d\delta_i}{dt} &= \omega_i - \text{Re}\left\{F_i(\hat{\overline{A}} + \overline{\rho}, \hat{\overline{\theta}} + \overline{\delta})\right\} \end{aligned} \tag{11}$$

where δ and ρ are small perturbations about the amplitude and phase solutions, respectively. Using the first two terms of a Taylor series expansion we can write

$$F_i(\hat{\overline{A}}+\overline{\rho},\hat{\overline{\theta}}+\overline{\delta}) \approx F_i(\hat{\overline{A}},\hat{\overline{\theta}})+\sum_k \left(\frac{\partial F_i}{\partial A_k}\rho_k + \frac{\partial F_i}{\partial \theta_k}\delta_k\right) \quad (12)$$

where the derivatives are evaluated at the point $(\hat{\overline{A}},\hat{\overline{\theta}})$. Substituting (12) into (11) and using (10) gives linearized equations for the perturbations,

$$\frac{d\rho_i}{dt} = \hat{A}_i \sum_k \mathrm{Im}\left\{\frac{\partial F_i}{\partial A_k}\rho_k + \frac{\partial F_i}{\partial \theta_k}\delta_k\right\}$$
$$\frac{d\delta_i}{dt} = -\sum_k \mathrm{Re}\left\{\frac{\partial F_i}{\partial A_k}\rho_k + \frac{\partial F_i}{\partial \theta_k}\delta_k\right\} \qquad i = 1\cdots N. \quad (13)$$

This can be written as a matrix equation

$$\frac{d}{dt}\begin{bmatrix}\overline{\rho}\\ \overline{\delta}\end{bmatrix} = \overline{\overline{M}}\begin{bmatrix}\overline{\rho}\\ \overline{\delta}\end{bmatrix}. \quad (14)$$

A stable solution requires the perturbations to decay away in time, which occurs if the eigenvalues of the stability matrix $\overline{\overline{M}}$ have negative real parts [16]. In general the matrix elements can be difficult to evaluate, but certain simple coupling networks with nearest-neighbor coupling result in a stability matrix with mostly zero entries and exploitable symmetry properties [13].

A. Van der Pol Oscillators

Many oscillators can be adequately modeled by the single-tuned circuit shown in Fig. 2, which leads to the Van der Pol equation for certain nonlinearities. Consider an array of such oscillators, which have nearly identical Q-factors and load conductances but nonidentical free-running frequencies and amplitudes. The input admittance near the free-running frequency is

$$Y_{\mathrm{osc},i}(\omega, V_i) \approx G_L[1 - G_d(A_i)/G_L] + \jmath 2C(\omega - \omega_i). \quad (15)$$

The variation of device impedance with frequency is neglected in (15). The term in brackets describes the saturation of negative conductance with increasing oscillator output amplitude. A simple expression which leads to sinusoidal oscillations is

$$[G_d(A_i)/G_L - 1] \Rightarrow \mu(1 - A_i^2/\alpha_i^2). \quad (16)$$

where μ is a positive, dimensionless quantity that determines the rate of amplitude saturation. Using the substitution $S_i(A_i) = 1 - A_i^2/\alpha_i^2$, the total admittance and its derivative at each terminal are then

$$Y_i(\omega_i, A_i) = -\mu G_L S_i(A_i) + \sum_{j=1}^{N} Y_{ij}\frac{A_j}{A_i}e^{\jmath(\theta_j-\theta_i)}$$
$$\left.\frac{\partial Y_i}{\partial \omega}\right|_{\omega_i} = 2\jmath C + \sum_{j=1}^{N}\left.\frac{\partial Y_{ij}}{\partial \omega}\right|_{\omega_i}\frac{A_j}{A_i}e^{\jmath(\theta_j-\theta_i)}. \quad (17)$$

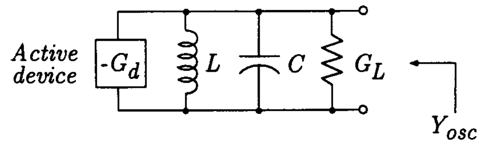

Fig. 2. Single tuned oscillator model, which leads to the Van der Pol equation.

Defining normalized coupling parameters κ_{ij} as

$$\kappa_{ij} \equiv Y_{ij}/G_L \quad (18)$$

and recalling that $Q = \omega_o C/G_L$ for a parallel resonant circuit, gives

$$F_i(\overline{A},\overline{\theta}) = \jmath\frac{\omega_i}{2Q}\left[\frac{\mu S_i(A_i) - \sum_{j=1}^{N}\kappa_{ij}\frac{A_j}{A_i}e^{\jmath(\theta_j-\theta_i)}}{1 - \jmath\frac{\omega_i}{2Q}\sum_{j=1}^{N}\frac{\partial\kappa_{ij}}{\partial\omega}\frac{A_j}{A_i}e^{\jmath(\theta_j-\theta_i)}}\right]. \quad (19)$$

This expression is then substituted into (7) to analyze the amplitude and phase relationships for a given coupling network. Condition (8) indicates that (19) is valid unless the coupling becomes too strong (numerator becomes comparable to $2Q$) or the coupling network is very frequency sensitive (denominator becomes small compared with $1/2Q$)

B. Relationship to Earlier Work

The form of (19) gives a considerably more complicated set of equations than those used in the previous array analyses [12], [13], but does reduce to the previous results when the sum in the denominator of (19) is negligible. This occurs when

$$\frac{\omega_i}{2Q}\sum_{j=1}^{N}\frac{\partial\kappa_{ij}}{\partial\omega}\frac{A_j}{A_i} \ll 1. \quad (20)$$

When this condition is true, (7) and (19) give

$$\frac{dA_i}{dt} = \frac{\mu\omega_i}{2Q}S_i(A_i)A_i - \frac{\omega_i}{2Q}\sum_{j=1}^{N}A_j\,\mathrm{Re}\left\{\kappa_{ij}e^{\jmath(\theta_j-\theta_i)}\right\}$$
$$\frac{d\theta_i}{dt} = \omega_i - \frac{\omega_i}{2Q}\sum_{j=1}^{N}\mathrm{Im}\left\{\kappa_{ij}\frac{A_j}{A_i}e^{\jmath(\theta_j-\theta_i)}\right\}. \quad (21)$$

These equations are of exactly the same form as used in [13]. Furthermore, the coupling parameters used in our previous work were viewed as empirically determined quantities, but are now seen to be identical to the normalized Y-parameters given by (18).

The condition (20) is a "broadband" constraint, forcing the characteristics of the coupling network to exhibit much slower variation with frequency than that of the oscillator circuits. For a coupling network with reactive elements, (20) implies that the Q-factor of the coupling network must be smaller than that of the oscillators. This could be accomplished by adding loss in the coupling network. Note that a lossy coupling network does not necessarily imply weak coupling. The theory is still valid for large coupling strengths, subject to (8) and the proviso that there are no multiple-frequency oscillations. In our previous arrays, the self- and mutual-impedance of the patch antennas provided the loss mechanism, but for

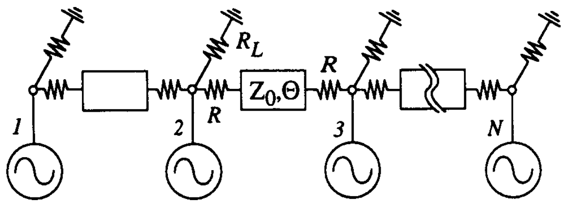

Fig. 3. A simple resistively-loaded nearest-neighbor coupling network suitable for oscillator arrays.

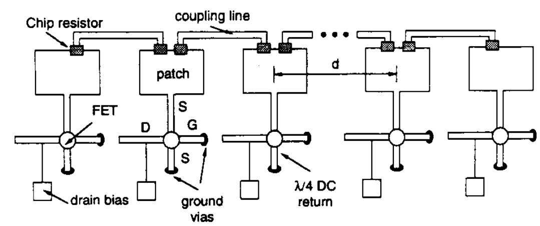

Fig. 4. Experimental patch array using the coupling network of Fig. 3(a).

transmission-line coupling networks other resistive elements must be used. These considerations suggest that our previous work [13], [18] must apply strictly to broadband coupling networks, where the term broadband is interpreted relative to the free-running oscillators.

Since the present formulation of coupled oscillator dynamics includes the load as part of the oscillator circuit (rather than as part of the coupling network), it is clear why the past experimental results with active patch arrays conform well to the simpler theory. Only the off-diagonal elements of the coupling matrix, $\overline{\overline{\kappa}}$, are nonzero in that configuration, because the patch acts as both the resonant element and the load, and elementary antenna theory confirms that the mutual coupling between antennas satisfies (20) for all but very close spacings.

III. A Practical Example and Results

A Simple nearest-neighbor coupling network that can be designed to satisfy (20) is shown in Fig. 3. Many other coupling structures are conceivable; we selected that of Fig. 3 for simplicity. Chip resistors are used to introduce loss and lower the Q-factor. This circuit has desirable properties for the special case of $R = Z_0$, for which the Y-parameters become

$$Y_{ij} = \begin{cases} \frac{\eta_i}{2Z_0} & i = j \\ \frac{-e^{-j\beta L}}{2Z_0} & |i-j| = 1 \\ 0 & \text{otherwise} \end{cases}$$
$$\frac{\partial Y_{ij}}{\partial \omega} = \begin{cases} 0 & i = j \\ \frac{j\tau_g e^{-j\beta L}}{2Z_0} & |i-j| = 1 \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

where βL is the electrical length of the transmission-line, $\eta_i = (2 - \delta_{i1} - \delta_{iN})$, δ_{ij} is the Kronecker delta function, and τ_g is the group delay through the transmission-line. For TEM or quasi-TEM lines, $\tau_g = \beta L/\omega$, and when $\beta L \leq 2\pi$ then condition (20) is satisfied if $\pi R_L/QZ_0 \ll 1$ ($R_L = 1/G_L$). Assuming this constraint holds, and letting $\varepsilon = R_L/2Z_0$ and $\Phi = \beta L$, then (21) becomes

$$\frac{dA_i}{dt} = \frac{\omega_i}{2Q}[\mu S_i(A_i) - \eta_i \varepsilon]A_i + \frac{\varepsilon\omega_i}{2Q} \sum_{\substack{j=i-1 \\ j\neq i}}^{i+1} A_j \cos(\Phi + \theta_i - \theta_j)$$
$$\frac{d\theta_i}{dt} = \omega_i - \frac{\varepsilon\omega_i}{2Q} \sum_{\substack{j=i-1 \\ j\neq i}}^{i+1} \frac{A_j}{A_i} \sin(\Phi + \theta_i - \theta_j) \qquad i = 1 \cdots N \tag{23}$$

which holds for the end elements provided that the substitution $A_0 = A_{N+1} = 0$ is used. These equations are nearly identical to those used in [13] for nearest-neighbor coupling, except for a small change in the amplitude equation. Since the coupling network perturbs the amplitudes of end-elements differently than the central elements, parasitic elements on the array periphery may be required to ensure uniformity across the array.

The quantities ε and Φ are the coupling strength and coupling phase, respectively, from [13]. Therefore all of our previous results regarding mode stability, beam-scanning, etc., apply to this coupling structure. This analysis shows that the coupling phase is simply the electrical length of the transmission-line (an intuitive result) and that the coupling strength is the ratio of load resistance to the characteristic impedance of the transmission-line. Note that the coupling strength can be quite large, subject to (8) and the constraint $\varepsilon > \mu S_i(A_i)$ (the latter is a condition for oscillation derive from (23)). A large coupling strength would result in a large locking range and less stringent manufacturing tolerances.

The coupling network of Fig. 3 thus provides a simple and flexible alternative to antenna-coupled arrays. This structure was experimentally investigated using the six-element patch array shown in Fig. 4. To be consistent with the theory, the coupling network is connected at the radiating edge of the patch. At resonance, the load impedance at this point is a parallel combination of the two edge radiation resistances. The oscillator and patch circuits were designed on 31 mil (0.787 mm) Rogers Duroid 6810 with $\epsilon_r = 10.8$, for operation at 4.0 GHz with an array period of $d = 20$ mm. The patch width gave a radiation resistance of 800 Ω at each edge. A one-wavelength, 50 Ω coupling line with 50 Ω chip resistors were used, which should provide a coupling angle of $\Phi = 0°$ and a coupling strength of $\varepsilon \approx 4.0$. A low-noise packaged MESFET, NEC 32184A, was used.

When the oscillators were all powered on and mutually locked, the measured output frequency was centered in the range of 4.3 GHz, or 7% above the design frequency. Because of this, the coupling angle was not exactly 0°. With all the oscillators near a common free-running frequency the measured radiation pattern was close to the broadside position (approximately −5°) as shown in Fig. 5. This is consistent with the presence of randomness in the free-running frequencies [13], as well as some measurable differences in the free-running amplitudes of the oscillators, which both conspire to produce a small average phase progression along the array. Combining the 6-element array factor (calculated using a phase shift of 8° along the array) with the patch antenna

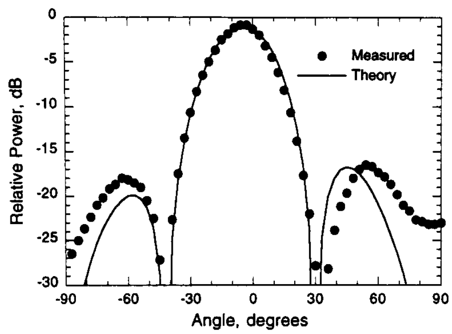

Fig. 5. Measured radiation pattern for the array of Fig. 4 with nearly identical free-running frequencies, and comparison to theory.

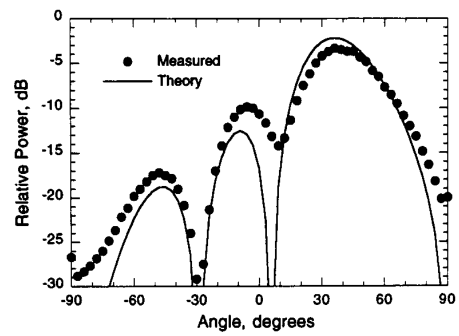

Fig. 6. Measured radiation pattern when the end-element frequencies are detuned according to [18]. The beam could be continuously scanned from −30° to +40°.

element pattern (essentially given by $\cos\theta$ for the H-plane) gives the theoretical pattern shown in Fig. 5 for comparison with the measurements. Good agreement is observed with the measured pattern.

By slightly de-tuning the free-running frequencies of the end-elements, the beam can be scanned away from broadside (this is described in more detail in [18]). This result can be derived directly from (23) by solving for the free-running frequency distribution that will produce a constant phase progression. Theoretically the phase progression can be continuously adjusted over a 180° range, which for this particular array would correspond to a scan range of ±60° off-broadside. Our measurements indicated that the beam could be scanned from an angle of $-30°$ to $+40°$ away from broadside, using the drain bias voltage to control the end-element frequencies. The $+40°$ measurement is shown in Fig. 6, and again shows good agreement to the theory. We were not able to achieve the full scan range predicted by the theory because one of the array end-elements suffered a dramatic decrease in power as the frequency was tuned beyond that leading to Fig. 6, and consequently the array could not lock to a common frequency in this regime. Presumably the use of wide-band VCO's for the end-elements would correct this problem. At no time were multi-moding problems encountered during the experiments, since the device was terminated for a narrow band negative resistance region.

IV. Conclusion

A nonlinear description of oscillator array dynamics with arbitrary coupling networks was developed. This theory reduces to previous work when the electrical characteristics of the coupling network vary slowly with frequency relative to the oscillator embedding network. This is significant because it provides insight into the design of coupling networks to increase the operating bandwidth of the array (increased coupling strength), while maintaining the phase behavior described in [13], [18]. A simple and practical coupling structure, suitable for microstrip patch arrays, was designed and verified experimentally at 4 GHz. Currently we are applying this work in collaboration with the Jet Propulsion Laboratory to the development of a practical, compact X-band scanning array.

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Robert A. York (S'86–M'91) received the B.S. degree in electrical engineering from the University of New Hampshire, Concord, and the M.S. and Ph.D. degrees in electrical engineering from Cornell University, Ithaca, NY, in 1987, 1989, and 1991, respectively.

In November 1991, he joined the Department of Electrical and Computer Engineering at the University of California, Santa Barbara, where he is involved with the design and fabrication of microwave and millimeter-wave circuits and devices, quasi-optical device arrays and other nonlinear or nonreciprocal components, and quasi-optical measurement techniques.

Dr. York is a member of the Compound Semiconductor Research Group and the ational Science Foundation Center for High-speed Image Processing at UCSB. He received an Army Research Office Young Investigator Award in 1993 for research in quasi-optical arrays. He was also the recipient of a 1990 MTT-S Graduate Fellowship Award, and corecipient of the Ban Dasher award for best paper at the 1989 IEEE Frontiers in Education Conference.

Peter Liao (S'92) received the B.S. degree from the Cooper Union, New York, NY, and the M.S. degree in electrical engineering from the University of California at Santa Barbara in 1989 and 1991, respectively. He is currently pursuing the Ph.D. at the latter university.

His research interests include quasi-optical power combining arrays, microwave oscillator and circuit design, and coupled oscillator theory.

Jonathan J. Lynch received the B.S. and M.S. degrees in electrical engineering from the University of California at Santa Barbara in 1987, 1993, respectively. He is currently pursuing the Ph.D. degree at the same university.

His current research interests includes high power millimeter wave pulse generation using mode locked arrays of microwave oscillators.