

# A Mode Locked Array of Coupled Phase Locked Loops

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**Abstract**—An array of coupled phase locked loops is configured to produce a comb spectrum, and therefore pulsed microwave power, using a frequency locking technique analogous to the mode locked optical laser. The basic principles of operation are derived and measurements of an 11-GHz system, built from commercially available microwave components, demonstrate the frequency and time domain characteristics.

## I. INTRODUCTION

MANY quasioptical systems utilize arrays of frequency locked microwave oscillators to produce electronically steerable radiation beams [1]–[3]. However, one of the authors has shown that operating the array elements outside the region of direct synchronization can result in a periodic train of microwave pulses or in a continuously sweeping transmitting beam [2], [3]. The authors designed and built a five element mode locked array to demonstrate the concept as a means of pulse power production and to show that such systems can be designed using commercially available components. Phase locked loops were used in place of conventional microwave oscillators to enhance the locking ability and to improve various aspects of array performance. This paper describes the general operating principles behind the array and presents time and frequency domain measurements.

## II. GENERAL OPERATING PRINCIPLES

The quasioptical oscillator array (as well as the optical laser) depends on frequency locking, or synchronization, to maintain output signal coherence. Engineers familiar with non-linear oscillations understand that when two coupled nonlinear oscillators are tuned within a certain bandwidth, called the “locking range,” that the oscillators synchronize and produce a single output frequency. This synchronization phenomenon persists when either oscillator is slightly tuned and represents a “preferred” state of the system. A less familiar locking effect is referred to in the laser community as mode locking. This effect occurs when three or more coupled oscillators are tuned so that the output frequencies are *almost* evenly spaced, but far enough apart to avoid direct synchronization. The oscillators lock to a state where the frequencies are *exactly* evenly spaced and, again, maintain this spacing when the oscillators are slightly tuned. Fig. 1(a) shows an example of the spectrum of a mode locked array with and without coupling. The

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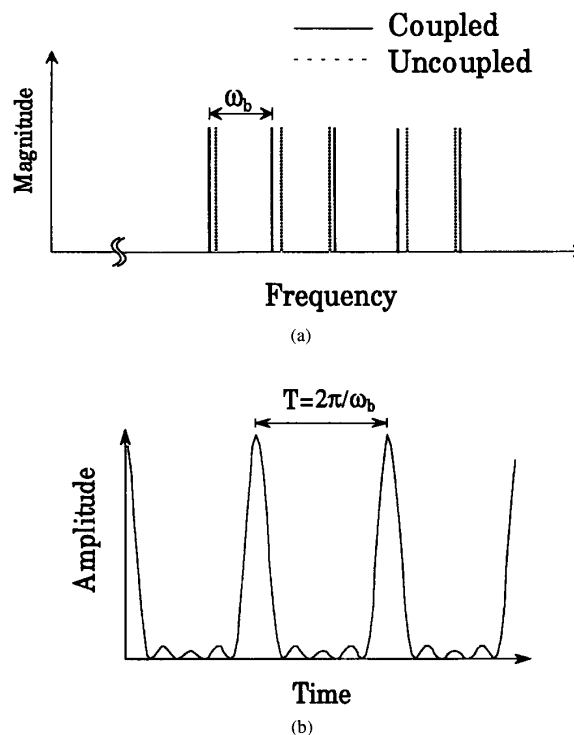


Fig. 1. (a) Frequency spectrum of a five element mode locked array. The solid line show evenly spaced steady state frequencies for the coupled system, and the dotted lines show the frequencies that would exist if the inter-element coupling were removed. (b) Time domain waveform that results from summing the oscillator outputs, assuming the proper phase relation exists between oscillators.

comb spectrum produced by a large mode locked array can produce a periodic pulse train in the time domain, depending on the relative phasing, and is the principal method used to produce fast optical pulses. Fig. 1(b) shows the time domain waveform obtained from the spectrum of Fig. 1(a), assuming the oscillator phasing is ideal.

A block diagram of the mode locked array is shown in Fig. 2. The array consists of five voltage controlled oscillators (VCO's) and two levels of mixers. The first level produces four outputs that oscillate at the beat frequency,  $\omega_b$ , and these feed into the second level whose outputs, assuming a mode locked state exists, are DC signals that control the oscillation frequencies of the three central VCO's. The first and fifth VCO frequencies control the outer two spectral components, as shown in Fig. 2, and therefore control the spectral location

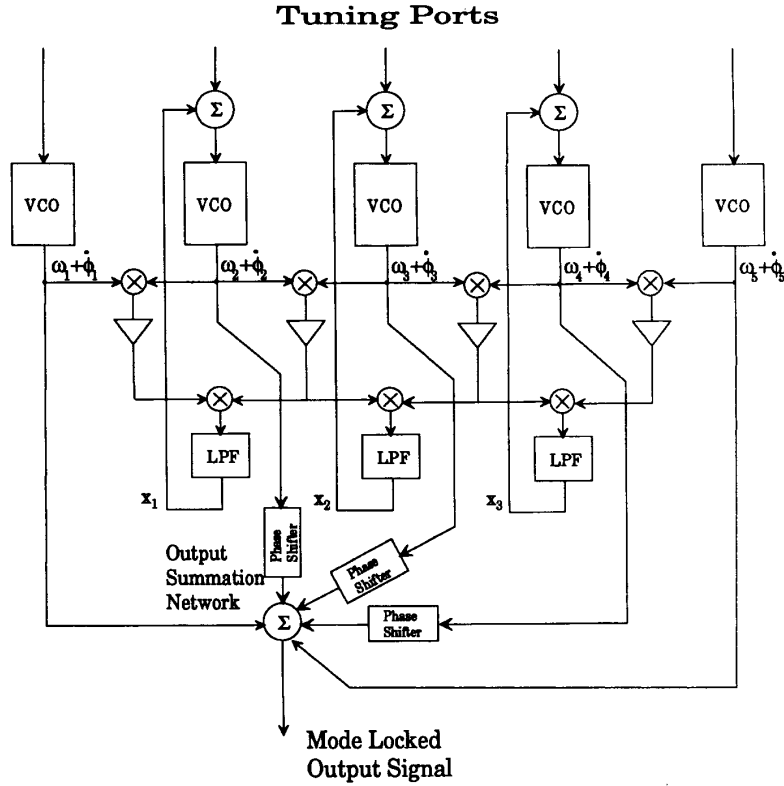


Fig. 2. Block diagram of the mode locked array of phase locked loops. The outputs of the first stage of mixing oscillate at the beat frequency. The outputs of the second stage are DC signals used to control the VCO frequencies thus maintaining equal frequency spacing. The first and fifth tuning ports control the spectral location and beat frequency.

and separation. For example, tuning only the fifth VCO causes the entire comb spectrum to fold and unfold like an accordion, as long as the mode locked state is maintained. One cannot define a unique locking bandwidth for an array of more than three elements since there are many combinations of oscillator tunings that can cause loss of lock. For a full discussion of the tuning of mode locked arrays see [6]; although the reference discusses a different type of oscillator, the analytical methods can be applied to the present case.

The relation between the relative phases and the oscillator tunings can be analyzed using the methods of [7] and [8] as follows. The equations for the instantaneous frequencies of the oscillators are

$$\omega_n + \dot{\phi}_n = \omega_{on} + Sx_n, \quad n = 1, 2, \dots, 5 \quad (1)$$

where  $S$  is the VCO sensitivity in  $\text{rad/V} \cdot \text{sec}$ , and the control signals  $x_n$  are given by

$$\begin{aligned} x_1 &= x_5 = 0 \\ x_n &= V_o \sin(\Delta\phi_n - \Delta\phi_{n-1}) \\ &= V_o \sin(\Delta\Delta\phi_{n-1}), \quad n = 2, 3, 4. \end{aligned} \quad (2)$$

The first and second differences are defined as  $\Delta\phi_n = \phi_{n+1} - \phi_n$  and  $\Delta\Delta\phi_n = \Delta\phi_{n+1} - \Delta\phi_n$  the first and fifth control signals are zero since these oscillators are tuned independently. Combining (1) and (2) gives the dynamic equations

for the phases. In addition, the steady state frequencies can be eliminated and the system order reduced by forming the second difference of the equations. The result is

$$\begin{aligned} \Delta\Delta\dot{\phi}_1 &= \Delta\Delta\omega_{o1} \\ &\quad + SV_o[\sin(\Delta\Delta\phi_2) - 2\sin(\Delta\Delta\phi_1)] \\ \Delta\Delta\dot{\phi}_2 &= \Delta\Delta\omega_{o2} \\ &\quad + SV_o[\sin(\Delta\Delta\phi_3) - 2\sin(\Delta\Delta\phi_2) + \sin(\Delta\Delta\phi_1)] \\ \Delta\Delta\dot{\phi}_3 &= \Delta\Delta\omega_{o3} \\ &\quad + SV_o[-2\sin(\Delta\Delta\phi_3) + \sin(\Delta\Delta\phi_2)] \end{aligned} \quad (3)$$

where  $\Delta\Delta\omega_{on} = \Delta\omega_{o,n+1} - \Delta\omega_{on}$ . It may seem strange that there are only three degrees of freedom for the five element array, but when one considers that the time domain waveform consists of a periodic carrier modulated by a periodic envelope, the existence of two arbitrary phases becomes evident.

The algebraic equations describing the relation between the oscillator tunings (that is, the frequencies  $\omega_{on}$ ) and the phases are found by noting that in the steady state the phases  $\Delta\Delta\phi_n$  are constant. Setting the time derivatives to zero gives

$$\begin{aligned} \Delta\Delta\omega_{o1} &= -SV_o[\sin(\Delta\Delta\phi_2) - 2\sin(\Delta\Delta\phi_1)] \\ \Delta\Delta\omega_{o2} &= -SV_o[\sin(\Delta\Delta\phi_3) - 2\sin(\Delta\Delta\phi_2) + \sin(\Delta\Delta\phi_1)] \\ \Delta\Delta\omega_{o3} &= -SV_o[-2\sin(\Delta\Delta\phi_3) + \sin(\Delta\Delta\phi_2)]. \end{aligned} \quad (4)$$

Thus, to find the phases for a given a set of five oscillator tunings  $\omega_{on}$  we must form the three second differences  $\Delta\Delta\omega_{on}$  and then solve the linear system (4) for the three quantities  $\sin(\Delta\Delta\phi_n)$ . Again, only three steady state frequencies affect the phases. If the first and fifth oscillator tunings are fixed, the spectral location and beat frequency are also fixed, and tuning the central elements changes only the phases. Taking the inverse sine of each  $\sin(\Delta\Delta\phi_n)$  gives two possible choices for each  $\Delta\Delta\phi_n$  for a total of six possible states. Applying the stability criterion developed in [7] shows that stable states exist only when all phases satisfy  $|\Delta\Delta\phi_n| < \pi/2$  and therefore only one stable state exists for a given set of tunings. Summing the outputs of the five oscillators produces the time domain mode locked waveform, and the pulses are sharpest when the phases satisfy  $\Delta\Delta\phi_n = 0$  [9]. From (4) we see that this occurs when the oscillators are tuned as  $\Delta\Delta\phi_{on} = 0$ , that is, when the tunings are evenly spaced.

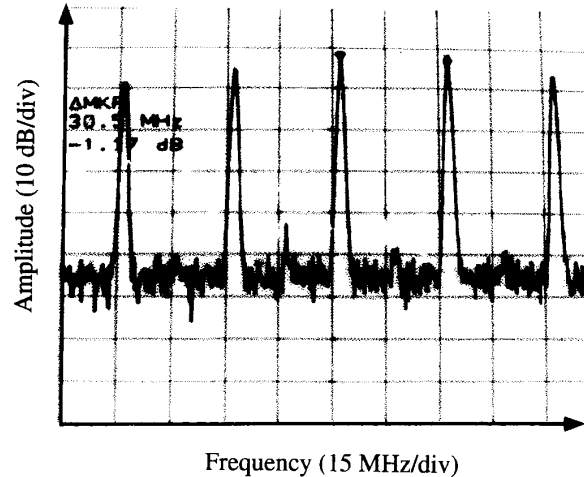
### III. EXPERIMENTAL RESULTS

An array of mode locked PLL's was designed and built at a carrier frequency of 11 GHz and beat frequency ranging from 20–50 MHz. All of the components were catalogue parts with the exception of the VCO's donated by Watkins-Johnson (no special characteristics were required—the donation allowed us to avoid long lead times). Referring to the block diagram of Fig. 2, amplifiers were required after the first stage of mixers in order to provide the proper level of LO drive to the second set of mixers. Each VCO bias could be adjusted manually via a tuning pot thus allowing control of individual oscillator tunings. The VCO outputs were summed through a simple resistive summing network. The high insertion loss in the summing network was not a problem since the pulse shape was the most important figure of merit. The phases of the central elements were adjusted via mechanical phase shifters to maximize the peak level and minimize sidelobes. The mode locked signal envelope was split between a spectrum analyzer and a high speed detector for oscilloscope display.

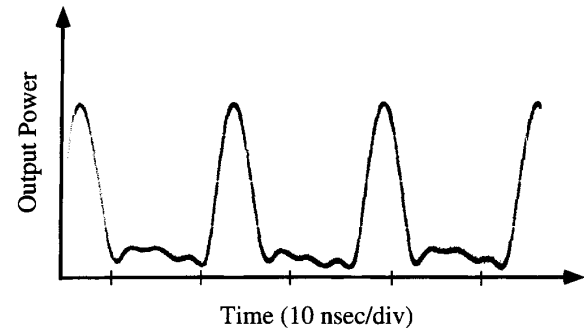
The time domain envelope of the RF carrier is shown in Fig. 3(a), and the frequency spectrum in Fig. 3(b). The beat frequency was set to 30.5 MHz which produced a pulse repetition period of 32.8 nsec. The DC to RF efficiency was not measured since the purpose of this proof of concept design was to verify the pulse shape. One may notice that additional spectral energy exists halfway between the main spectral components. This unanticipated effect produces a period doubling in the time domain waveform, and can be seen in Fig. 3(a). The ripple near the center of the display is different than the ripple on either side. Thus the true period of the waveform is actually 65.6 nsec.

### IV. CONCLUSION

A new type of quasioptical pulse generation system was developed using phase locked loops organized into a mode locked array. This proof of concept design demonstrates that the pulse generation method can be implemented using off the shelf components. Mode locked array theory gives the



(a)



(b)

Fig. 3. (a) Measured spectrum of mode locked output. The carrier is near 11 GHz and the beat frequency is 30.5 MHz. (b) Time domain waveform of detected mode locked output.

basic properties of the array and measurements in the time and frequency domains confirm the basic theory.

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