Phase Noise in Coupled Oscillator Arrays

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ABSTRACT

A simple method to evaluate the phase noise properties of the coupled oscillators for arbitrary coupling and injection-locking topologies is presented [1,2]. The method is based on the phase dynamics equation and the concept of noise admittance [3,4] to represent the noise source. The PM noise of the coupled oscillators can be easily analyzed by solving the coupling matrix related to the coupling topologies, after neglecting amplitude noise and AM-to-PM conversion. An 8.5 GHz prototype array was constructed to verify the phase noise reduction of the total PM noise and the array injection locked by the external low phase noise source.

I. INTRODUCTION

Coupled oscillator arrays (figure 1) have been used in quasi-optical power combining and beam scanning applications[2]. In this paper we present the coupled oscillator noise theory with the noise source expressed by the noise admittance. The noise admittance is equivalent to the concept of the noise voltage or current. Assuming that phase noise sources of the oscillators are mutually uncorrelated and have equal power spectral density[1,3,4], we find total PM noise of N coupled parallel resonator oscillators with coupling phase $\Phi = 0$ reduced to $1/N$, independent of the phase progression along the array. We also prove that N oscillator coupled through the reciprocal coupling network always lead to $1/N$ reduction in total phase noise. Though the total phase noise shows $1/N$ reduction for the bilaterally reciprocal coupled oscillators, the effect is not sufficient to overcome the poor noise property for small array. In our previous work the array can be locked to the external low phase noise source to reduce the total phase noise[6]. Here we also study and measure the total phase noise of the array locked to an external source. The array noise can only be reduced by the external injection source near the carrier, and noise offset frequency far from the carrier is not affected by the injection source.

II. PHASE NOISE OF MUTUALLY SYNCHRONIZED ARRAYS

We can express the noisy oscillators by adding an independent noise admittance[1,3,4] $Y_{noise} = G_L Y_n = G_L(G_n + \jmath B_n)$ to the parallel resonator oscillator model. Assuming the oscillators have identical amplitudes, the phase dynamics equation of N coupled noisy oscillator array becomes

$$\frac{\partial \theta_i}{\partial t} = \omega_i - \frac{\omega_i}{2Q}\sum_{j=1}^{N} \epsilon_{ij}\sin(\theta_i - \theta_j + \Phi_{ij}) - \frac{\omega_i}{2Q}B_{ni}(t) \tag{1}$$

where θ_i, ω_i, and Q are the phase, free-running frequency, and Q-factor of the ith oscillator, respectively. ϵ_{ij} and Φ_{ij} are the coupling parameters between the ith and jth oscillators. G_n and B_n describe the in-phase and quadrature components of the noise source, respectively. For phase noise analysis we perturb (1) by substituting $\theta_i \Rightarrow \hat{\theta}_i + \delta\theta_i$ where $\hat{\theta}$ is the steady-state solution of (1) without B_{ni}, and $\delta\theta_i$ is the phase fluctuation of the ith oscillator. Assuming small fluctuations , (1) being linearized around $\hat{\theta}$ and Fourier transformed gives

$$\left(\frac{\jmath\omega}{\omega_{3\text{dB}}}\right)\widetilde{\delta\theta}_i = -\sum_{j=1}^{N}\epsilon_{ij}(\widetilde{\delta\theta}_i - \widetilde{\delta\theta}_j)\cos(\hat{\theta}_i - \hat{\theta}_j) - \widetilde{B}_{ni} \tag{2}$$

where the tilde ($\tilde{}$) denotes a transformed or spectral variable, $\omega_{3dB} \equiv \omega_i/2Q$, half the 3 dB bandwidth of the oscillator tank circuits, and ω the noise offset frequency from the carrier. The matrix form of (2) can be written as

$$\overline{\overline{N}}\,\overline{\widetilde{\delta\theta}} = \overline{\widetilde{B}_n}; \qquad \overline{\widetilde{\delta\theta}} = \overline{\overline{P}}\,\overline{\widetilde{B}_n} \tag{3}$$

where

$$\overline{\widetilde{\delta\theta}} = \begin{pmatrix} \widetilde{\delta\theta}_1 \\ \widetilde{\delta\theta}_2 \\ \vdots \\ \widetilde{\delta\theta}_N \end{pmatrix}, \quad \overline{\widetilde{B}_n} = \begin{pmatrix} \widetilde{B}_{n1} \\ \widetilde{B}_{n2} \\ \vdots \\ \widetilde{B}_{nN} \end{pmatrix}, \quad \overline{\overline{P}} = \overline{\overline{N}}^{-1}. \tag{4}$$

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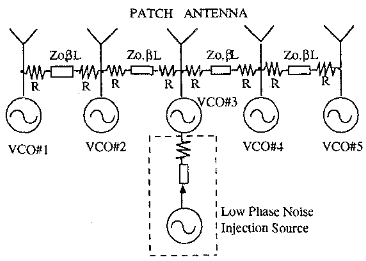

Figure 1–Diagram of the experimental five-element coupled oscillator array. The elements within the dashed blocked was used for the phase noise reduction of the array by low phase noise source HP8350B sweep oscillator

The matrix $\overline{\overline{N}}$ reflects the coupling topology of N-element coupled oscillator array. The phase fluctuations of the individual oscillators are then determined by (4). The individual noise fluctuation is given by $< \widetilde{\delta\theta}_i \widetilde{\delta\theta}_i^* >$, where the notation $<>$ represents an ensemble average. Assuming PM noise sources, $\widetilde{B}_{ni}(t)$, are random (ergodic) processes with zero time average, uncorrelated and have same power spectral density, it can be shown, using the Wiener-Khintchine theorem [5] that

$$< \widetilde{B}_{ni} \widetilde{B}_{nj}^* > = < |\widetilde{B}_n|^2 > \delta_{ij} \qquad (5)$$

where δ_{ij} is the Kronecker delta [4,5]. From (3) and (5), the PM noise of the ith oscillator is

$$< |\widetilde{\delta\theta}_i|^2 > = < |\widetilde{B}_n|^2 > \sum_{j=1}^{N} |p_{ij}|^2 \qquad (6)$$

where p_{ij} is the element of $\overline{\overline{P}}$. Furthermore, for notational convenience we will hereafter drop the notation $<>$ and write the power spectrum as $|\widetilde{\delta\theta}|^2$ or $|\widetilde{B}_n|^2$, with the ensemble or time average being implicitly understood. Assuming the outputs of the array are combined efficiently, the combined output signal is $V(t) = A\sum_{j=1}^{N} \cos(\omega_o t + \delta\theta_j) = NA\cos(\omega_o t + \delta\theta_{total})$, where

$$\delta\theta_{total} = \frac{1}{N}\sum_{j=1}^{N} \delta\theta_j \qquad (7)$$

With (3),(5) and (7), the total phase noise becomes

$$|\widetilde{\delta\theta}_{total}|^2 = \frac{|\widetilde{B}_n|^2}{N^2} \sum_{j=1}^{N} \left| \sum_{i=1}^{N} p_{ij} \right|^2 \qquad (8)$$

The single uncoupled oscillator phase noise can be found by substituting $\epsilon_{ij} = 0$ into (2), and

$$|\widetilde{\delta\theta}_i|^2_{uncoupled} = \frac{|\widetilde{B}_{ni}|^2}{(\omega/\omega_{3dB})^2} \qquad (9)$$

For the scanning nearest-neighbor coupled oscillator chain with uniform phase progression $\Delta\hat{\theta}$[2], the coupling matrix is

$$\overline{\overline{N}} = \epsilon \cos \Delta\hat{\theta}$$

$$\begin{pmatrix} -1-jx & 1 & 0 & \dots & 0 \\ 1 & -2-jx & 1 & \dots & \vdots \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -2-jx & 1 \\ 0 & \dots & 0 & 1 & -1-jx \end{pmatrix} \qquad (10)$$

where $x = \omega/(\Delta\omega_{lock}\cos\Delta\hat{\theta})$. From the relation $\overline{\overline{P}}\,\overline{\overline{N}} = \overline{\overline{I}}$, we can write $\sum_{j=1}^{N} p_{ij} n_{jk} = \delta_{ik}$ and $\sum_{k=1}^{N}\sum_{j=1}^{N} p_{ij} n_{jk} = \sum_{j=1}^{N} p_{ij} (\sum_{k=1}^{N} n_{jk}) = 1$. By inspecting (10) we can easily see that the sum of the jth column, is simply $-j\omega/\omega_{3dB}$ for all j. Therefore $\sum_{j=1}^{N} p_{ij} = \frac{-1}{j\omega/\omega_{3dB}}$, and the total output phase noise is

$$|\widetilde{\delta\theta}_{total}|^2 = \frac{1}{N} |\widetilde{\delta\theta}_i|^2_{uncoupled} \qquad (11)$$

The total phase noise is reduced by a factor of $1/N$, independent of the phase difference $\Delta\hat{\theta}$. Recently we have shown that the total phase noise reduction factor $1/N$ is true for any N reciprocally coupled oscillator array [1], and the nearest-neighbor coupled oscillator chain is just a special case of the reciprocal coupled oscillator array.

III. EXPERIMENTAL RESULTS FOR 5-ELEMENT ARRAY

A five-element scanning coupled oscillator array was built for experimental verification of the theory (figure 1).The oscillators use NE32184A packaged MESFETs and MA-COM 46600 varactor diodes. These 8.5 GHz VCOs are coupled by one wavelength microstrip transmission lines ($\Phi = 0$) and are resistively loaded with two 75Ω chip resistors. The maximum end-element detuning for the locked array is $\pm$125MHz. Each oscillator can either deliver the power to patch antenna or SMA connectors in order to measure the total and individual PM noise. Because of

VCO's poor phase noise behavior and comparatively large thermal drift, the frequency discriminator method was used for phase noise measurement. The experimental apparatus can be found in [1]. For total phase noise, the output signal was measured with a detector in the far field. For the individual oscillator PM noise, the oscillator was connected directly to the measurement system using SMA cables. Figure 2 shows the PM noise of each element when free-running, and the total array output under synchronized conditions. The total PM noise is clearly reduced as compared to those of free-running oscillators.

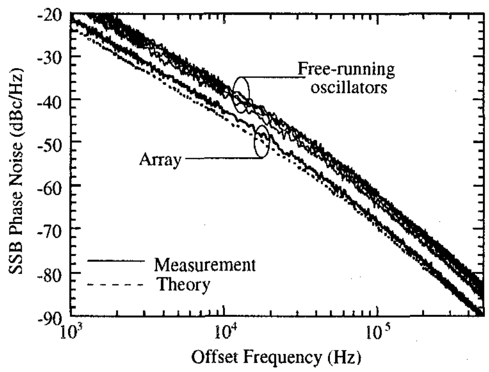

Figure 2–Comparison of free-running PM noise for each of the five oscillators in the experimental array with the total PM noise measured in the far-field. The theoretical noise reduction is shown for comparison, which is the average free-running noise divided by 5.

IV. PHASE NOISE OF ARRAY WITH EXTERNAL LOCKING SIGNAL

For phase noise analysis of the array locked to an external source, we can modify (1) and (2) to get

$$(\overline{\overline{N}} + \overline{\overline{N'}})\overline{\delta\theta} = \overline{\widetilde{B_n}} + \overline{\widetilde{B'_n}} \tag{12}$$

where

$$\overline{\overline{N'}} = \begin{pmatrix} -\epsilon'_1 \cos\psi_1 & & & & 0 \\ & \ddots & & & \\ & & -\epsilon'_k \cos\psi_k & & \\ & & & \ddots & \\ 0 & & & & -\epsilon'_N \cos\psi_N \end{pmatrix} \tag{13}$$

$$\overline{\widetilde{B'_n}} = \begin{pmatrix} -\epsilon'_1 \cos\psi_1 \widetilde{\delta\theta}_{inj} \\ \vdots \\ -\epsilon'_k \cos\psi_k \widetilde{\delta\theta}_{inj} \\ \vdots \\ -\epsilon'_N \cos\psi_N \widetilde{\delta\theta}_{inj} \end{pmatrix} \tag{14}$$

ϵ'_i is magnitude of coupling coefficient between the ith VCO and injection source and $\psi_i = \hat{\theta}_i - \hat{\theta}_{inj}$. Here we only focus on the array with one element locked to the external source and $\psi_i = 0$ [9]. Following the same previous procedures, as noise offset frequency is near the carrier, we get

$$|\widetilde{\delta\theta}_{total}|^2 \rightarrow |\widetilde{\delta\theta}_{inj}|^2 \tag{15}$$

When offset frequency is far from the carrier,

$$|\widetilde{\delta\theta}_{total}|^2 \rightarrow \frac{1}{N}|\widetilde{\delta\theta}_i|^2_{uncoupled} \tag{16}$$

The injection source can only reduce the array total noise near the carrier. As the noise offset frequency is far from the carrier, the total phase noise returns to the value without the external injection source. If the array is injection-locked at one of the end element, the array phase noise become noisier as the number of the coupled oscillators increases within some offset frequency range. When the external injection power increases, ϵ'_i also increases and the total array noise has larger offset frequency range with almost same noise property as the injection source. Figure 3 shows the simulation result of the array noise with end element injection locked to the low phase noise source. The upper dashed lines show the array noise without injection-locked for different coupled oscillator numbers. If the number of coupled oscillators increases, the total phase noise also increases with the offset frequency within certain range.

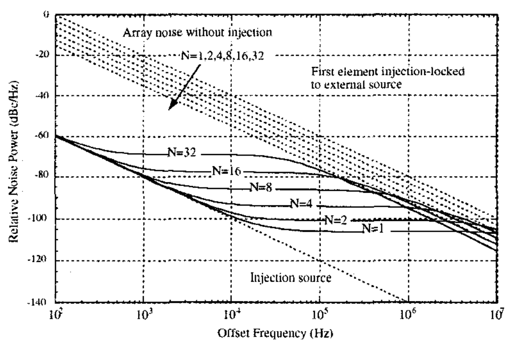

Figure 3–The total phase noise of the array with end element injection-locked by the external source. The dashed lines represent the array noise without injection and the injection source. The solid lines are the total PM noise of the array injection-locked. As the number of oscillators increases, the array phase noise increases for certain offset frequency range.

V. EXPERIMENTAL RESULTS WITH EXTERNA LOCKING SIGNAL

We also use the same scanning coupled oscillator array with an extra injection port at the center element for the experiment. The array is synchronized at the same frequency as the injection source HP8350B. The array shows the noise reduction near the carrier with almost same noise property as the injection source. As the offset frequency far from the carrier, the total phase noise of the array returns to the value not affected by the injection source. This agrees with our theory.

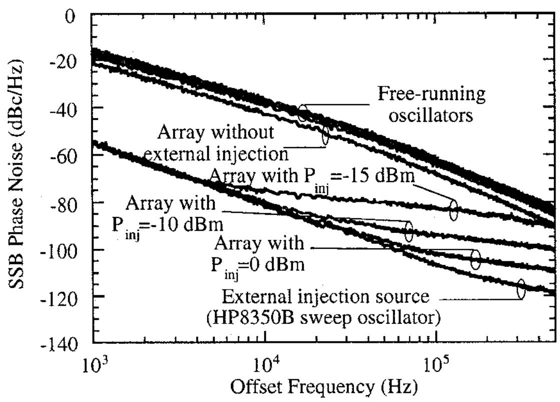

Figure 4–Comparison of the array PM noise with different external injection power at the same synchronized frequency. When the injection power is increased, the array noise has larger offset frequency range with almost same noise property as the injection source.

VI. CONCLUSIONS

The phase noise near the carrier in scanning coupled oscillator array has been analyzed, and noise reduction proportional to the number of oscillators is found. Recently we have proved that the total phase noise reduction of $1/N$ for the oscillators coupled through the reciprocal coupling network, and N nearest-neighbor coupled chain is a special case of it. We also analyze the phase noise of the array with elements locked to the external injection source. Measurements for a small MESFET oscillator array at X-band confirm the noise reduction. This work has neglected the AM noise influence, which affects PM noise in the case of nonzero phase progression. The analysis also neglects possible correlations between the oscillator noise sources, and influence of non-uniform amplitude and phase distributions, but these are minor effects.

VII. ACKNOWLEDGMENTS

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VIII. REFERENCES

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