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A Simplified Theory of Coupled Oscillator Array Phase Control

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Linear and planar arrays of coupled oscillators have been proposed as means of achieving high power rf sources through coherent spatial power combining.[1][2] In such applications, a uniform phase distribution over the aperture is desired. However, it has been shown that by detuning some of the oscillators away from the oscillation frequency of the ensemble of oscillators, one may achieve other useful aperture phase distributions.[3] Notable among these are linear phase distributions resulting in steering of the output rf beam away from the broadside direction. The theory describing the operation of such arrays of coupled oscillators is quite complicated since the phenomena involved are inherently nonlinear. This has made it difficult to develop an intuitive understanding of the impact of oscillator tuning on phase control and has thus impeded practical application. In this work a simplified theory is developed which facilitates intuitive understanding by establishing an analog of the phase control problem in terms of electrostatics.

We begin by reviewing the nonlinear equations describing the behavior of an array of loosely coupled oscillators.[2] The behavior of the phase of a single oscillator injection locked to an input signal,

$$V_{inj} = A_{inj} e^{j(\omega_{inj} t + \psi_{inj})} = A_{inj} e^{j\theta_{inj}}$$

can be described by the following differential equation.

$$\frac{\partial \theta}{\partial t} = \omega_o + \Delta\omega_{lock} \sin(\theta_{inj} - \theta)$$

where $\theta = \omega t + \phi$, ϕ is the phase of the oscillator oscillating at frequency, ω, and

$$\Delta\omega_{lock} = \frac{\omega_o}{2Q}\frac{A_{inj}}{A}$$

the locking bandwidth which is inversely proportional to the Q of the oscillator and A, the amplitude of the oscillation. Now, for an array of N coupled oscillators, the injection signals are just the outputs of the other oscillators and the phase of the i^{th} oscillator is described by a differential equation of the form,

$$\frac{\partial\theta_i}{\partial t} = \omega_o - \frac{\omega_i}{2Q}\sum_{j=1}^{N}\varepsilon_{ij}\frac{A_j}{A_i}\sin(\Phi_{ij} + \theta_i - \theta_j)$$

and $\varepsilon_{ij}e^{j\Phi_{ij}}$ is the coupling between oscillators i and j. Limiting the coupling to nearest neighbors and taking the continuum limit as the number of oscillators increases to infinity and the spacing decreases to zero (i becomes a continuous variable, x), results in,

$$\frac{\partial\phi}{\partial t} = \omega(x) - <\omega> + \Delta\omega_m \frac{\partial}{\partial x}\sin(\frac{\partial\phi}{\partial x})$$

where $\Delta\omega_m$ is the mutual locking bandwidth of the coupled oscillators and $<\omega>$ is the average of the oscillator tuning frequencies, $\omega(x)$. In steady state with small phase differences between neighboring oscillators, one has,

$$\frac{\partial^2\phi}{\partial x^2} = \frac{<\omega> - \omega(x)}{\Delta\omega_m} = \rho(x)$$

which is Poisson's equation of electrostatics! Similarly for a two dimensional array one obtains a two dimensional Poisson equation. From this point, all of the familiar results of electrostatics apply if one merely identifies the oscillator tuning with charge density and the phase distribution with electrostatic potential.

For example, suppose that we detune the oscillators at each end of a linear array in opposite direction with respect to the average tuning frequency with the intention of steering the beam as described by Liao and York.[3] This can be represented as two delta function charge densities of opposite sign one at each end of the aperture. The solution for the phase distribution is merely a linear function as shown in Figure 1. yielding the desired steering of the beam. This linear solution may, of course, be recognized as the potential in a parallel plate capacitor. For comparison, the dots in Figure 1 represent the solution of the full nonlinear equations with no approximation.

Note that if the two delta functions have the same sign, the average of the tuning frequencies is changed resulting in a constant charge distribution in addition to the deltas. This constant term yields a quadratic solution for the phase distribution as shown in Figure 2. Of course, various ratios of delta function amplitudes yield corresponding combinations of linear and quadratic solutions such as the one indicated in Figure 3. Similar results obtain for two dimensional arrays wherein, for example, various detunings of the oscillators on the perimeter of the array yield phase distributions which are solutions of the two dimensional Poisson equation with delta functions and constants as sources. Such a phase distribution is illustrated in Figure 4. This resulted from detuning of all the perimeter oscillators by the same amount.

Finally, it is noted that this simplified theory makes clear the fact that any desired slowly varying phase distribution can be realized if one is willing to detune all of the oscillators. The appropriate tuning can be ascertained by substituting the desired phase distribution into Poisson's equation and determining the resulting charge distribution.

References

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3. P. Liao and R. A. York, "A New Phase-Shifterless Beam-Scanning Technique Using Arrays of Coupled Oscillators," IEEE Trans. . Microwave Theory Tech., vol. MTT-41, pp.1810-1815, Oct. 1993.

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Figure 1. Equal and opposite detuning of the end oscillators.

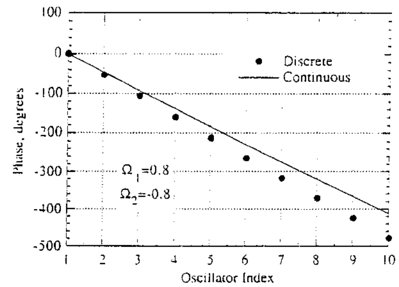

Figure 2. Equal detuning of the end oscillators.

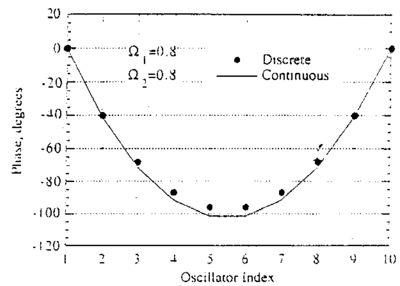

Figure 3. Unequal detuning of end oscillators.

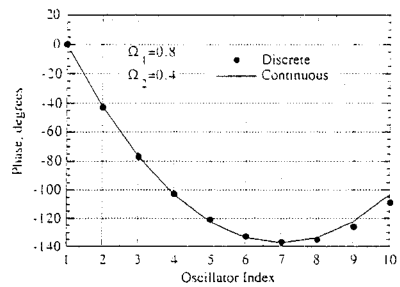

Figure 4. Equal detuning of perimeter oscillators.