Inductor Design –
with Continuous/DC Current

The circuit design process will/should specify:

- \( L \): required inductance at full load
- \( I_{dc} \): average DC current
- \( \Delta I \): peak current variation at full load
- \( P_{loss} \): maximum allowed loss

For the continuous current case the AC current swing is small, so the AC flux density is also small. Consequently, most of the loss is DC copper winding loss unless the frequency is very high.

Winding loss is related to the wire gauge and length (number of turns). Both of these parameters are constrained by the core geometry and thermal considerations. In the absence of a detailed thermal analysis, it is common to specify a certain maximum rms current density,

\[
\frac{I_{rms}}{A_{bw}} \leq J_{max} \quad \Leftrightarrow \quad d_{bw} \geq \sqrt{\frac{4I_{rms}}{\pi J_{max}}} \quad J_{max} \approx 400 - 600 \text{ A/cm}^2 \text{ (rms)}
\]

It is important to emphasize that this is just a rule-of-thumb expedient. The actual maximum allowed rms current density depends on the particular thermal environment. We will consider a better treatment of thermal issues later.

The next step is to choose the core material and geometry, and the correct number of turns in the winding.

The core selection must satisfy several constraints:

- Must achieve the correct inductance
- Keep the loss below a specified level
- Keep the inductor size as small as possible without saturating the core

Most manufacturers now provide some software tools to help select the correct core for a given application and circuit conditions.

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Typical Core Geometries

Some Basic Core shapes

“C” or “U” core

E-core

“Pot” core

Toroid

There are many variants on the above, here are a few examples

“EP”

“ETD”

“RM”

“PQ”

“EQ”
### FERRITE CORE COMPARATIVE GEOMETRY CONSIDERATIONS

<table>
<thead>
<tr>
<th></th>
<th>Pot Core</th>
<th>Double Slab, RM Cores</th>
<th>E Core</th>
<th>EC, ETD, EER Cores</th>
<th>PQ Core</th>
<th>EP Core</th>
<th>Toroid</th>
</tr>
</thead>
<tbody>
<tr>
<td>See Catalog Section</td>
<td>6</td>
<td>7-8</td>
<td>11</td>
<td>12</td>
<td>10</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>Core Cost</td>
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<td>medium</td>
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<td>medium</td>
<td>very low</td>
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<td>low</td>
<td>medium</td>
<td>high</td>
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<td>high</td>
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<td>excellent</td>
<td>excellent</td>
<td>good</td>
<td>good</td>
<td>fair</td>
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<td>medium</td>
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<td>poor</td>
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<td>fair</td>
<td>excellent</td>
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</tr>
</tbody>
</table>

** Hardware is required for clamping core halves together and mounting assembled core on a circuit board or chassis.
Single-Layer Toroid

The inductance is given by:

\[ L = \mu_0 N^2 \frac{A_e}{1_e} = N^2 A_L \]

To keep the flux density below some specified maximum, the peak current must satisfy:

\[ I_{\text{max}} \leq \frac{NB_{\text{max}} A_e}{L} \]

The number of turns that will fit on the toroid is constrained by the inner diameter and wire size:

\[ 2(a - d_w) \sin \frac{\theta}{2N} > d_w \quad \theta = 2\pi = \frac{2N}{N} \quad Nd_w \leq 2\pi a \]

The copper loss is given by:

\[ P_{Cu} = \rho_{Cu} \frac{N(MLT)}{A_{bw}} I_{\text{rms}}^2 \]

(Note: DC resistance is justified in the continuous current case where the AC current variation is small)

The core loss can be estimated from the Steinmetz equation:

\[ P_{\text{core}} = k_{\text{core}} f^n \Delta B^m V_e \quad \Delta B \approx \Delta I \frac{B_{\text{max}}}{I_{\text{max}}} \]

\[ \theta = 2\pi - \phi: \quad \text{winding angle} \]

\[ d_{bw}: \quad \text{bare wire diameter} \]

\[ A_{bw}: \quad \text{bare wire area} \]

\[ d_w: \quad \text{diameter with insulation} \]

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Note that the first two constraints on the previous page scale in opposite ways with the number of turns

\[ A_e \geq \frac{LI_{\text{max}}}{NB_{\text{max}}} \quad \text{I.D.} \geq \frac{Nd_w}{\pi} \]

The product of these two numbers gives

\[ A_e \times \text{I.D.} \geq \frac{LI_{\text{max}}d_w}{\pi B_{\text{max}}} \]

This gives simple guidance for choosing the correct core dimensions: we must use a toroid with an area-diameter product that satisfies the above constraint.

Once the core is selected, all the geometrical parameters will be known, so we just need to calculate the number of turns:

\[ N = \sqrt{\frac{L}{A_L}} \]

Often the manufacturers will provide several choices for \( A_L \) for a given core dimension. The remaining constraint on copper loss can help us choose:

\[ A_L > \frac{\rho_{\text{Cu}}(MLT)I_{\text{rms}}^2}{P_{\text{Cu}}A_{bw}L} \]

This is a simple method for core selection that closely parallels the area-product method to be developed later
Single-Layer Toroid: A Closer Look at Core Selection

There are often many possible core sizes and materials that can be selected for a given design. Is there a “best” design?

Note that toroid dimensions are not entirely independent: the inner diameter must scale with the outer diameter, and the height is also constrained. Let’s “parameterize” the core dimensions as follows:

\[ \xi \equiv \frac{\text{I.D.}}{\text{O.D.}} \quad \gamma \equiv \frac{h}{w} \quad \xi \approx 0.5 - 0.6 \quad \gamma \approx 1 - 2 \]

Catalog cores (chart at right):

With these definitions the various geometrical parameters can all be expressed in terms of the outer radius \( b \):

\[
\begin{align*}
    a &= \xi b \\
    h &= \gamma(1 - \xi) b \\
    A_e &= \gamma(1 - \xi)^2 b^2 \\
    l_e &\approx 2\pi \sqrt{ab} = 2\pi \sqrt{\xi} b \\
    \text{MLT} &\approx 2(\gamma + 1)(1 - \xi) b
\end{align*}
\]

Eliminating the number of turns from the equations on the previous page gives these three constraints that relate permeability to core size:

\[
\mu_e \leq \left( \frac{B_{\text{max}}}{I_{\text{max}}} \right)^2 k_1 k_2^2 b^3 \\
\mu_e \geq k_2 k_3^2 L \left( \frac{\rho_{\text{Cu}} I_{\text{rms}}^2}{A_{bw} P_{\text{Cu}}} \right)^2 b \\
\mu_e \geq k_2 \frac{L}{k_1} \left( \frac{d_w}{\theta \xi} \right)^2 \frac{1}{b^3}
\]

To limit the peak flux density
To keep the Ohmic loss below \( P_{\text{Cu}} \)
To insure that the required number of turns will fit the core size
Example Calculations

The three constraints are plotted at right for a particular design example:

Circuit Spec:  
\[ L = 500 \, \mu H \]  
\[ I_{dc} = 2 \, A \]  
\[ \Delta I = 0.4 \, A \]

Assumptions:  
\[ \bar{\xi} = 0.5 \]  
\[ \gamma = 2 \]  
\[ B_{\text{max}} = 0.3 \, T \]  
\[ J_{\text{max}} = 400 \, A/cm^2 \]

The shaded region is the range of allowed permeability and core sizes that will satisfy the constraints on number of turns and \( B_{\text{max}} \).

The minimum possible core size and associated permeability and copper loss can be found as:

\[
b_{\text{min}} = \left( \frac{L I_{\text{max}}}{k_1 B_{\text{max}}} \frac{d_w}{2\pi \bar{\xi}} \right)^{1/3} \]
\[
\mu_{e,\text{crit}} = k_2 \frac{B_{\text{max}}}{I_p} \frac{d_w}{2\pi \bar{\xi}} \]
\[
P_{\text{crit}} = k_3 \left( \frac{L I_{\text{max}}}{k_1 B_{\text{max}}} \right)^{2/3} \left( \frac{2\pi \bar{\xi}}{d_w} \right)^{1/3} \left( \frac{\rho}{A_{bw}} \right) \]

This approach gives a quick estimate of the core outer diameter that is required (2 \( b_{\text{min}} \)). This is useful because most manufacturers list or label toroids in terms of the O.D.

Key tradeoff: reducing ohmic loss requires a larger core size and higher permeability.
Single-Layer Toroid: Optimization

The parameterization allows us to explore the possibility of “optimal” toroid geometries:

\[ b_{\text{min}} \propto \left[ \gamma (1 - \xi)^2 \xi \right]^{-1/3} \quad \Rightarrow \quad \text{Core has minimum outer radius when:} \quad \xi = \frac{1}{3} \quad \gamma = \text{large as possible} \]

\[ P_{\text{Cu}} \propto 2 \left( \frac{\gamma + 1}{\gamma^{2/3}} \right) \left( \frac{\xi}{1 - \xi} \right)^{1/3} \quad \Rightarrow \quad \text{Winding loss minimized when:} \quad \xi = \text{small as possible} \quad \gamma = 2 \]

This suggest a possible design goal of: \[ \xi = \frac{1}{3} \quad \gamma = 2 \]

But these are not the only possible considerations. For example the designer might want to minimize the footprint on a PC board. For a vertical toroid the device consumes an area of about 2hb, and this is minimized when

\[ A_{\text{PCB}} \propto 2hb = 2 \left( \frac{\gamma}{\xi^2 (1 - \xi)} \right)^{1/3} \quad \Rightarrow \quad \xi = \frac{2}{3} \quad \gamma = \text{small as possible} \]

Or we could look for a compromise by minimizing the area-loss product:

\[ A_{\text{PCB}} P_{\text{Cu}} \propto \left( \frac{\gamma + 1}{\gamma^{1/3}} \right) \left( \frac{1}{(1 - \xi)^2 \xi} \right)^{1/3} \quad \Rightarrow \quad \xi = \frac{1}{3} \quad \gamma = \frac{1}{2} \]

Cost is another important factor. Assuming cost is proportional to the volume of ferrite material in the core, the cost-loss product varies as:

\[ \text{Cost} \propto V_{\text{core}} = \pi h \left( b^2 - a^2 \right) \quad \text{Cost} \times P_{\text{Cu}} \propto 2 \left( \frac{\gamma + 1}{\gamma^{2/3}} \right) \left( \frac{1 + \xi}{\xi} \right) \left( \frac{\xi}{1 - \xi} \right)^{1/3} \quad \Rightarrow \quad \xi = 0.5 \quad \gamma = 2 \]

Catalog data shows that available core geometries are mostly clustered near this latter design goal, but there often other cores available for which the above considerations can help make a judicious choice.
Permability and Material Selection

Once a specific core geometry is chosen the design equations should be recalculate as needed, and this will give a range of permeabilities required for the design.

Once the desired permeability is known, the core material can be chosen. Powder cores are available in many different mixtures. Alternative a gapped ferrite core could be used.

Shown below are representative curves for available mixtures of Ferroxcube MPP material.

Note that the effective permeability falls off at high currents. The design calls for a certain permeability at full-load conditions. This means that the chosen material should have a low-field permeability that is significantly higher, to insure the correct inductance at full-load. It is common to choose the material such that the inductance has fallen to no less than 60-70% of its initial low-field value at full-load conditions.
Core Loss

As mentioned, the core loss is typically small for inductors with small AC current variations and low switching speeds, but this should be checked at the end of the design.

Coefficients in the Steinmetz equation can be estimated from manufacturers data. Note that high-mu materials are typically more lossy than low-mu materials.

Losses are related to frequency and the peak AC field, and can be read directly from these charts, where the peak AC field is

\[ \hat{B} = \frac{\Delta B}{2} \approx \Delta I \frac{B_{\text{max}}}{I_p} \]

Note that these charts assume sinusoidal variations. Typical switching waveforms are nonsinusoidal, so a more precise analysis should examine the influence of harmonics.